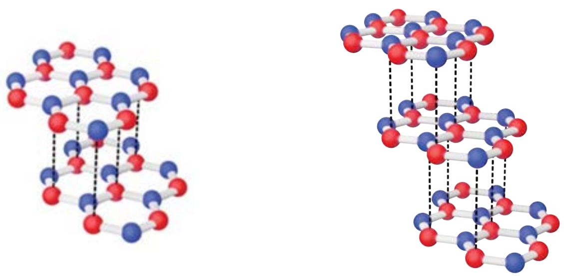
Exotic superconductivity in graphene multilayers

Erez Berg



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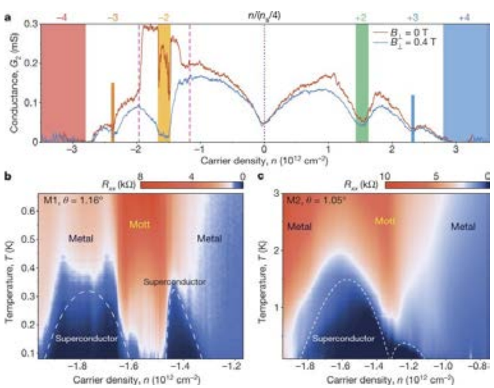
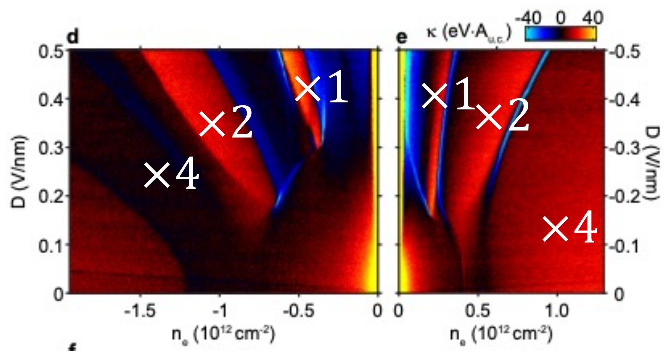
Novel correlated 2DEGs

- Twisted bilayer, n-layer graphene



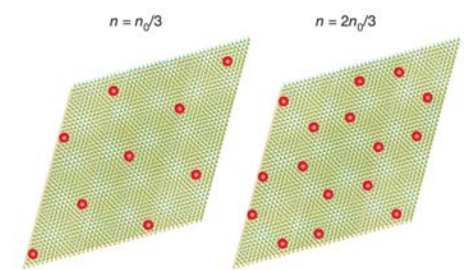
[Cao,...,Jarillo-Herrero, Nature (2018)]

- Rhombohedral multi-layer graphene



[Zhao,...,EB,...Young, Nature (2021)]

- Multi-layer TMD



[Regan,...,Wang, Nature (2020)]

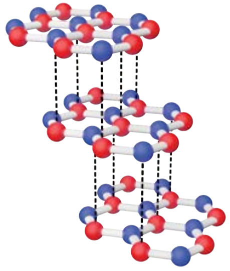
[Xu,...,Shan, Nature (2020)]

New features: Multi-valley, anisotropic dispersion, moiré lattice, Berry curvature,...

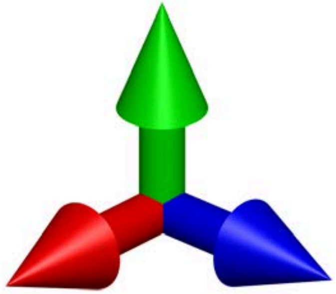
Rich phase structure: Spin/orbital ferromagnets, Mott insulators, superconductors, integer/fractional Chern insulators,...

Outline

- Superconductivity in rhombohedral bilayer and trilayer graphene
- Puzzles
- Electronic mechanism?

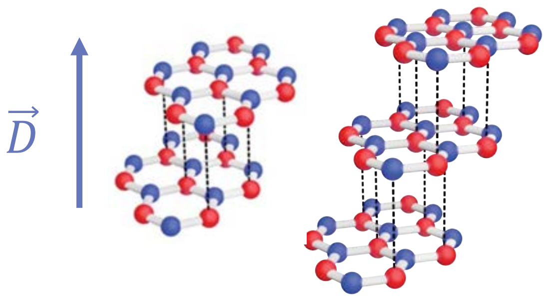


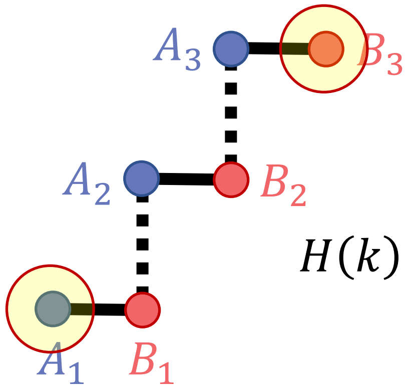
- Spin-polarized triplet superconductors: Order parameter topology and current dissipation

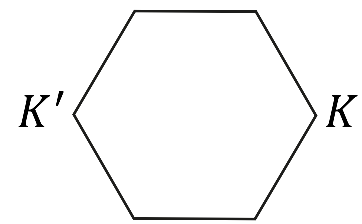


- Linear spectroscopy of collective modes

Rhombohedral AB and ABC trilayer graphene

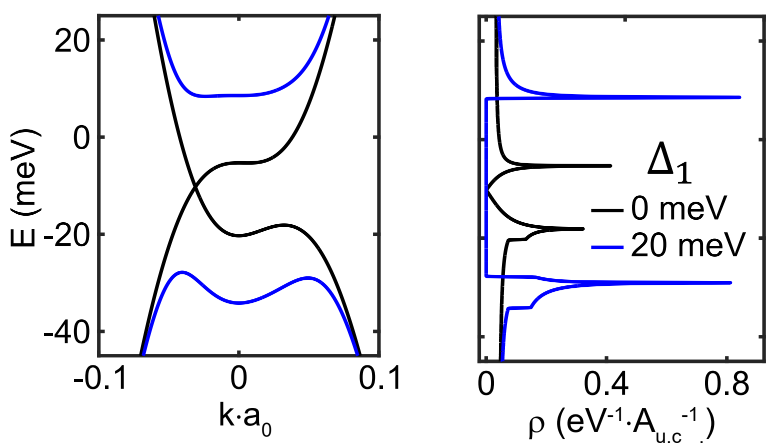


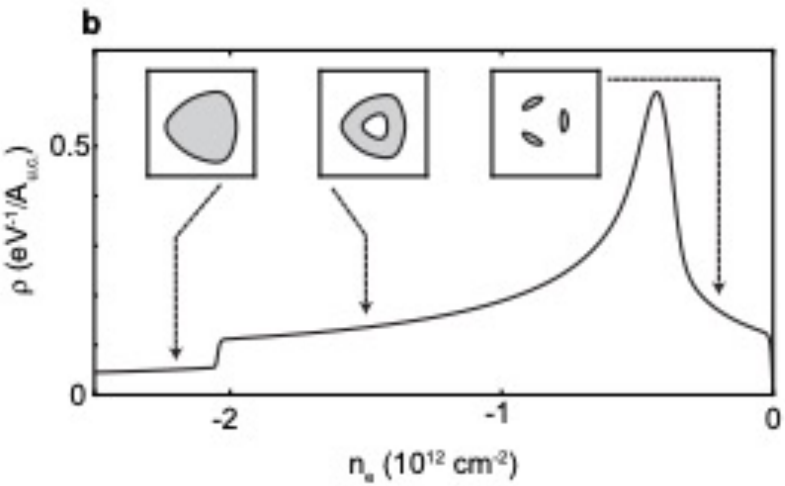




$$H(k) = \begin{pmatrix} \Delta_1 & \beta\left(k_x + \tau i k_y\right)^3 + \gamma \\ \beta\left(k_x - \tau i k_y\right)^3 + \gamma & -\Delta_1 \end{pmatrix}$$

$\Delta_1 = eDd_0$, $\tau = \pm 1$: valley index

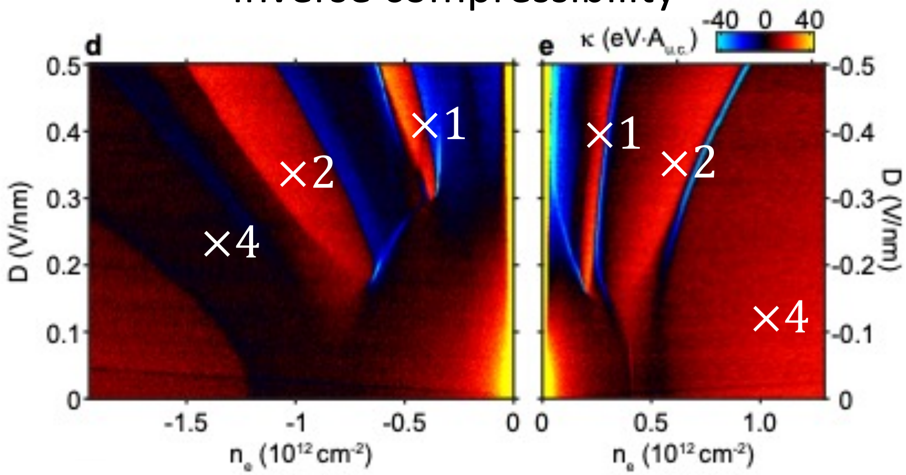




Phase diagram

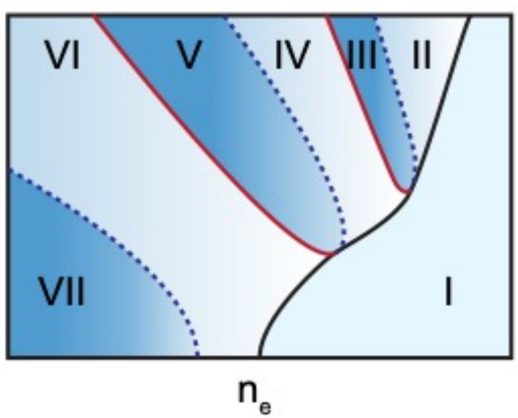
H. Zhou, …,A. Ghazaryan T. Holder, EB, M. Serbyn, A. Young (2021)

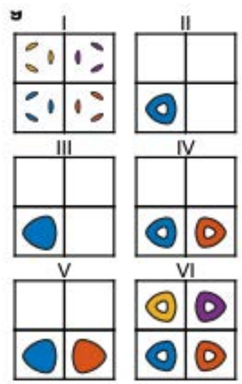
Inverse compressibility

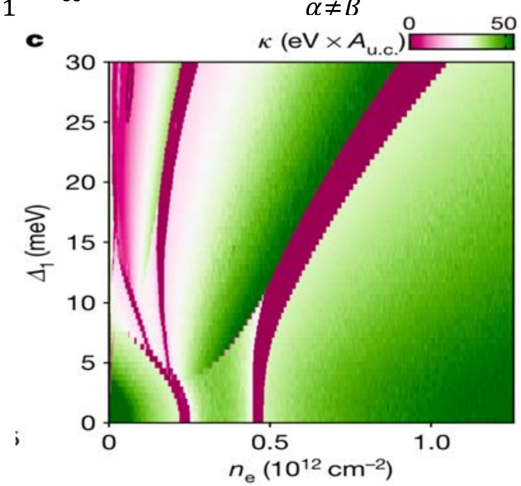


Fermi surface evolution
(SdH Oscillations)

$$E = \sum_{\alpha=1}^{4} \int_{-\infty}^{\mu_\alpha} d\varepsilon\, \varepsilon\, \rho(\varepsilon) + \frac{1}{2} \sum_{\alpha \neq \beta} n_\alpha n_\beta - J \vec{S}_K \cdot \vec{S}_{K'}$$

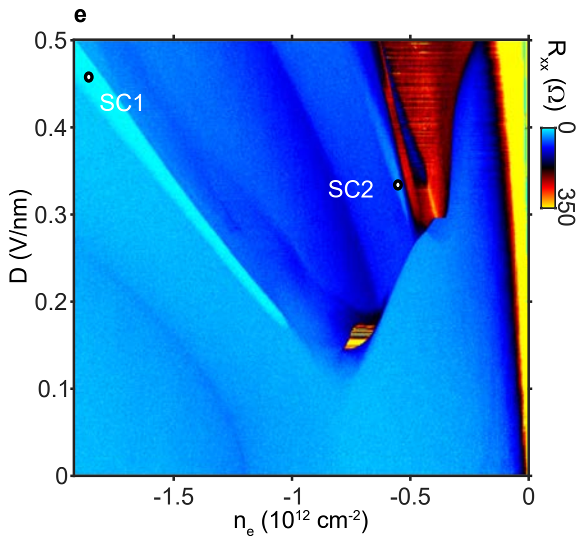


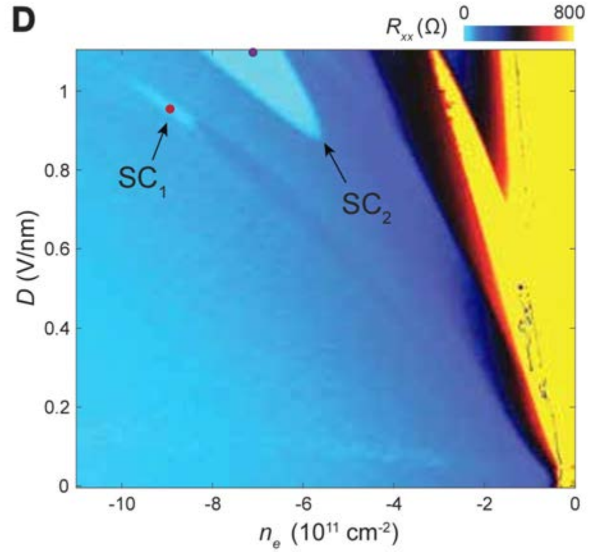




Similar phenomena in MATBG: Zondiner et al., Wong et al. (2020)

Superconductivity!





ABC graphene on HBN

Zhou, …, Young (Nature, 2021)

AB graphene on WSe_2

Zhang, …, Nadj-Perge (Nature, 2023); Holleis, …, Nadj-Perge, Young (2023)

Also: AB graphene on HBN with in-plane field

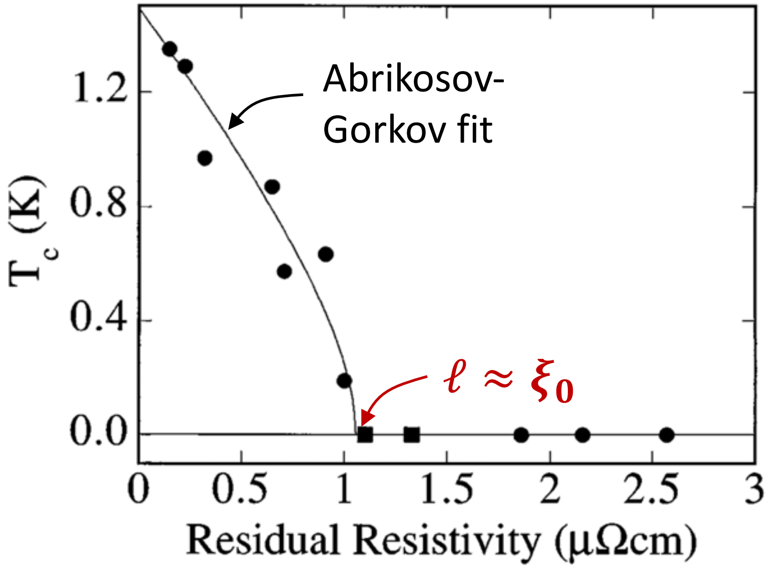
Zhou, …, Young (Science, 2022)

- $T_c \sim 40 - 400\text{mK}$
- **BN samples: both singlet and triplet SC observed! (Pauli limit violation)**
- **SC often near a phase transition to symmetry broken phase**

- **AB and ABC graphene SC:** $\ell/\xi \gtrsim 10$ **(Clean limit)**

⇒Unconventional pairing not ruled out (yet…)

c.f. Sr_2RuO_4 with controlled amounts of non-magnetic disorder



Mackenzie, …, Lonzarich, Maeno (PRL, 1998)

Puzzles

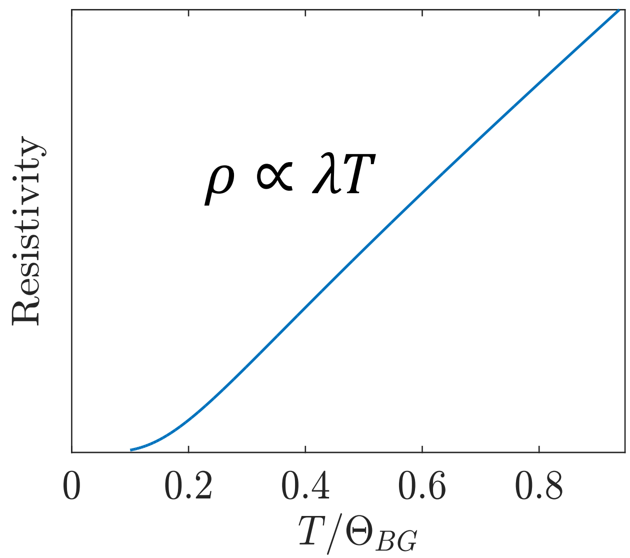
Conventional (acoustic phonon-mediated) s-wave?

Chou, Wu, Sau, Das Sarma (PRL, 2021)

$$\Theta_{BG} = 2v_s k_F$$

$T \gtrsim \Theta_{BG}/4$:

$$\rho = \frac{h}{e^2} \frac{1}{2\sum_i \varepsilon_{F,i}} 2\pi\lambda T$$



Experimentally: $T > 20\text{K}: \frac{\rho}{T} \approx 1 - 2\ \frac{\Omega}{K}$

Recall: $T_c \sim \Theta e^{-1/\lambda_{SC}}$

$$\Longrightarrow \lambda \approx \frac{1}{150} - \frac{1}{300}$$

Usually $\lambda_{SC} \approx \lambda$

Puzzles (2)

Singlet or triplet?

$$H_J = -J\, \vec{S}_K \cdot \vec{S}_{K'}$$

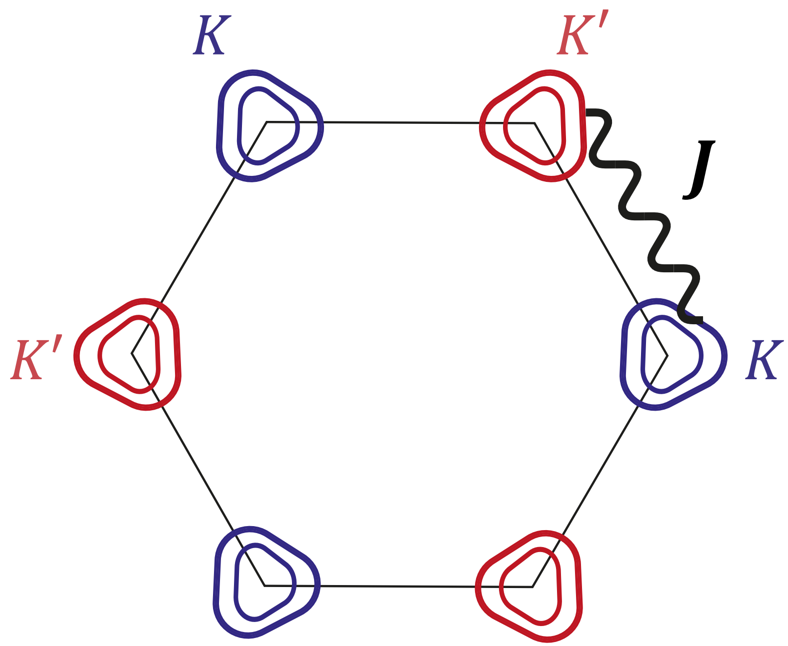
E

$E_c = \dfrac{e^2\sqrt{n}}{\epsilon} \sim 20 - 30\text{meV}$

$U_c(1)\times SU(2)\times SU(2)$

$J \sim 1\text{meV}$

$U_c(1)\times U_v(1)\times SU(2)$

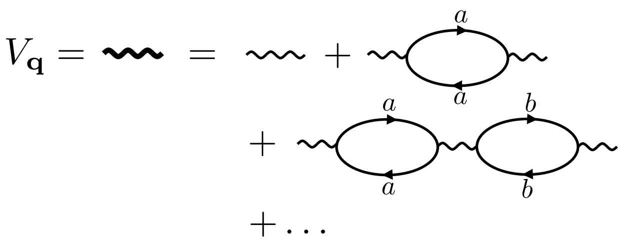


SC2: normal state is spin-polarized ⇒ $J > 0$, **triplet SC**

SC1: normal state is spin-unpolarized ⇒ **singlet SC??**

Electronic mechanisms

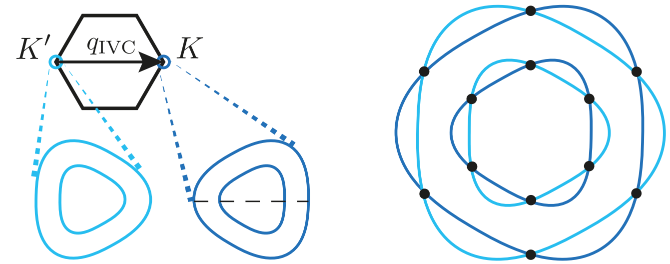
Charge fluctuations ("Kohn-Luttinger")

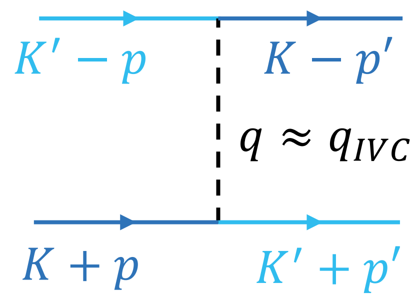


$$= \frac{V_{0,\boldsymbol{q}}}{1 + N\,\Pi_{0,\boldsymbol{q}}\,V_{0,\boldsymbol{q}}}$$

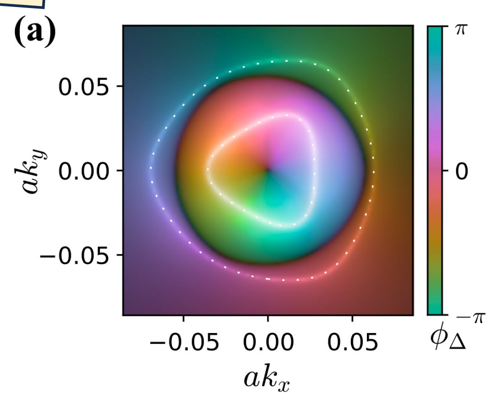
A. Ghazaryan, T. Holder, M. Serbyn, EB, PRL (2021); PRB (2023)

IVC fluctuations





S. Chatterjee, T. Wang, EB, M. Zaletel, Nature Comm. (2022)



Chiral $p_x + ip_y$ order parameter!

- e-e scattering conserves momentum: $\rho(T > T_c) \approx const.$
- Coulomb interactions favor spin singlet for SC1

Kohn-Luttinger mechanism

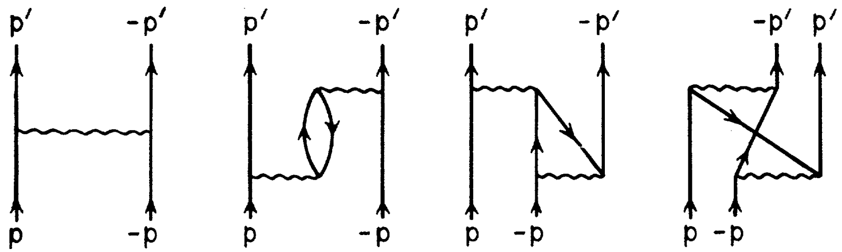
NEW MECHANISM FOR SUPERCONDUCTIVITY*

W. Kohn

University of California, San Diego, La Jolla, California

and

J. M. Luttinger (1965)

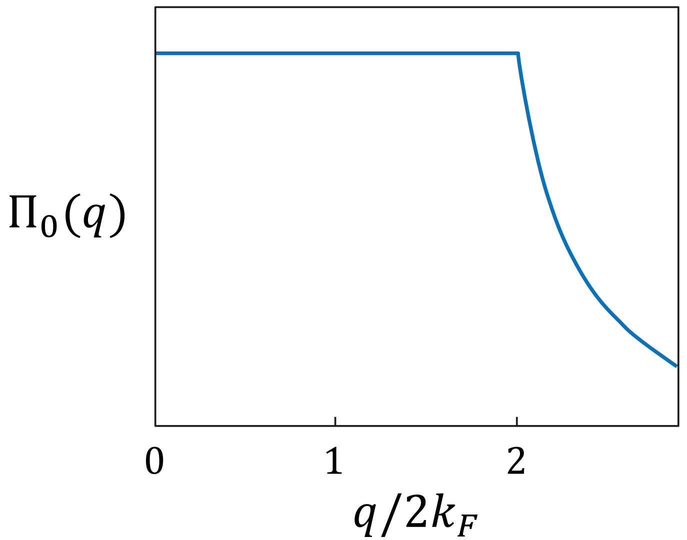


2D, parabolic dispersion:

$$\Pi_0(q < 2k_F) = const$$

No superconductivity to second order

A. Chubukov (1992)



Non-parabolic dispersion/multiple sub-bands:

Unconventional superconductivity!

E.g.: Raghu, Kivelson, Scalapino (2011); Raghu, Kivelson (2015); Chubukov, Kivelson (2017)

Kohn-Luttinger mechanism

NEW MECHANISM FOR SUPERCONDUCTIVITY*

W. Kohn

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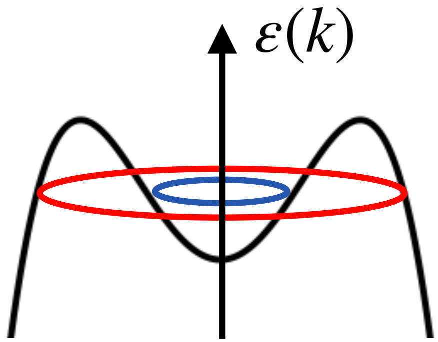
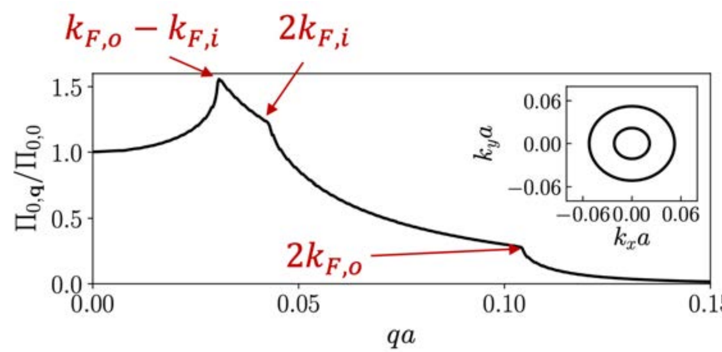
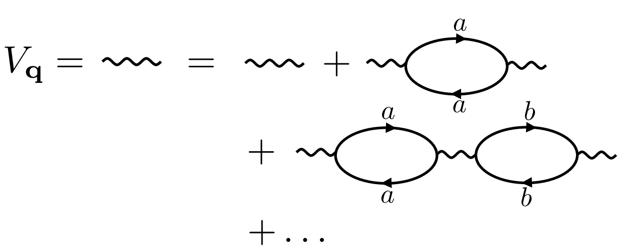
J. M. Luttinger (1965)

Annular 2D
Fermi surface

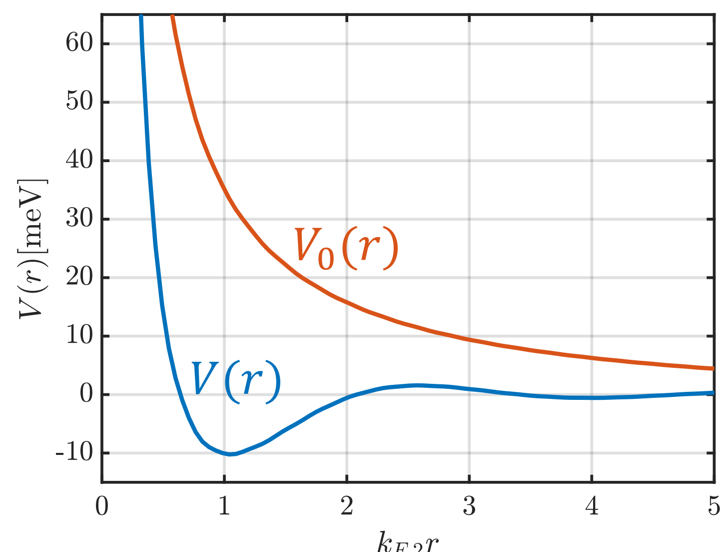




Screened Coulomb interaction (RPA):

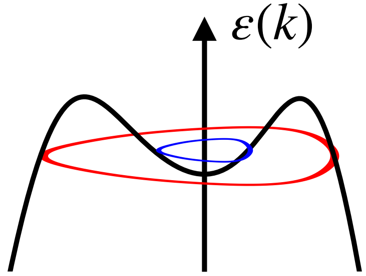


$$= \frac{V_{0,\boldsymbol{q}}}{1 + N\,\Pi_{0,\boldsymbol{q}}\,V_{0,\boldsymbol{q}}}$$



Model

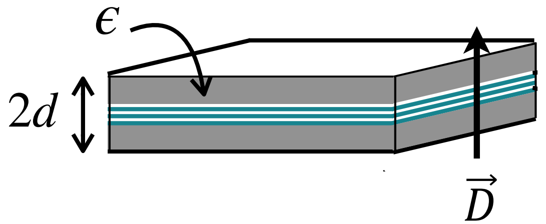
In both SC1,2: annular FS



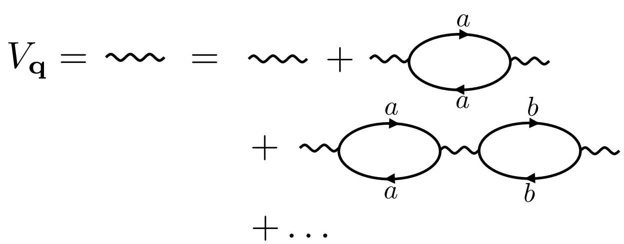
$$H = H_0 + H_C$$

$$H_0 = \sum_{\boldsymbol{k},\alpha=1,\ldots,4} \varepsilon_{\boldsymbol{k}} \psi^{\dagger}_{\alpha\boldsymbol{k}} \psi_{\alpha\boldsymbol{k}}$$

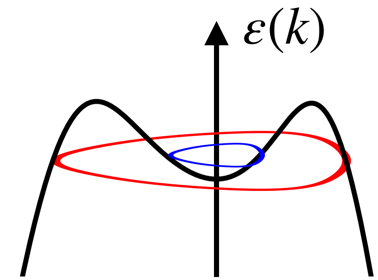
$$H_C = \frac{1}{2L^2} \sum_{\boldsymbol{q}} V_{0,\boldsymbol{q}}\, \rho_{\boldsymbol{q}} \rho_{-\boldsymbol{q}} \qquad V_{0,\boldsymbol{q}} = \frac{2\pi e^2}{\epsilon q} \tanh(qd)$$



Charge Fluctuations

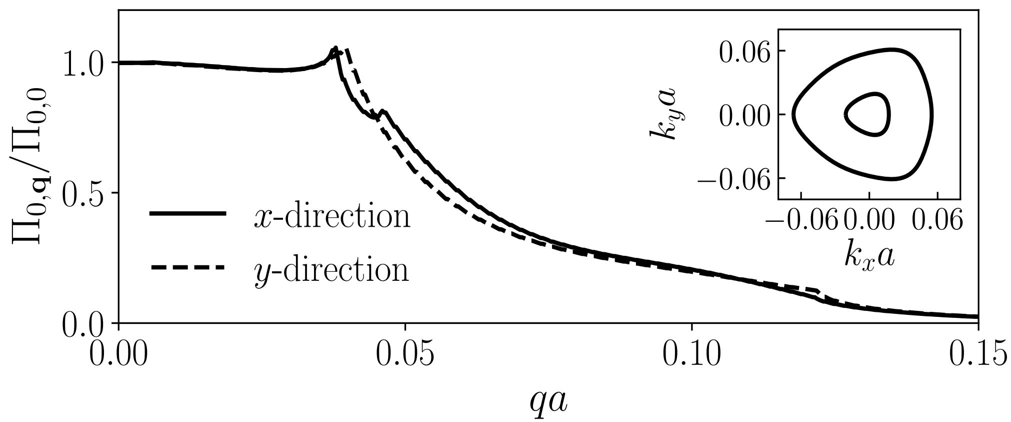


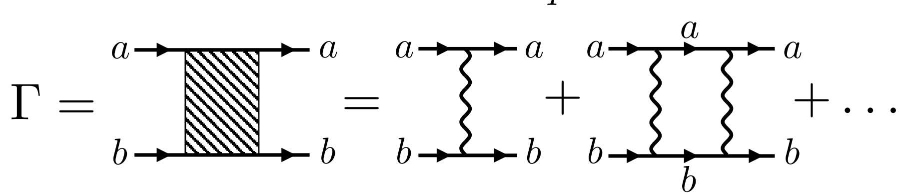
$$= \frac{V_{0,\boldsymbol{q}}}{1 + N\,\Pi_{0,\boldsymbol{q}}\,V_{0,\boldsymbol{q}}}$$



SC1: $N = 4$

SC2: $N = 2$ (spin polarized)



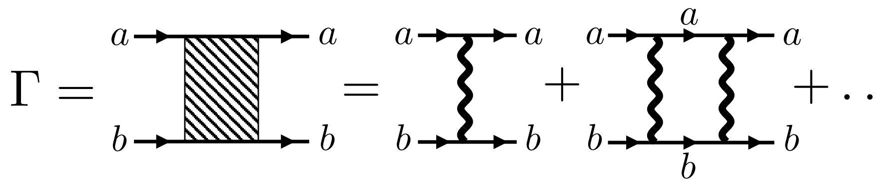


A. Ghazaryan, T. Holder, M. Serbyn, EB, PRL (2021)

See also: Dong, Chubukov, Levitov; Cea, Pantaleon, Phong, Guinea; Szabo, Roy; You, Vishwanath, Qin,MacDonald; Wagner, Kwan, Bultnick, Simon, Parameswaran; …

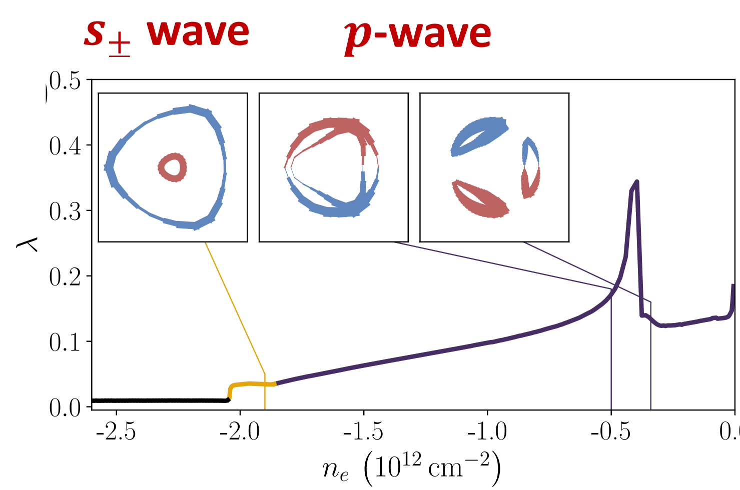
Charge Fluctuations

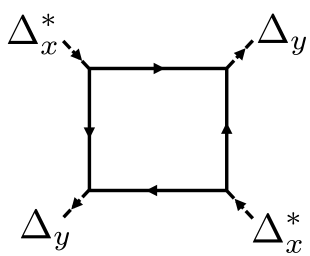
Solution to the linearized BCS gap equation with $V_{\boldsymbol{q}}$



$$T_c = W e^{-1/\lambda}$$

$$W \sim E_F$$





Beyond the linearized BCS equation:

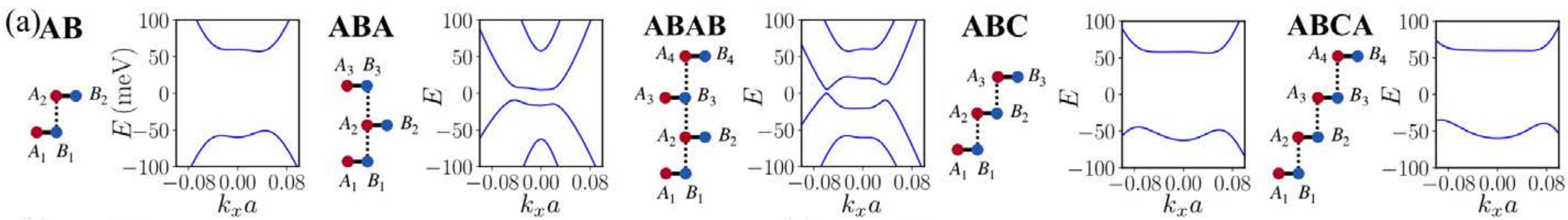
Chiral $\boldsymbol{\Delta_x + i\Delta_y}$

Topological SC?

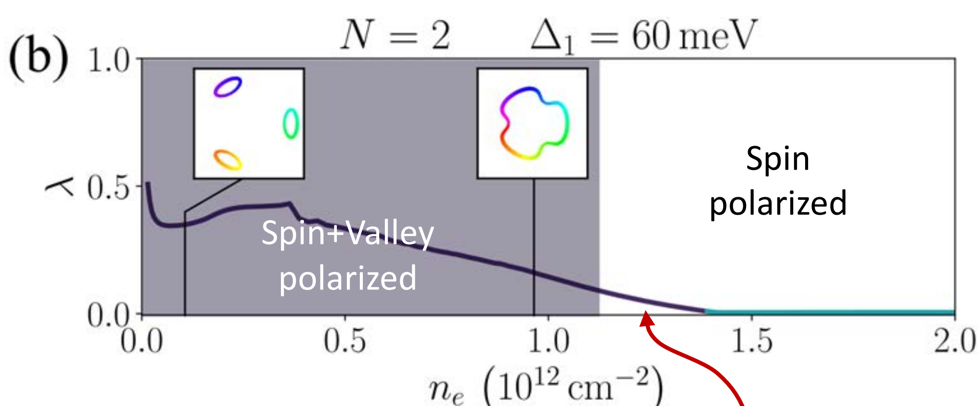
Total Chern number of BdG Hamiltonian vanishes!
(electron and hole pockets cancel)

Chiral $\Delta_x + i\Delta_y$

Consider up to 4 layers, all stackings



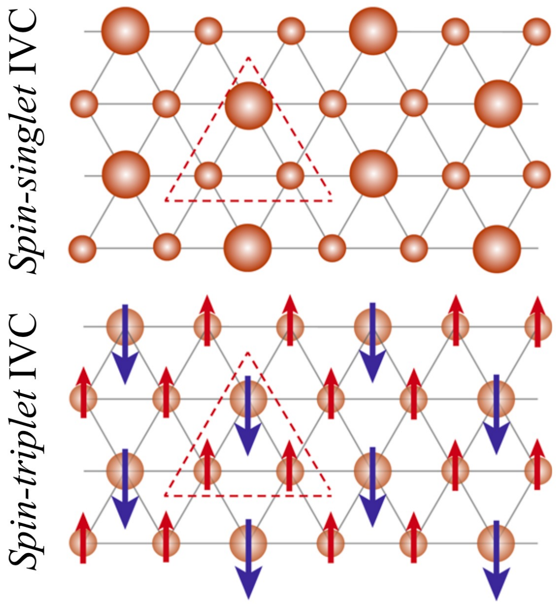
ABCA, electron side:

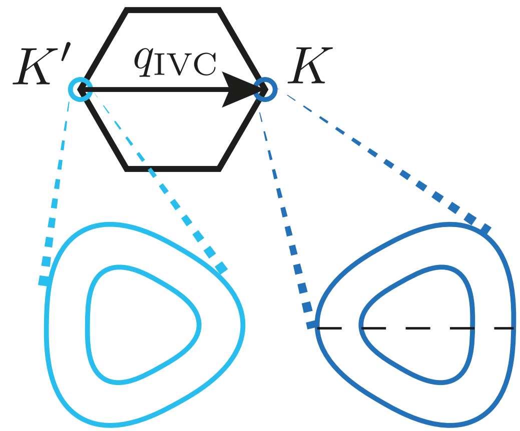


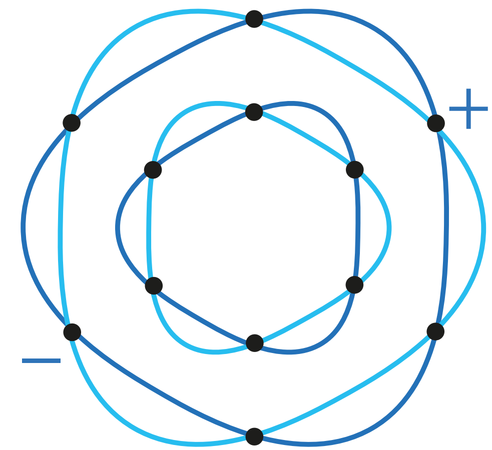
Topological SC!

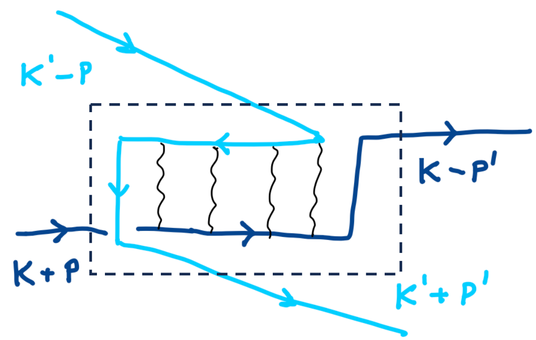
A. Ghazaryan, T. Holder, EB, M. Serbyn, PRB (2023)

Inter Valley Coherent (IVC) Mediated SC



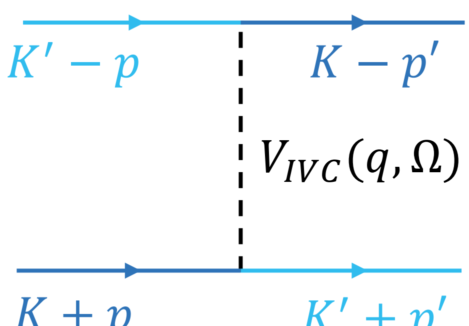






- Experimental evidence for IVC in RTG

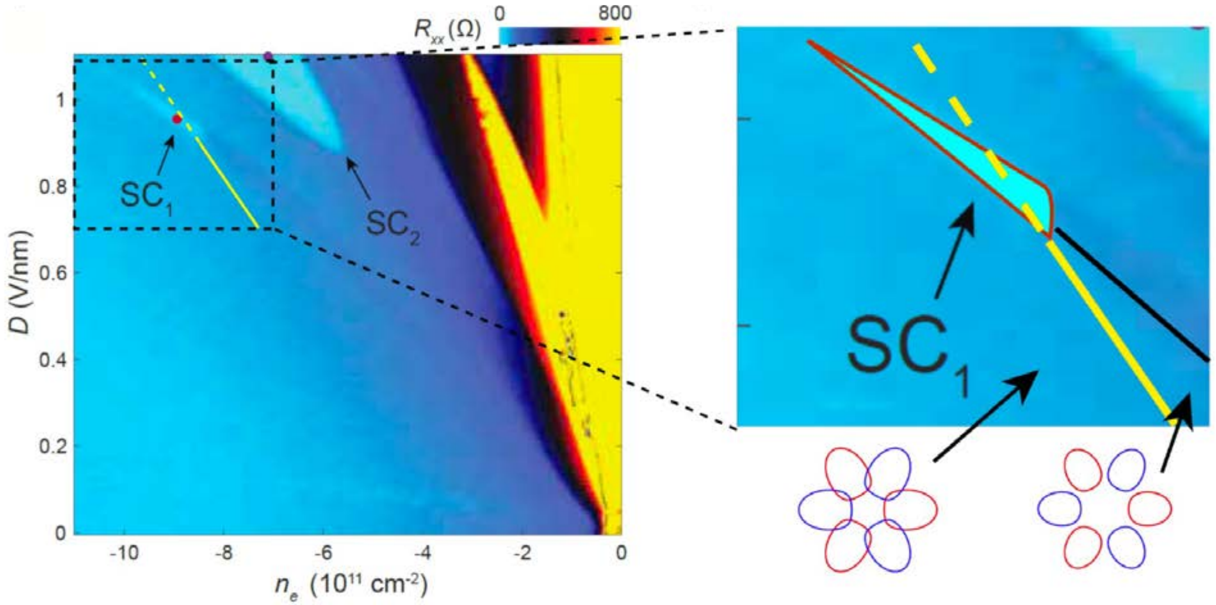
T. Arp, O. Sheekey…,Xiao, Vituri, Holder, EB, A. Young (arXiv:2310.0378)



$$V_{IVC}(q) = \frac{V_0}{1/\xi^2 + q^2 + i\gamma|\Omega|}$$

Inter Valley Coherent (IVC) Mediated SC

Evidence for dominant backscattering
pairing interaction in BLG

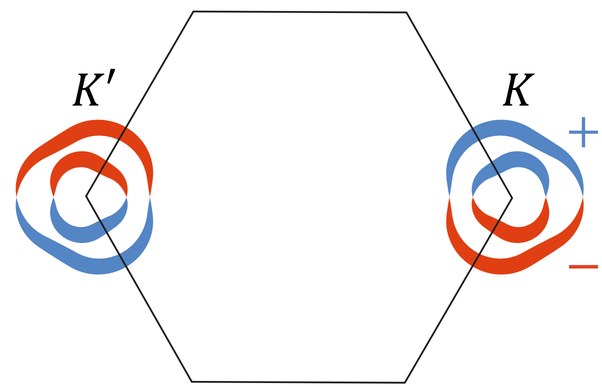


Dong, Lee, Levitov (2023)
[*Data: Holleis, ..., Nadj-Perge, Young (2023)*]

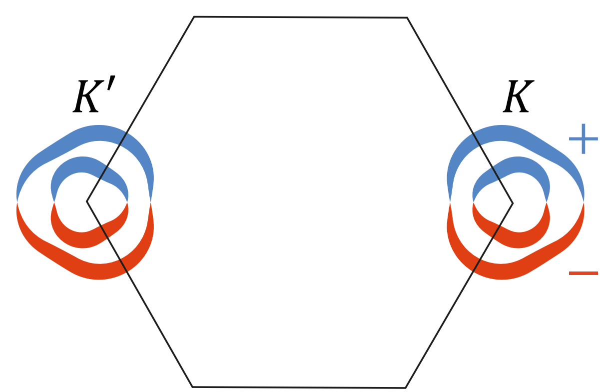
Singlet or Triplet: Hund's Term

Intra-valley p-wave can be either singlet or triplet

$$\left\langle \psi^{\dagger}_{K,\boldsymbol{k},\uparrow}\psi^{\dagger}_{K',-\boldsymbol{k},\downarrow}\right\rangle = \left\langle \psi^{\dagger}_{K',-\boldsymbol{k},\uparrow}\psi^{\dagger}_{K,\boldsymbol{k},\downarrow}\right\rangle = \phi_{\boldsymbol{k}} \neq 0$$



$$\left\langle \psi^{\dagger}_{K,\boldsymbol{k},s}(i\sigma_2\vec{\sigma})_{s,s'}\psi^{\dagger}_{K',-\boldsymbol{k},s'}\right\rangle = \vec{d}_{\boldsymbol{k}} \neq 0$$



Long-range Coulomb interactions: $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry

Singlet and triplet are degenerate!

Singlet or Triplet: Hund's Term

$$H_J = -J \int d^2 r\, \vec{S}_K \cdot \vec{S}_{K'} \qquad J \sim 10^{-1} - 10^{-2} \cdot \frac{e^2}{\epsilon k_F}$$

Chiral $\boldsymbol{\Delta_x + i\Delta_y}$: $\langle \psi_s^\dagger(\boldsymbol{r}) \psi_{s'}^\dagger(\boldsymbol{r}) \rangle = 0$

$\boldsymbol{H_J}$ drops out of gap equation!

$$\widetilde{H_J} = -\int_{\boldsymbol{r},\boldsymbol{r}'} J(\boldsymbol{r} - \boldsymbol{r}') \vec{S}_K(\boldsymbol{r}) \cdot \vec{S}_{K'}(\boldsymbol{r}')$$

$$\tilde{J}(q) = J_0 + J_2 (a_0 q)^2 + \cdots$$

E.g., in ABC trilayer:

$J_0 > 0, J_2 > 0$ favors: spin polarized, valley unpolarized state ✓

spin singlet $\Delta_x + i\Delta_y$ SC ✓

Order Parameter Topology and Current Dissipation and in Spin-Polarized Triplet Superconductors

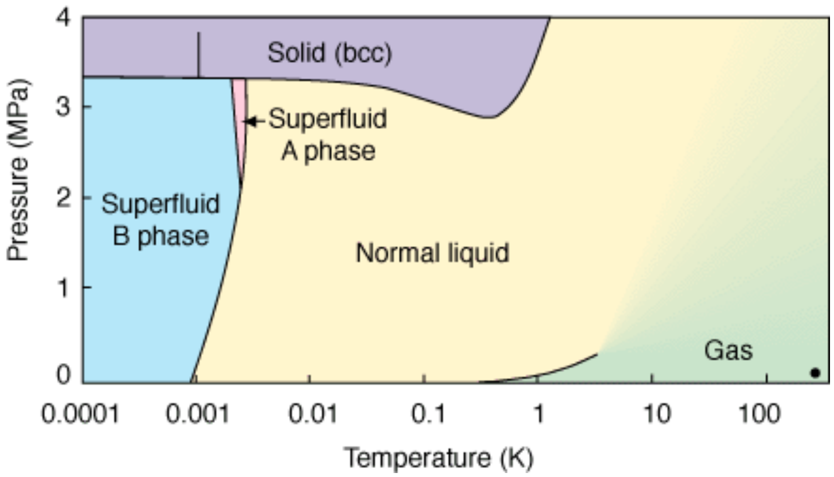
Eyal Cornfeld
(WIS→Classiq Technologies)

Mark Rudner
(Copenhagen→U. Washington)

E. Cornfeld, M. Rudner, EB, Phys. Rev. Research 3, 013051 (2021)

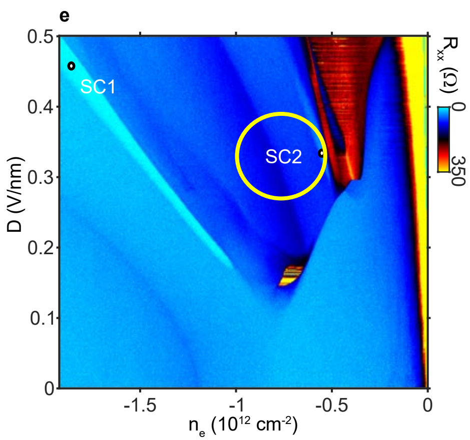
Triplet superconductivity in RTG

Solid state analogue of superfluid ^{3}He?



Triplet superconductivity:

- Strong electronic correlations ✅
- Nearby/coexisting ferromagnetism ✅
- Extremely clean ✅



Very small spin-orbit: SC and magnetism intertwined in interesting way?

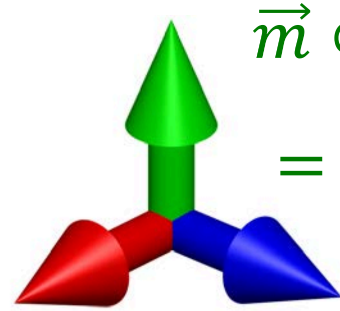
Order parameter of a spin-polarized superconductor

Order parameter of a spin-triplet SC:

$$\vec{d}_k = \langle c_k^\dagger i\sigma_2 \vec{\sigma} c_{-k}^\dagger \rangle \equiv \vec{d}_{1,k} + i\vec{d}_{2,k}$$

Fully spin polarized SC: $|\vec{d}_{1,k}| = |\vec{d}_{2,k}|$, $\vec{d}_{1,k} \perp \vec{d}_{2,k}$

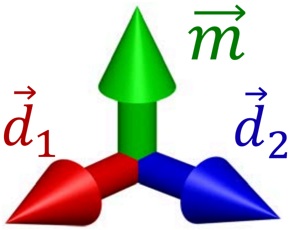
Order parameter space: $SO(3)$
(*Neglecting spin-orbit coupling)*



No finite T transition in $d = 2$
Mukerjee, Xu, Moore (2006)

Topological defects

Polarized triplet superconductor:



$\pi_1\big(SO(3)\big) = Z_2$ (Z_2 superconducting vortex)

"Dirac belt trick":

E. Cornfeld, M. Rudner, EB, Phys. Rev. Research 3, 013051 (2021)

Topological defects

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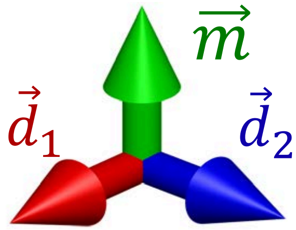

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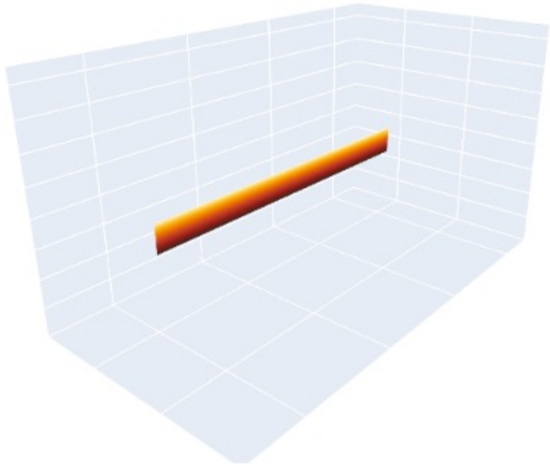
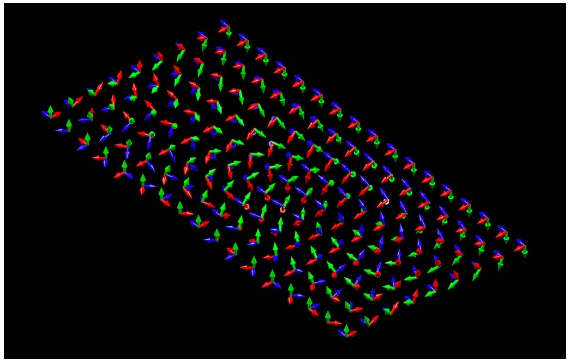


Consequence:
4π superconducting phase winding (two vortices) can be "unwound" by creating spin texture



E. Cornfeld, M. Rudner, EB, Phys. Rev. Research 3, 013051 (2021)

Consequences for current relaxation

Free energy density (assuming spin rotation invariance):

$$f = \frac{\kappa_d}{2}\left|\nabla\vec{d}\right|^2 + \frac{\kappa_m}{8}\left|\nabla\left(\vec{d}^*\times\vec{d}\right)\right|^2$$

Represent order parameter by 2×2 unitary matrix u:

$$\vec{d} = \mathrm{Tr}[u(\sigma_1 + i\sigma_2)u^\dagger\vec{\sigma}]$$

Spin rotation: $u \rightarrow e^{\frac{i}{2}\vec{\theta}\cdot\vec{\sigma}}u$, Gauge transformation: $u \rightarrow ue^{\frac{i}{2}\varphi\sigma_3}$

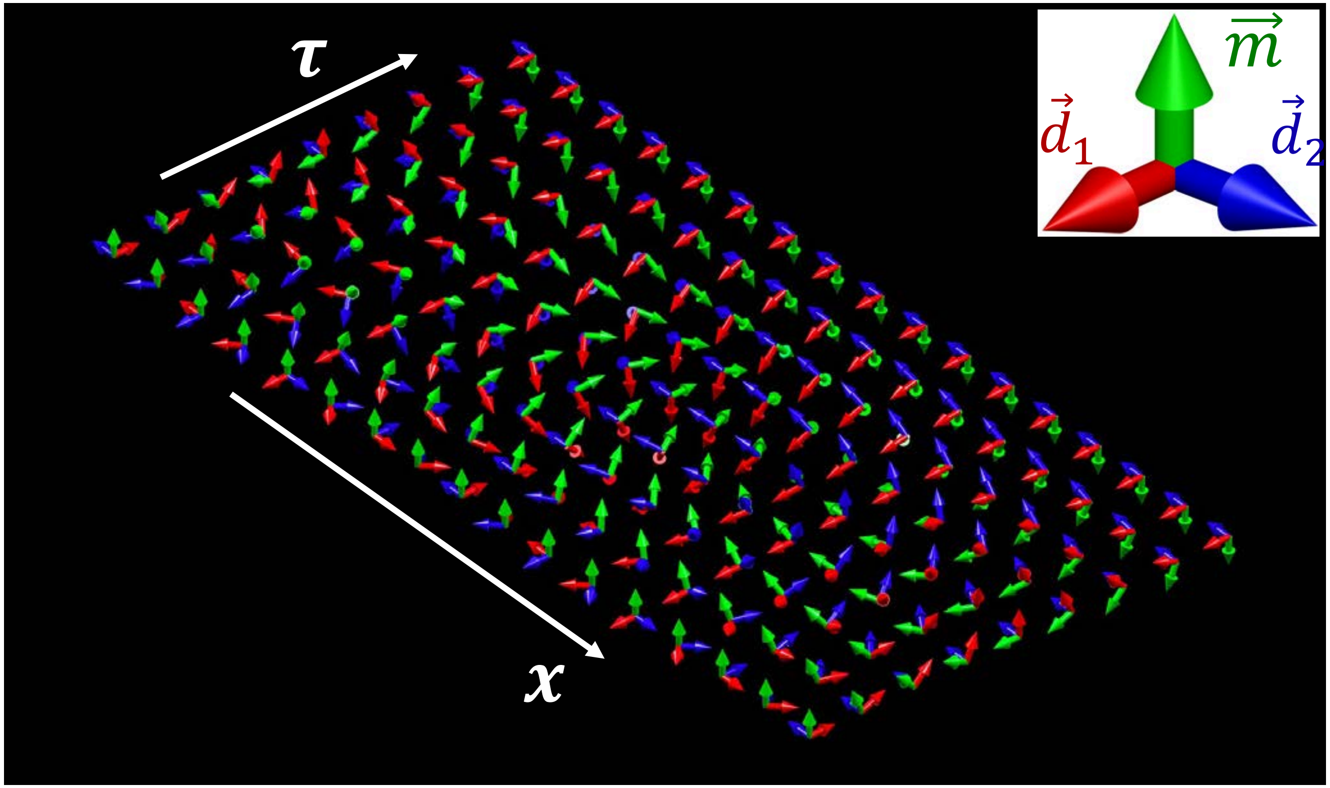
Supercurrent carrying state: $u(\vec{r}) = e^{i\pi n\sigma_3\frac{x}{L_x}}$

E. Cornfeld, M. Rudner, EB, Phys. Rev. Research 3, 013051 (2021)

Consequences for current relaxation

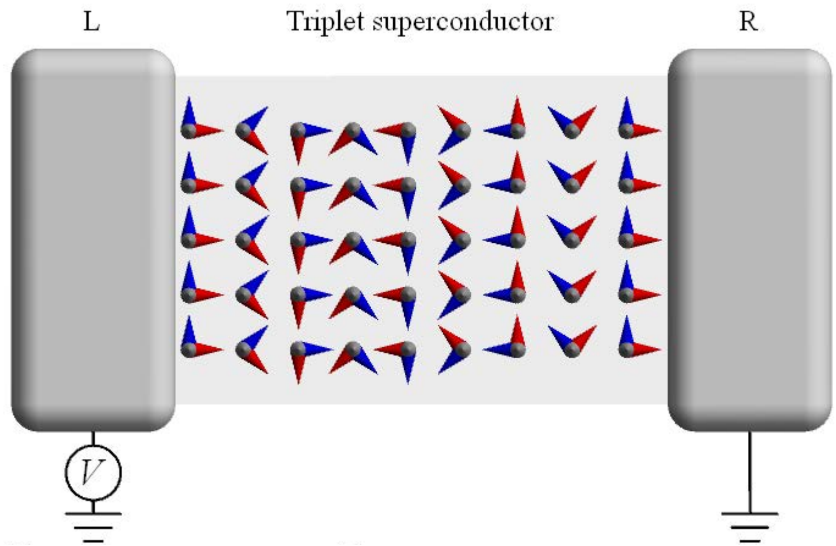
Unwinding a phase twist of 4π:

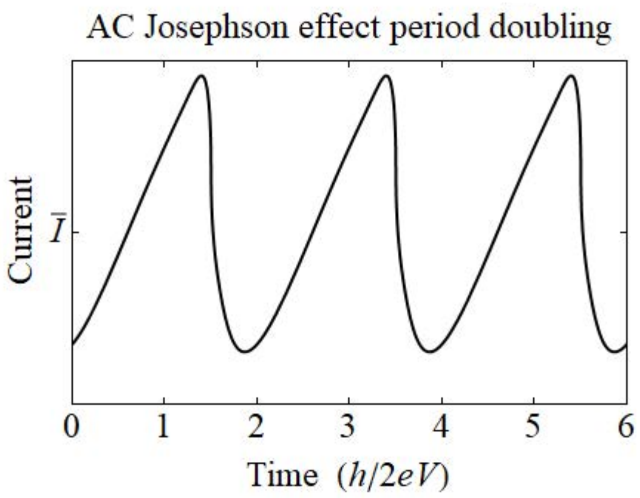
$$u(\vec{r}, 0 \le \tau \le 1) = e^{i\pi\sigma_3 \frac{x}{L_x}} e^{\frac{i\pi}{2}\sigma_1 \tau} e^{i\pi(n-1)\sigma_3 \frac{x}{L_x}}$$



E. Cornfeld, M. Rudner, EB, Phys. Rev. Research 3, 013051 (2021)

Double-period Josephson effect





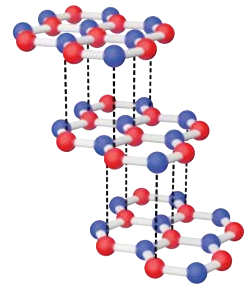
Half the usual Josephson frequency: $\omega = \frac{eV}{\hbar}$

Critical current: $J_c \sim 1/\lambda \sim \sqrt{B}$

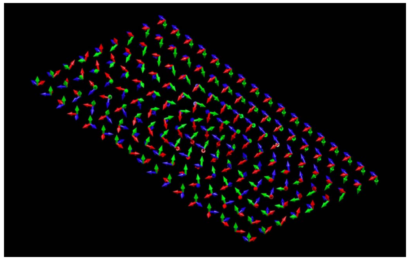
$^*T$ is low enough such that vortex-antivortex dissociation is suppressed.

Summary

- **Unconventional SC in AB and ABC rhombohedral graphene?**
  Possible state: chiral intra-valley p-wave



- **Fully spin polarized SC: fragility of supercurrent due to topology, double-period Josephson effect**



Thank you!