DIRECT MEASUREMENTS OF ANYONIC STATISTICS VIA INTERFEROMETRY

Michael Manfra

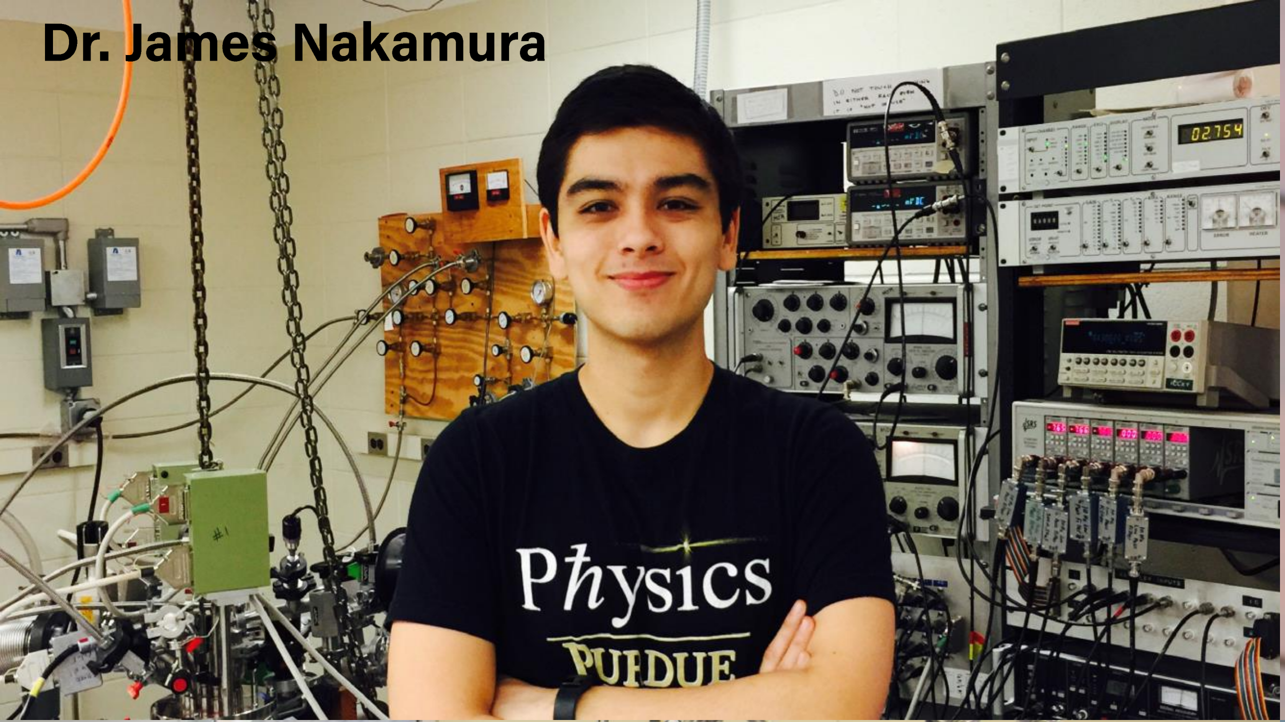
Department of Physics and Astronomy

Elmore Family School of Electrical and Computer Engineering

School of Materials Engineering

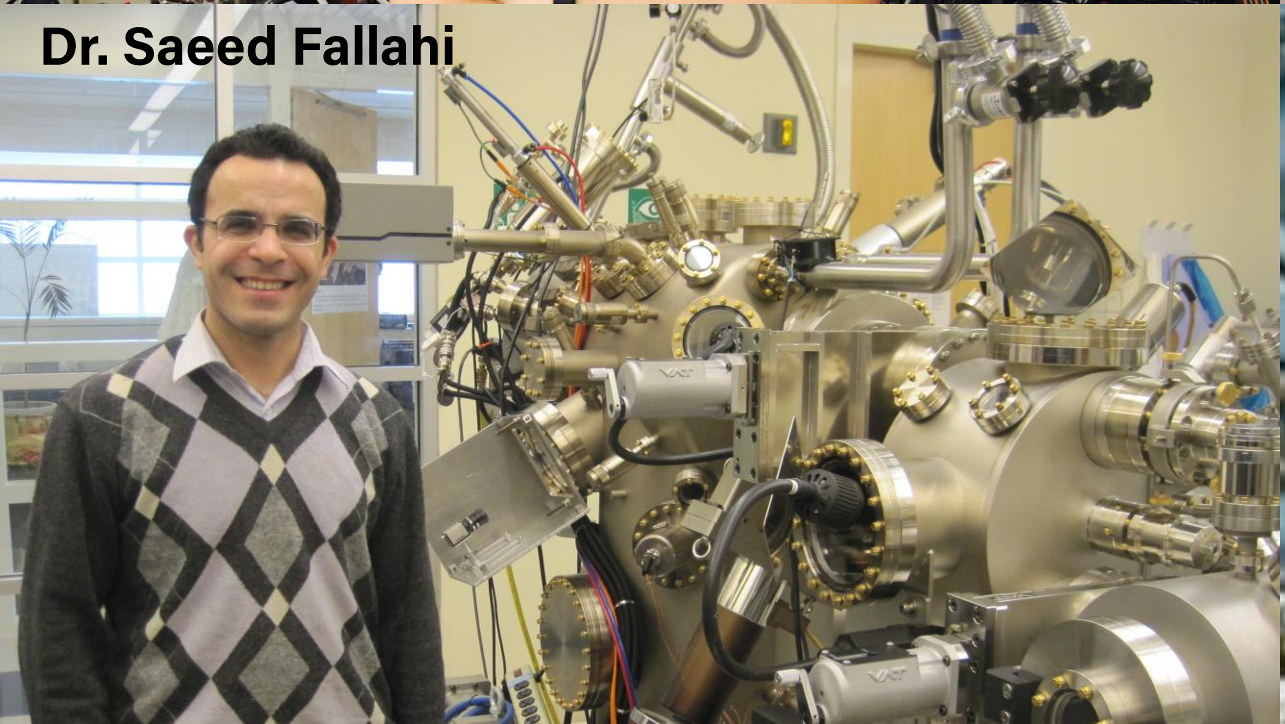
Microsoft Quantum Laboratory West Lafayette

PURDUE UNIVERSITY



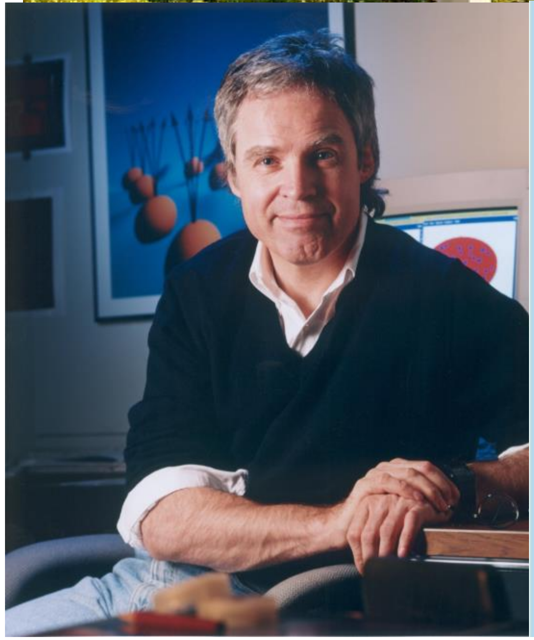
Dr. James Nakamura


Dr. Geoffrey Gardner


Dr. Saeed Fallahi


Shuang Liang

Semiconductor Physics Department: the early "aughts"



Horst Stormer

Aron Pinczuk

Early analysis of fractional statistics and the FQHE

Quantum Mechanics of Fractional-Spin Particles

Frank Wilczek
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 22 June 1982)

Composites formed from charged particles and vortices in (2 + 1)-dimensional models, or flux tubes in three-dimensional models, can have any (fractional) angular momentum. The statistics of these objects, like their spin, interpolates continuously between the usual boson and fermion cases. How this works for two-particle quantum mechanics is discussed here.

Statistics of Quasiparticles and the Hierarchy of Fractional Quantized Hall States

B. I. Halperin
Physics Department, Harvard University, Cambridge, Massachusetts 02138
(Received 9 November 1983)

Quasiparticles at the fractional quantized Hall states obey quantization rules appropriate to particles of fractional statistics. Stable states at various rational filling factors may be constructed iteratively by adding quasiparticles or holes to lower-order states, and the corresponding energies have been estimated.

Although practical applications of these phenomena seem remote, I think they have considerable methodological interest and do shed light on the fundamental spin-statistics connection.

The appearance of fractional statistics in the present context is strongly reminiscent of the fractional statistics introduced by Wilczek to describe charged particles tied to "magnetic flux tubes" in two dimensions.[6]

Early analysis of fractional statistics and the FQHE

VOLUME 53, NUMBER 7 PHYSICAL REVIEW LETTERS 13 AUGUST 1984

Fractional Statistics and the Quantum Hall Effect

Daniel Arovas
Department of Physics, University of California, Santa Barbara, California 93106

and

J. R. Schrieffer and Frank Wilczek
Department of Physics and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

(Received 18 May 1984)

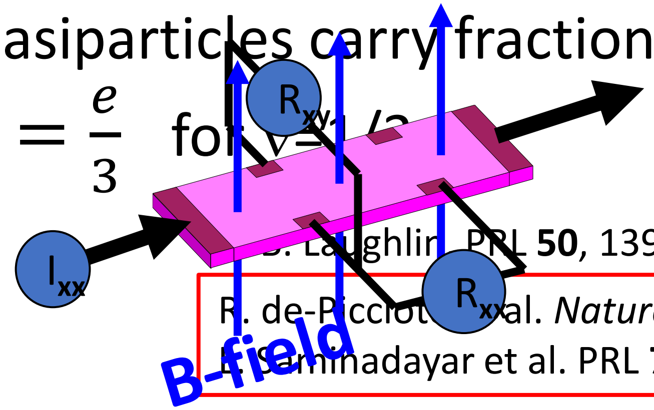
The statistics of quasiparticles entering the quantum Hall effect are deduced from the adiabatic theorem. These excitations are found to obey fractional statistics, a result closely related to their fractional charge.

PACS numbers: 73.40.Lq, 05.30.–d, 72.20.My

Excitations of the FQHE are anyons: fractional charge and statistics

- Quasiparticles carry fractional charge

$e^* = \frac{e}{3}$ for ν=1/3

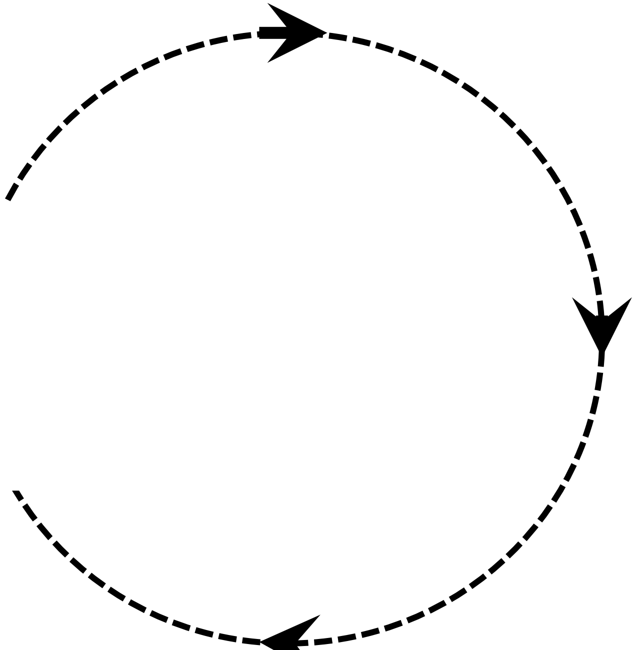


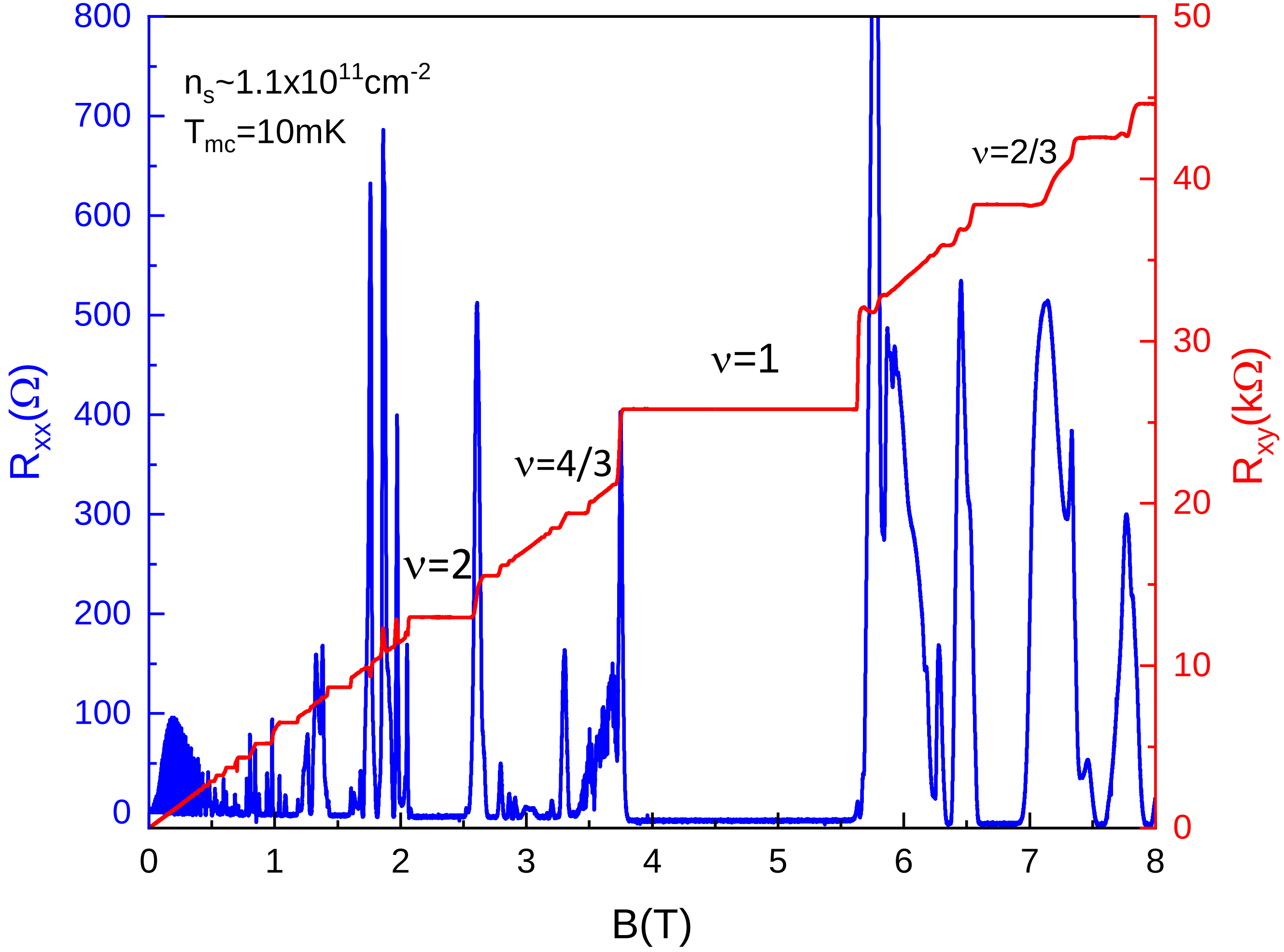
R. B. Laughlin, PRL **50**, 1395 (1983)

R. de-Picciotto et al. *Nature* **389**, 162 (1997)
L. Saminadayar et al. PRL **79,** 2526 (1997)

- Anyonic braiding statistics: $\theta_{anyon} = 2\pi \frac{e^*}{e}$

B. I. Halperin, PRL **52**, 1583 (1984)
Arovas, Schrieffer, and Wilczek, PRL **53**, 722 (1984)



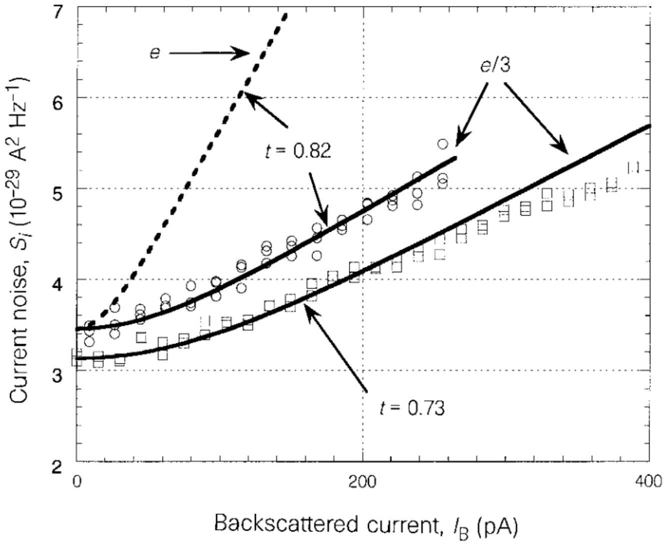


ν=1/3: $\theta_{anyon} = \frac{2\pi}{3}$

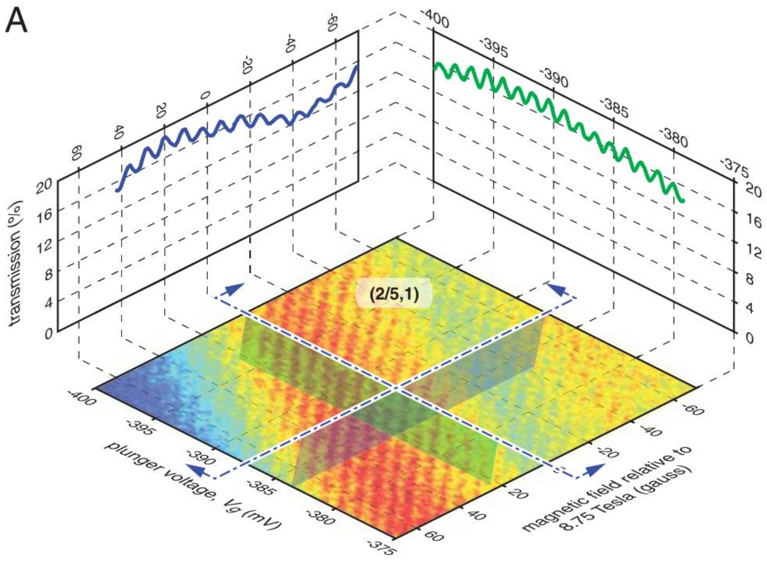
Building upon years of experimental progress

- Observation of fractional charge via shot noise
- Interference of integer QHE modes
- Evidence for interference at $\nu = 5/2$
- Observation of anyon exchange statistics through quasiparticle collisions

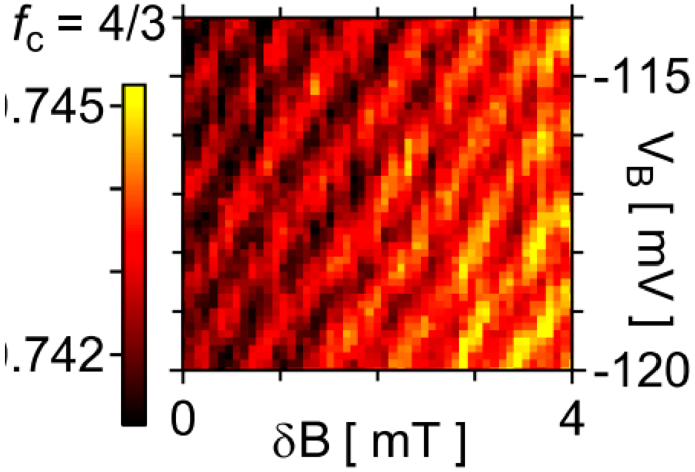
R. de-Picciotto et al. *Nature* 389, 162 (1997)



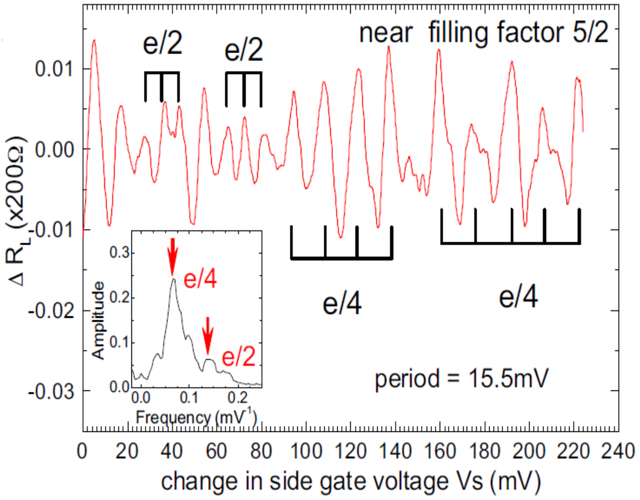
N. Ofek at al. *PNAS* 107, 5276 (2010)



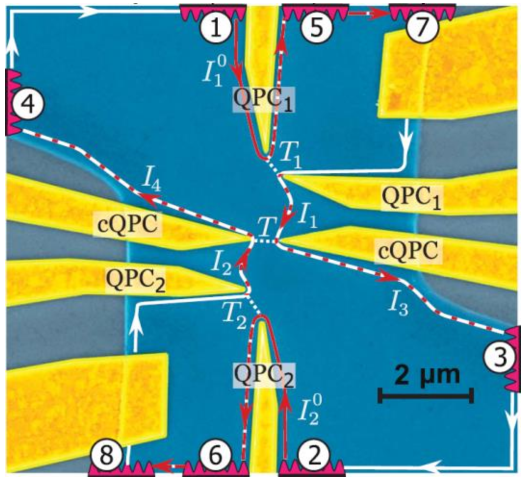
D. T. McClure et al. *PRL* 108, 256804 (2012)



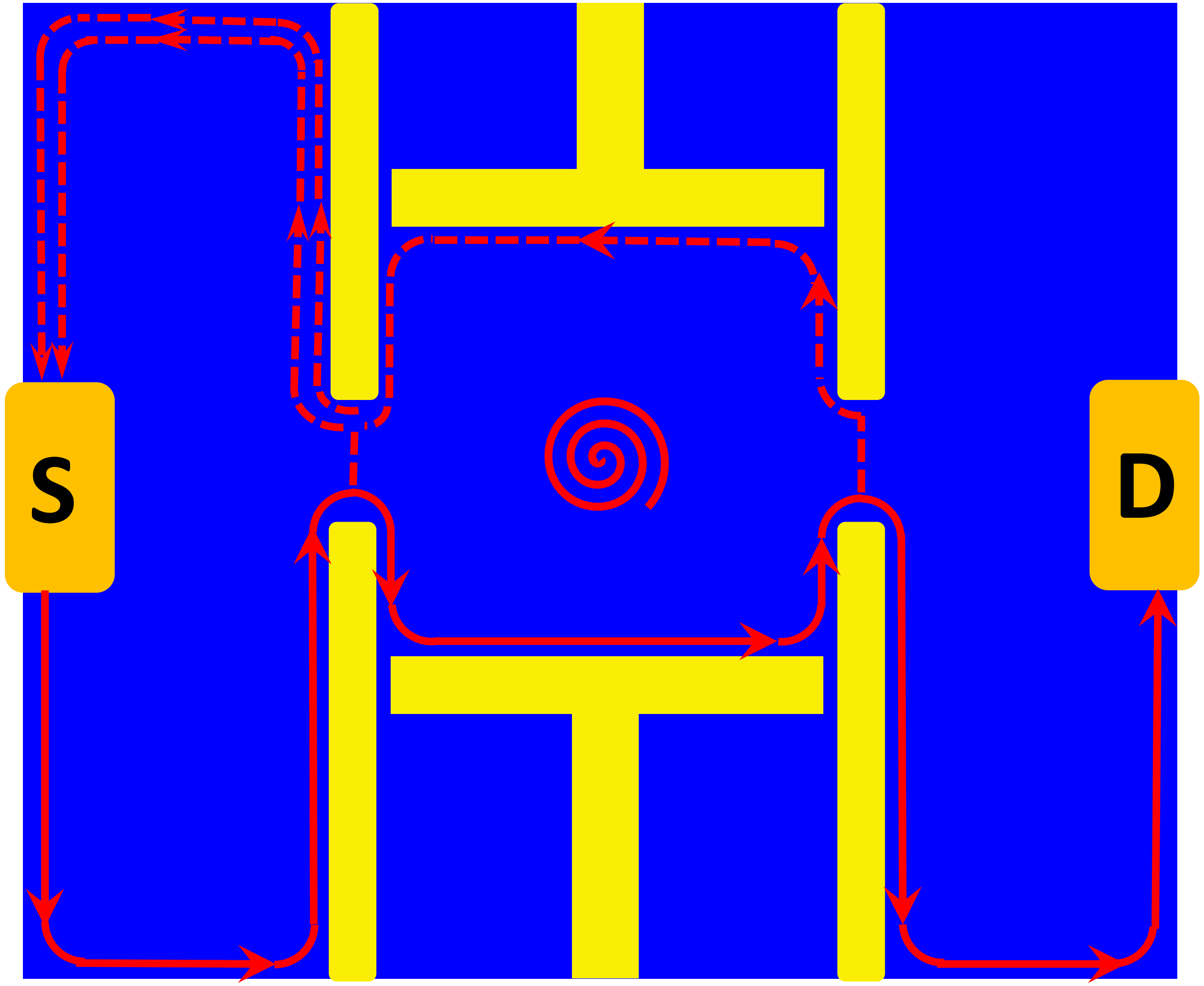
R. L. Willet at al.
PNAS 106, 8853 (2009)



H. Bartolomei et al.
Science 368, 173 (2020)



Electronic Fabry-Perot interferometry in the Quantum Hall regime

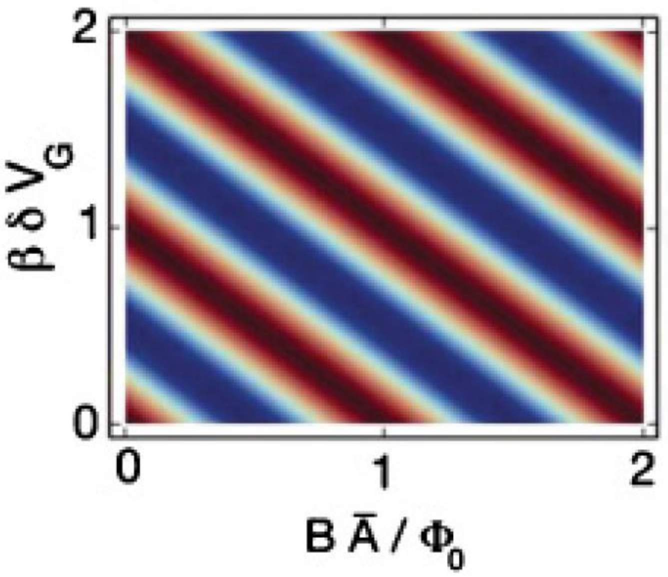


$B \otimes$

- Surface gates define electron interference path
- Quantum point contacts (QPCs) act as beam splitters

$$I \sim |t_1|^2 + |t_2|^2 + |t_1||t_2|\cos(\theta)$$

$$\theta = 2\pi \underbrace{\left(\frac{AB}{\Phi_0}\right)\frac{e^*}{e}}_{\text{Aharonov-Bohm phase}} + \underbrace{N_L \theta_{anyon}}_{\text{Braiding phase}} \qquad \Phi_0 \equiv \frac{h}{e}$$

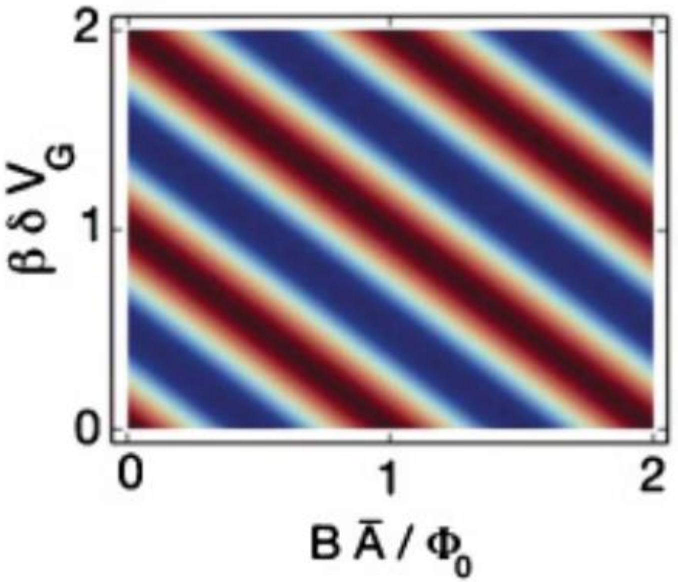


C. de C. Chamon, D. Freed, S. Kivelson, S. Sondhi, X. Wen Phys. Rev. B **55**, 2331 (1997)

B.I. Halperin, A. Stern, I. Neder, and B. Rosenow PRB **83**, 155440 (2011)

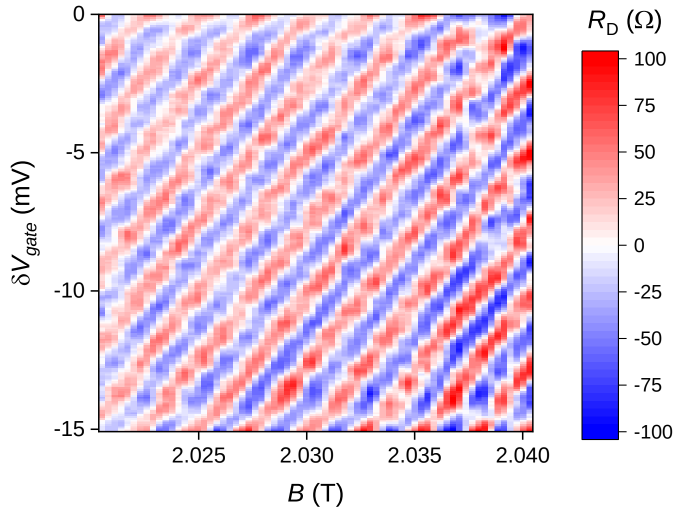
Problem: strong bulk-edge interaction

Expectation: Aharonov-Bohm Interference



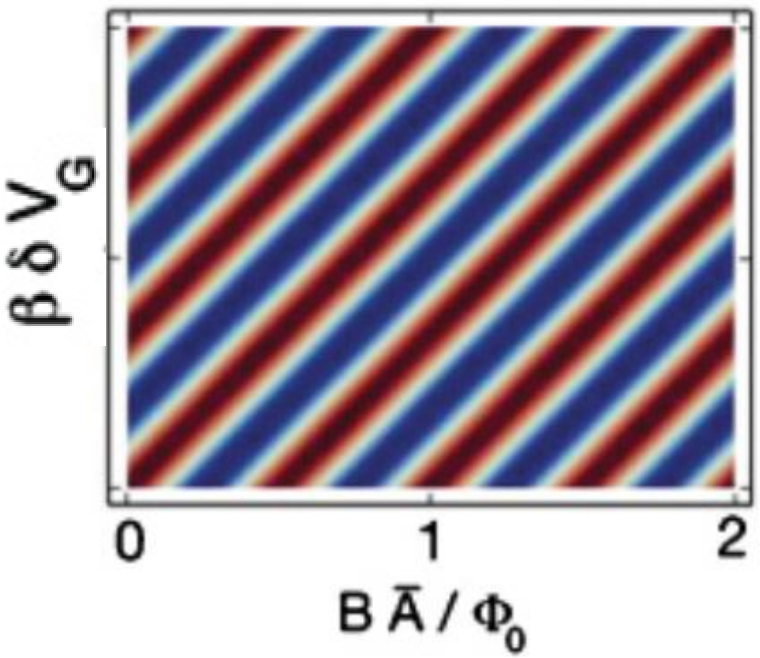
$$\theta_I = 2\pi \frac{AB}{\Phi_0}$$

Reality: Coulomb-dominated oscillations



Wrong slope!

- Bulk-edge interactions cause area to change with magnetic field
- Cannot change A and B independently – flux *decreases* when increase B!
- Makes braiding unobservable

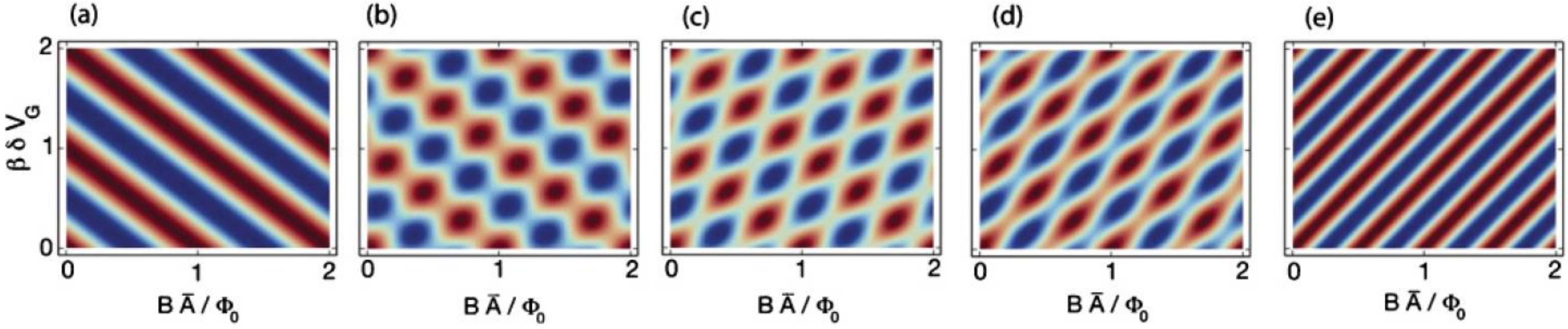


$$\delta A = -\nu \bar{A} \delta B$$

B.I. Halperin, A. Stern, I. Neder, and B. Rosenow. PRB 83, 155440 (2011)

Aharonov-Bohm vs. Coulomb dominated regime

Aharonov-Bohm **Coulomb dominated**



- **Regime of operation depends on the ratio of K_{IL}/K_I, where K_{IL} parameterizes bulk-edge interaction and K_I parameterizes the energy cost to add charge to the edge**

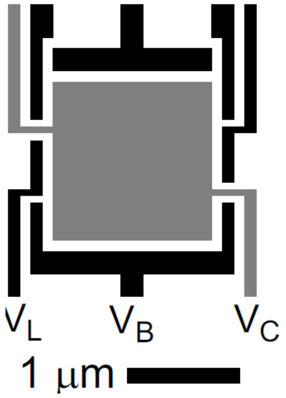
- **Critically, θ_{anyon} is unobservable in the Coulomb dominated regime: phase change is multiple of 2π.**

B. I. Halperin, A. Stern, I. Neder, and B. Rosenow. PRB **83**, 155440 (2011)
C. W. von Keyserlingk, S. H. Simon, B. Rosenow, ***PRL*** 115, 126807 (2015)

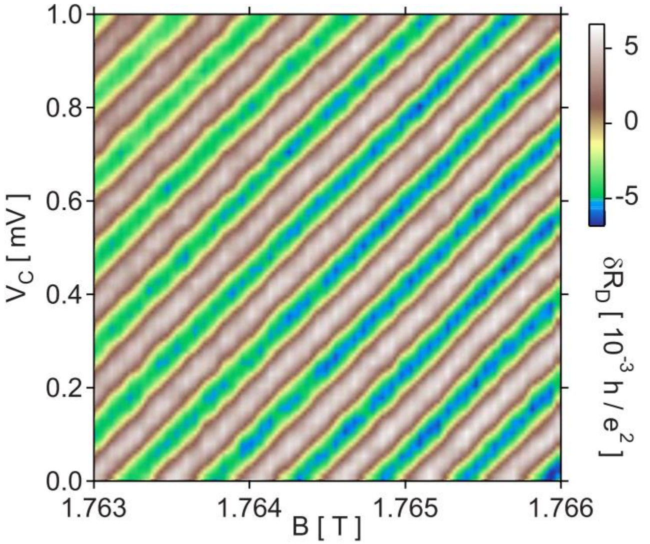
AB vs. CD in early experiments: a valuable lesson

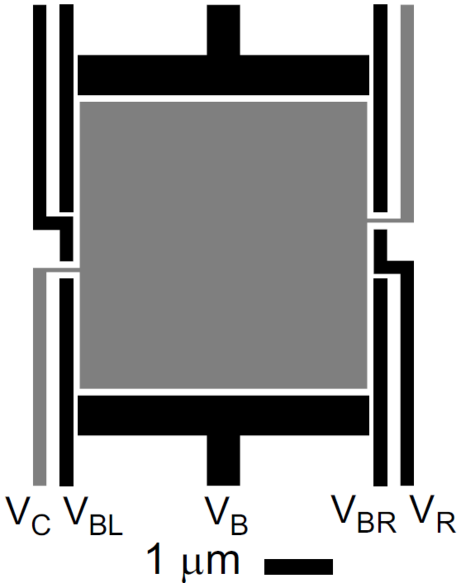
- Many early experiments observed Coulomb dominated behavior
- C. Marcus group observed AB behavior (negative slope) in devices with large area which included a metal screening gate
- Coherence was poor due to large path length
- Need better way to screen to observe AB interference in smaller devices

Zhang et al. PRB **79**, 241304 (2009)

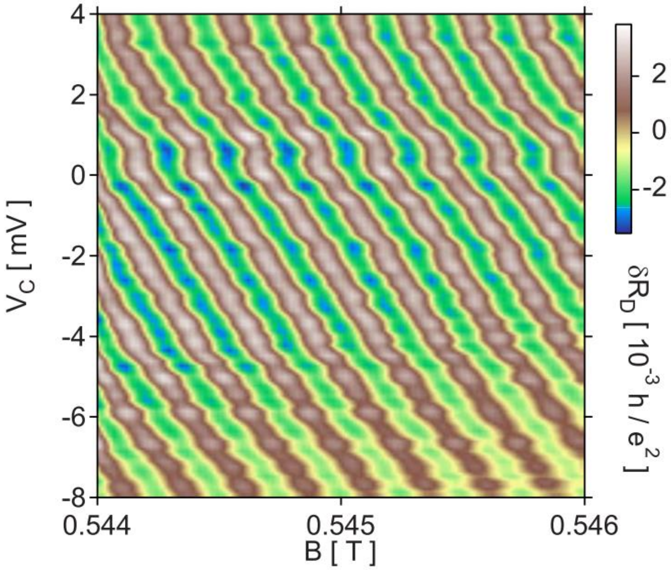


$2\mu m^2$
Device





$18\mu m^2$
Device



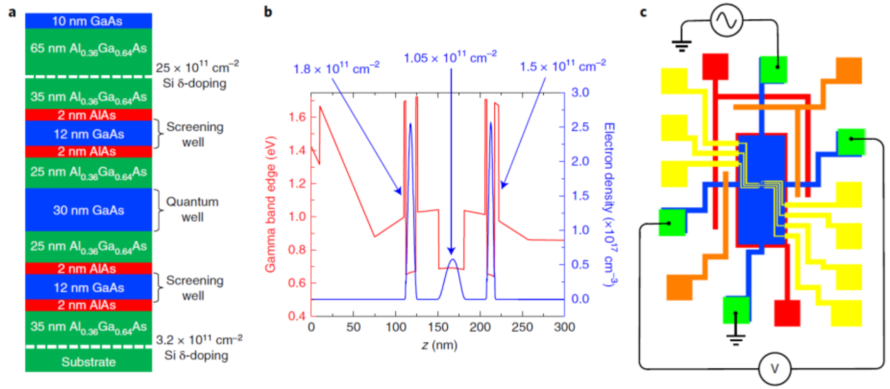
Demonstration Experiments

nature physics 2019 ARTICLES
https://doi.org/10.1038/s41567-019-0441-8

Aharonov–Bohm interference of fractional quantum Hall edge modes

J. Nakamura[1,2], S. Fallahi[1,2], H. Sahasrabudhe[1], R. Rahman[3], S. Liang[1,2], G. C. Gardner[2,4] and M. J. Manfra[1,2,3,4,5]*

Aharonov–Bohm Interference in the FQHE via novel heterostructure and device

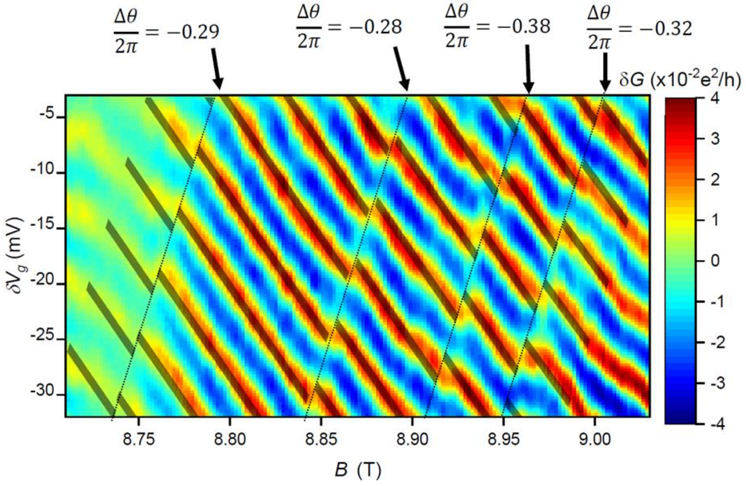


nature physics 2020 ARTICLES
https://doi.org/10.1038/s41567-020-1019-1
Check for updates

Direct observation of anyonic braiding statistics

J. Nakamura[1,2], S. Liang[1,2], G. C. Gardner[2,3] and M. J. Manfra[1,2,3,4,5]

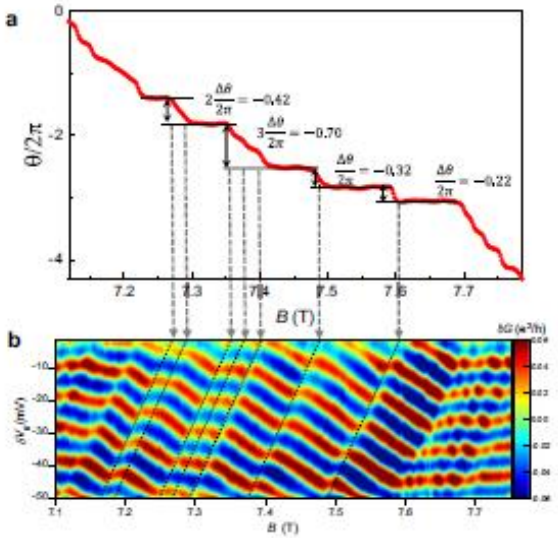
Measured $\theta_{anyon} = \frac{2\pi}{3}$ at ν=1/3



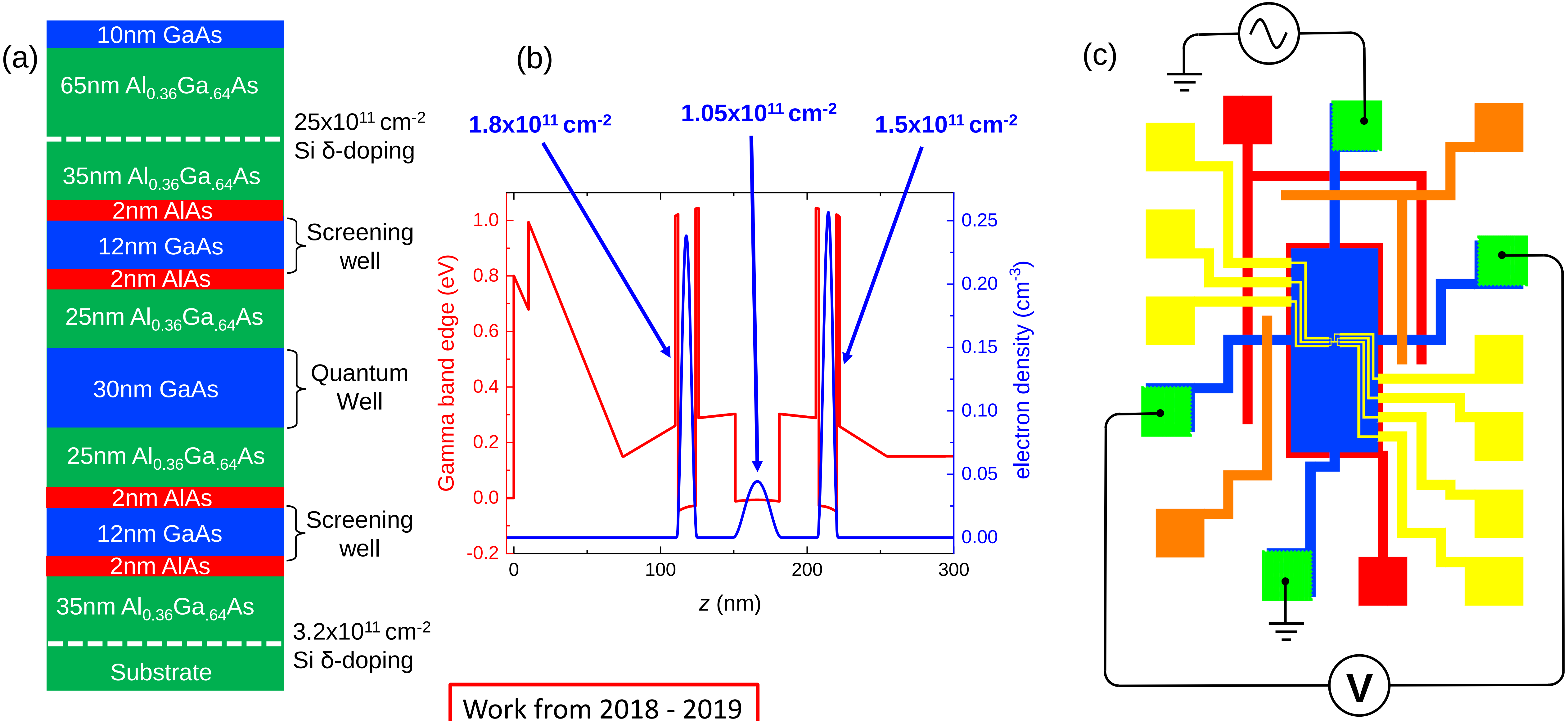
nature COMMUNICATIONS 2022
ARTICLE
https://doi.org/10.1038/s41467-022-27958-w OPEN
Check for updates

Impact of bulk-edge coupling on observation of anyonic braiding statistics in quantum Hall interferometers

J. Nakamura[1,2], S. Liang[1,2], G. C. Gardner[2,3] & M. J. Manfra[1,2,3,4,5]

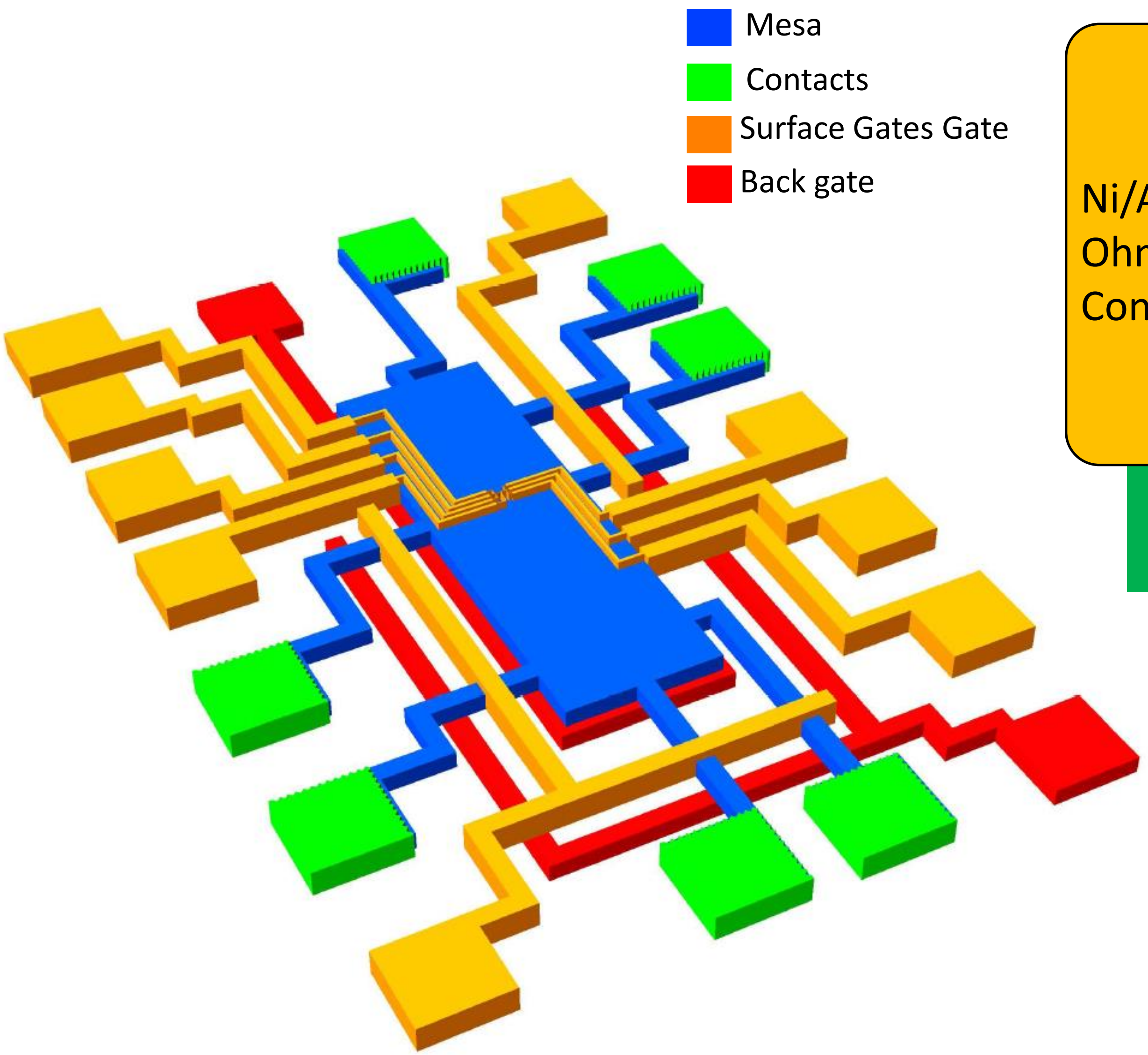


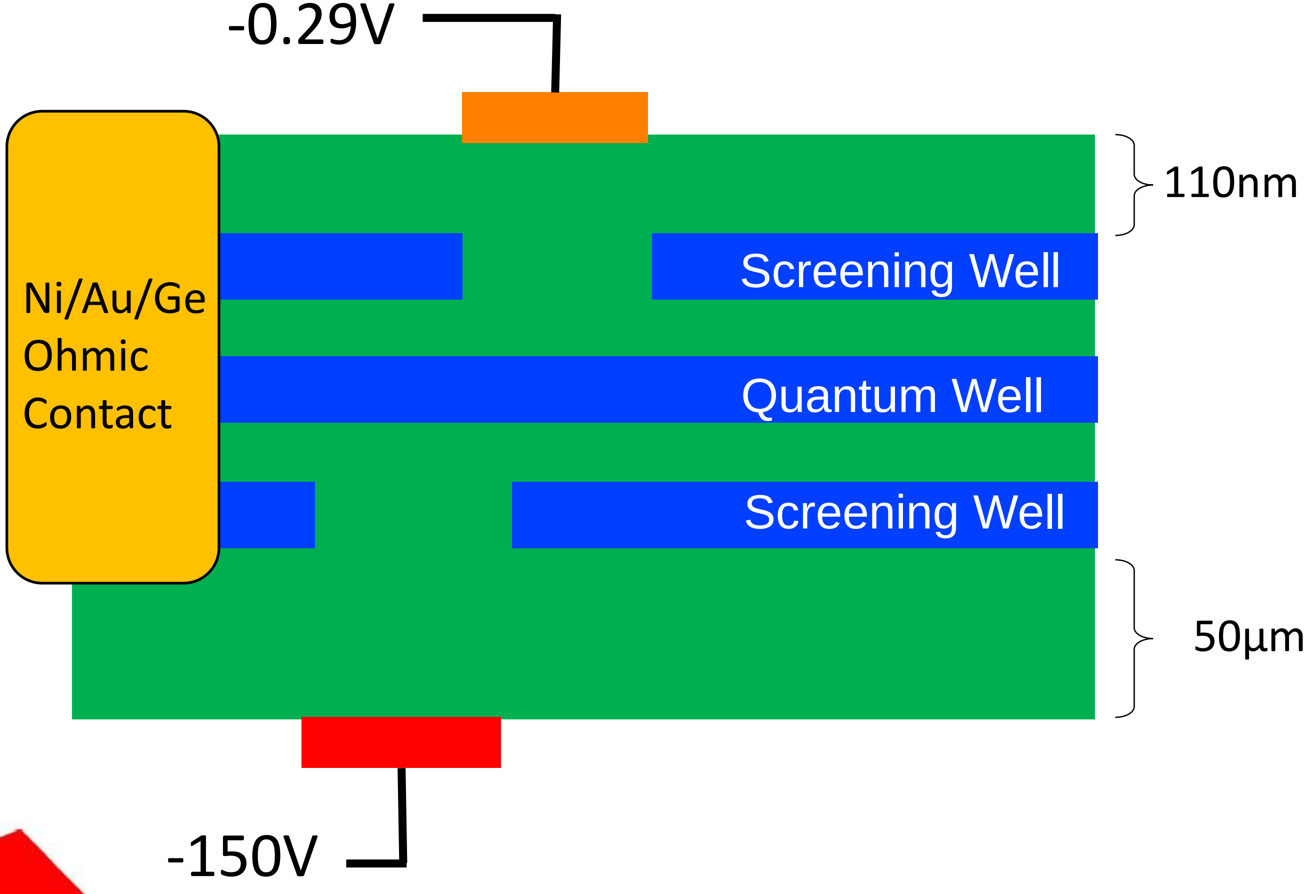
Screening well heterostructure and device design



Work from 2018 - 2019

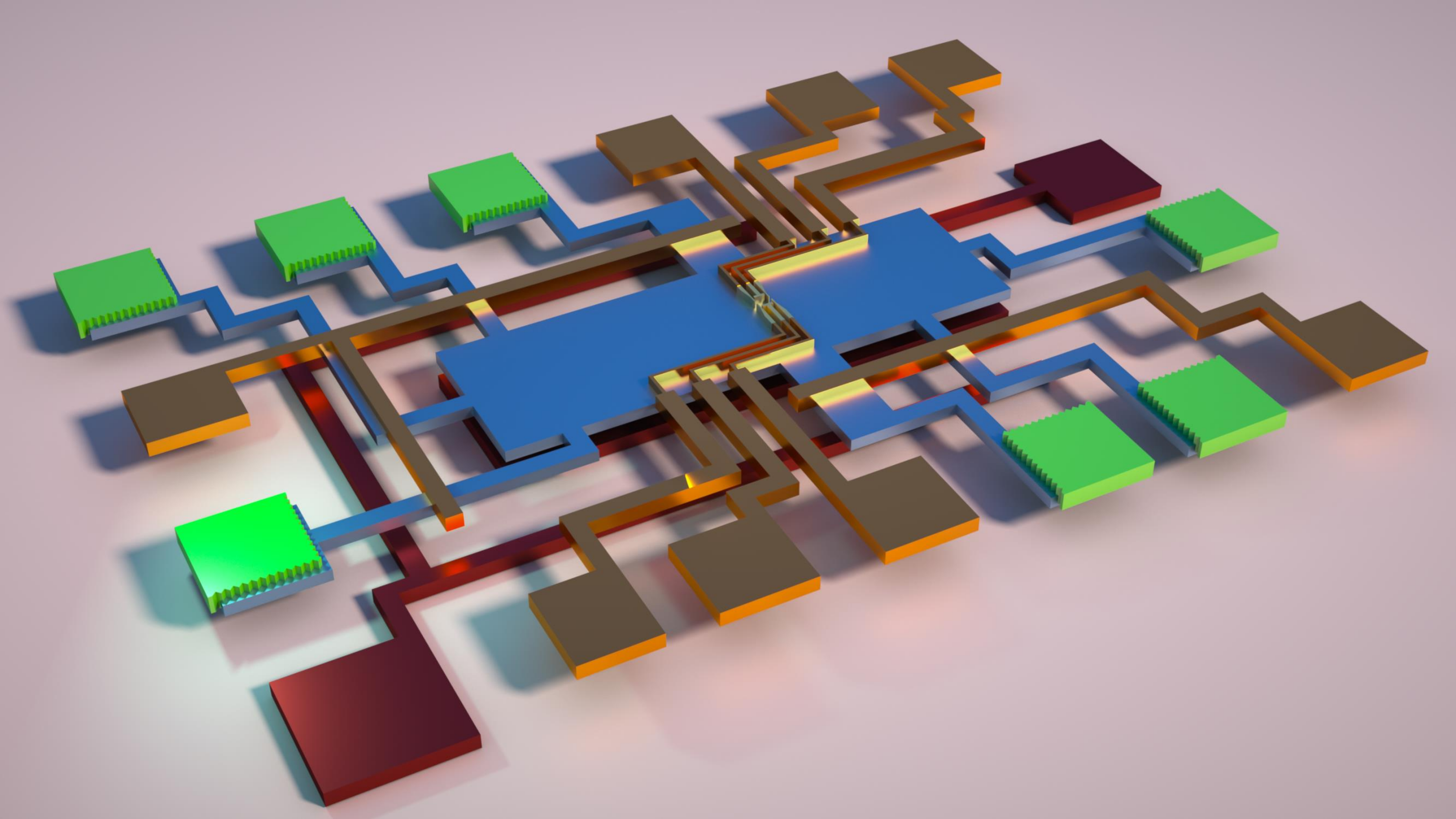
Top and back-gated interferometer operation

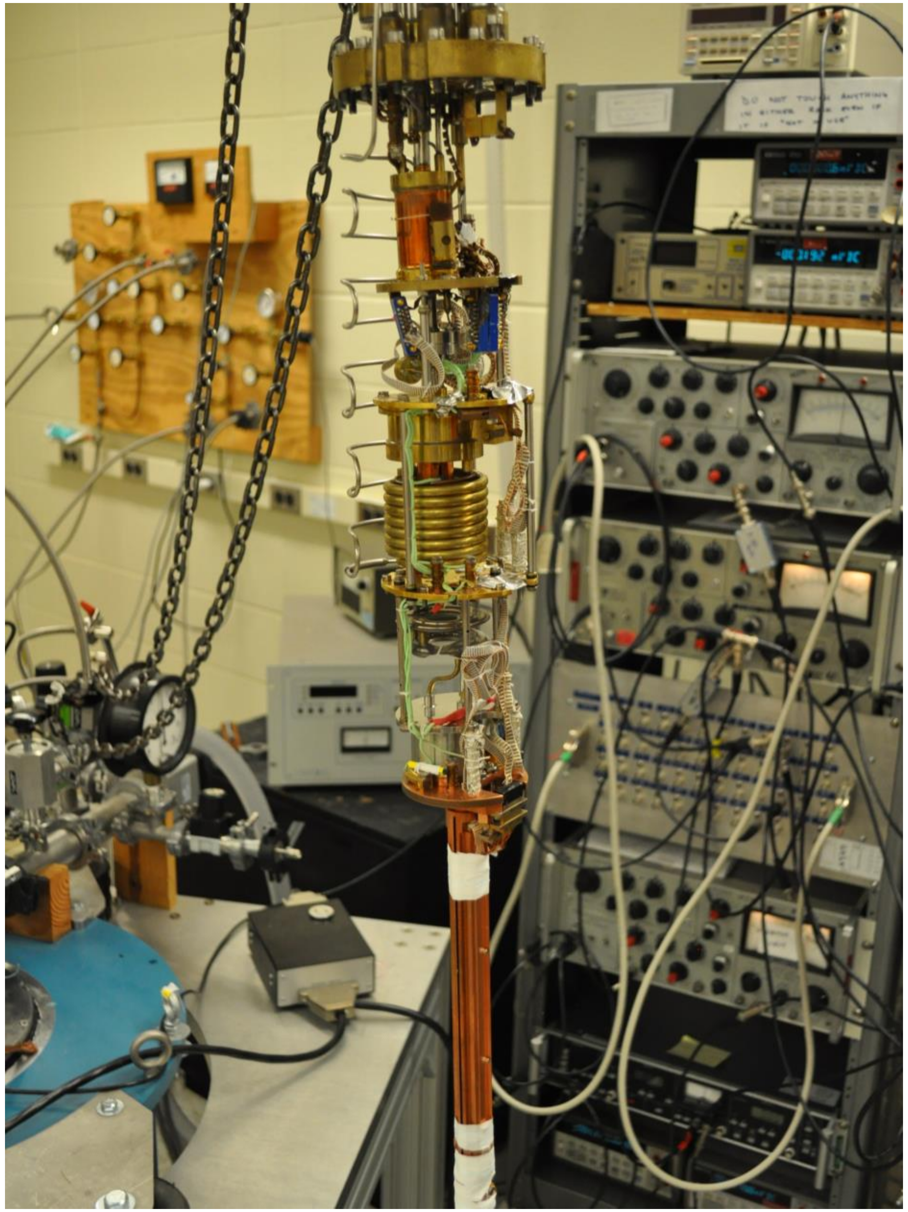


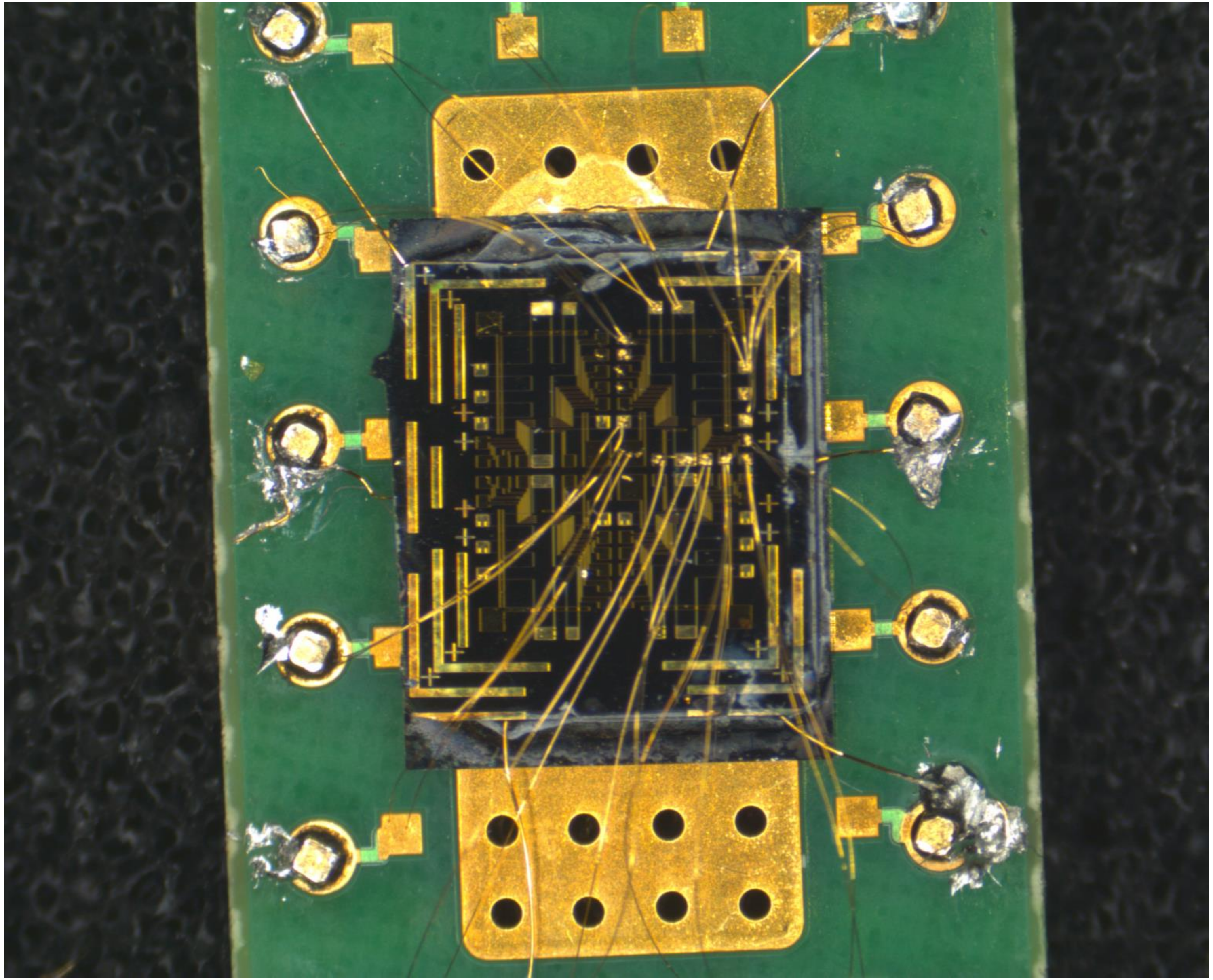


- Need to eliminate parallel conduction through screening wells
- Adapt technique used in bilayer systems – use gates around Ohmics to disconnect SWs from contacts

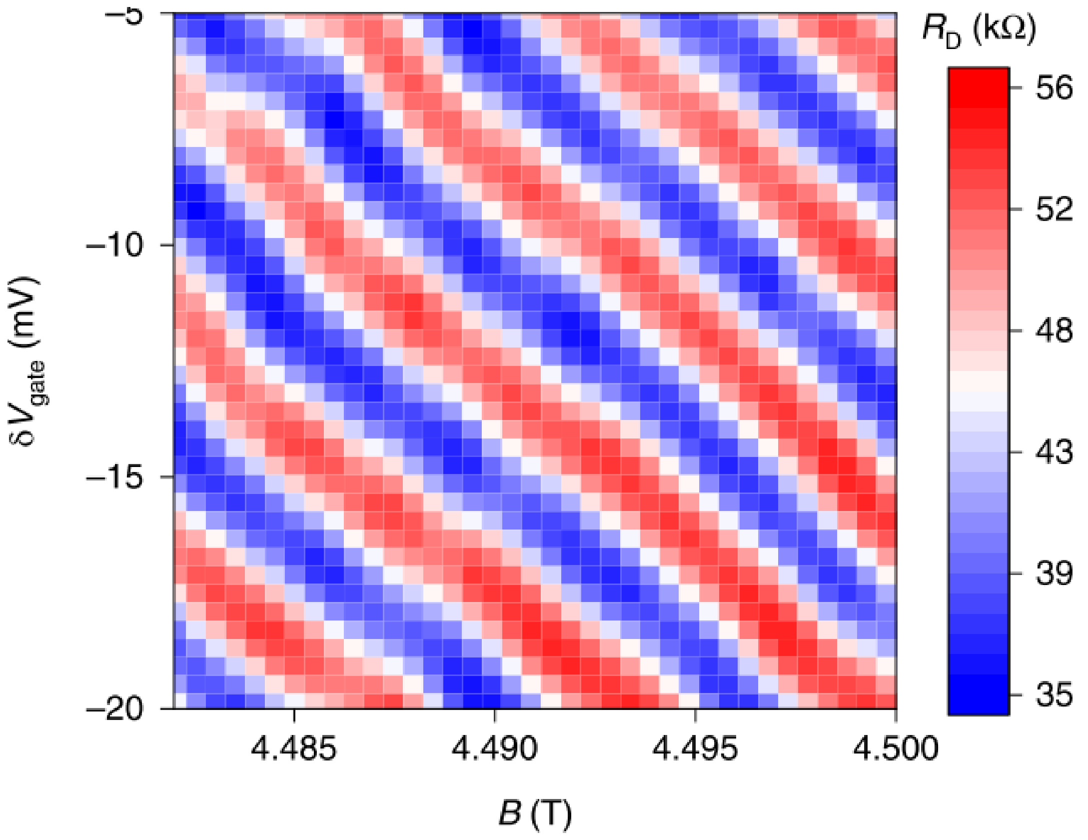
J. P. Eisenstein, L. N. Pfeiffer, & K. W. West. APL 57, 2324 (1990)

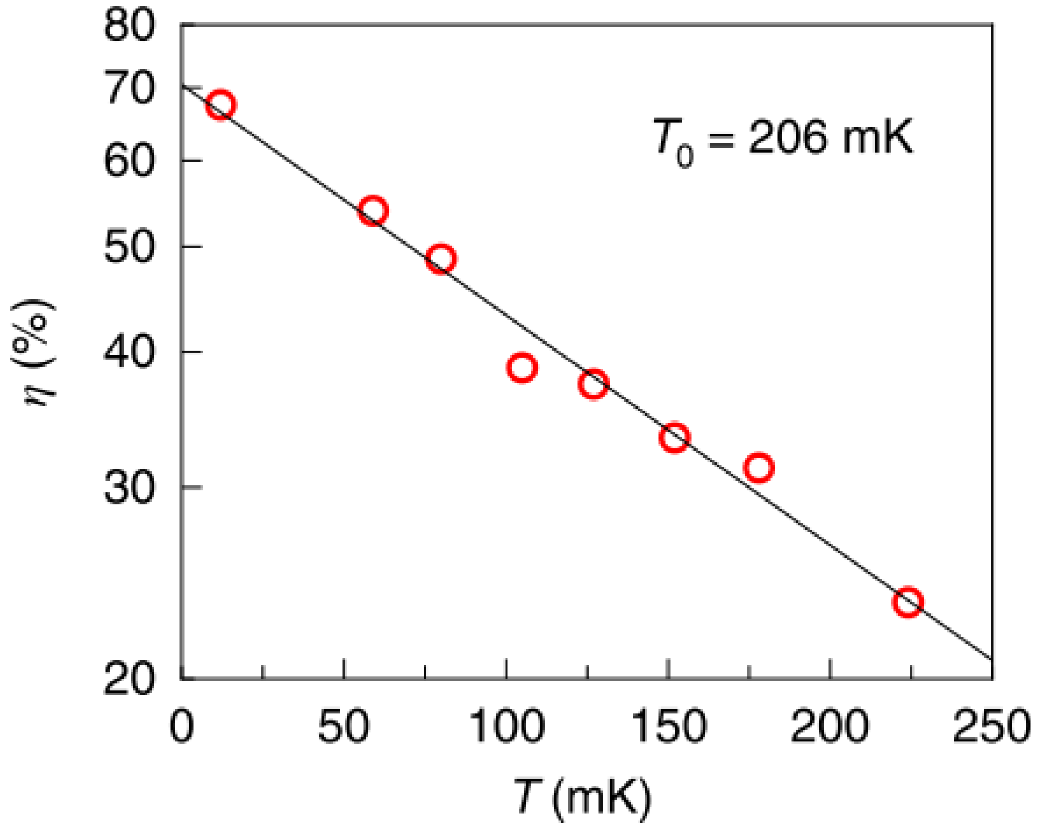






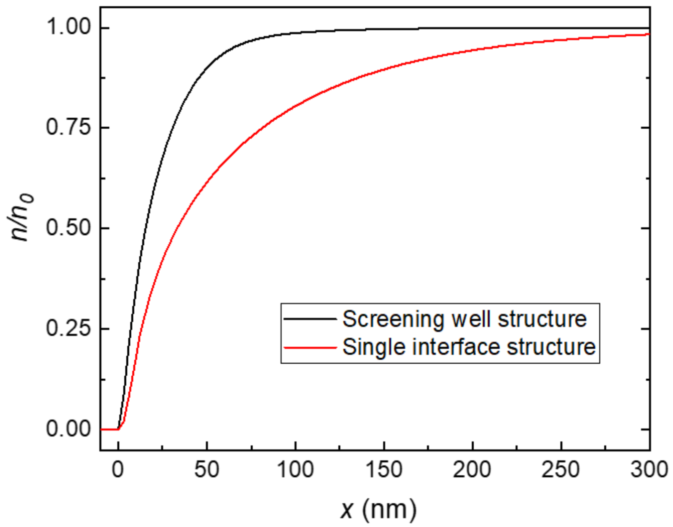
Aharonov-Bohm interference at $\nu = 1$

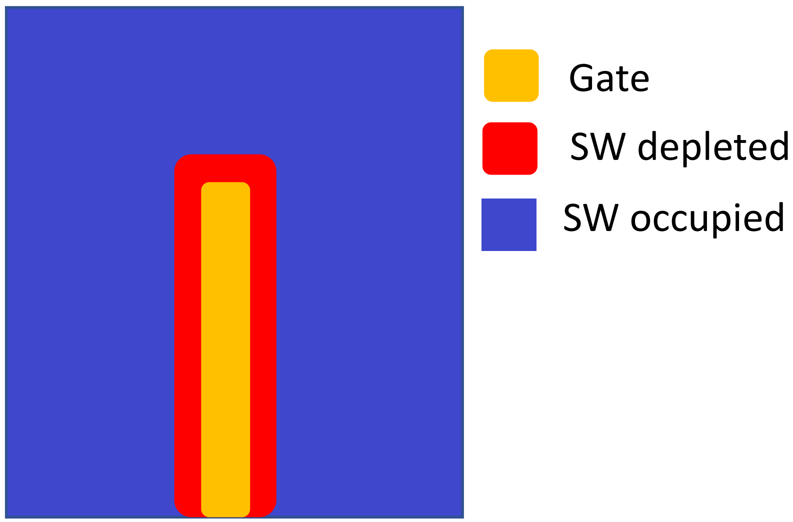




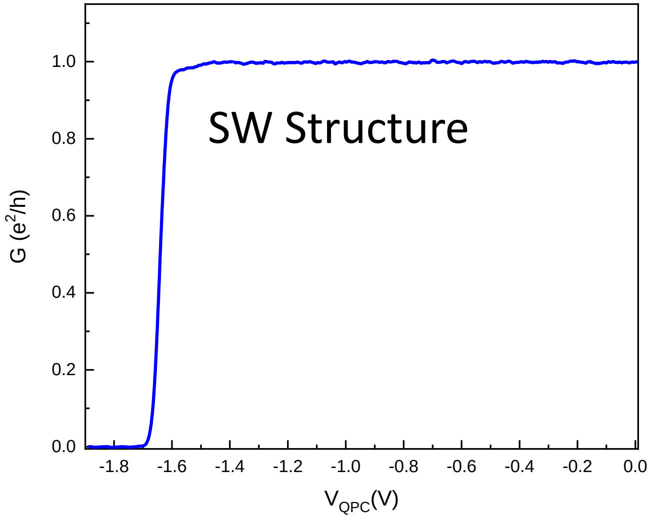
- Negative slope to constant phase lines – Coulomb charging suppressed
- Aharonov-Bohm interference in device ~20x smaller than possible with surface metal screening gate
- Interference is large amplitude and robust (survives up to hundreds of mK)

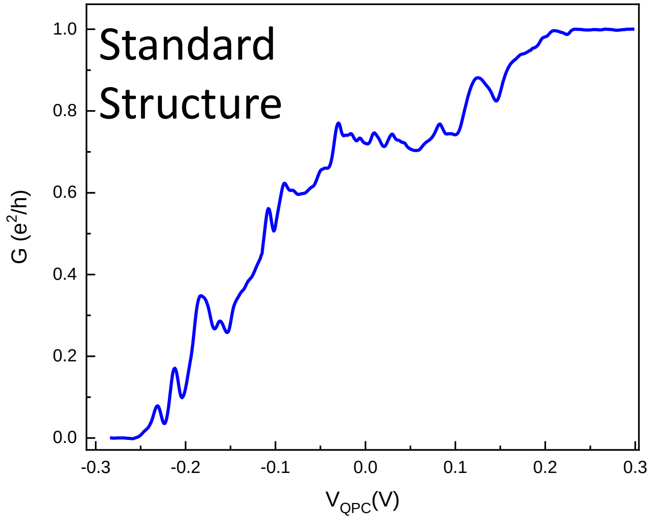
Sharper Confining Potential due to SW structure





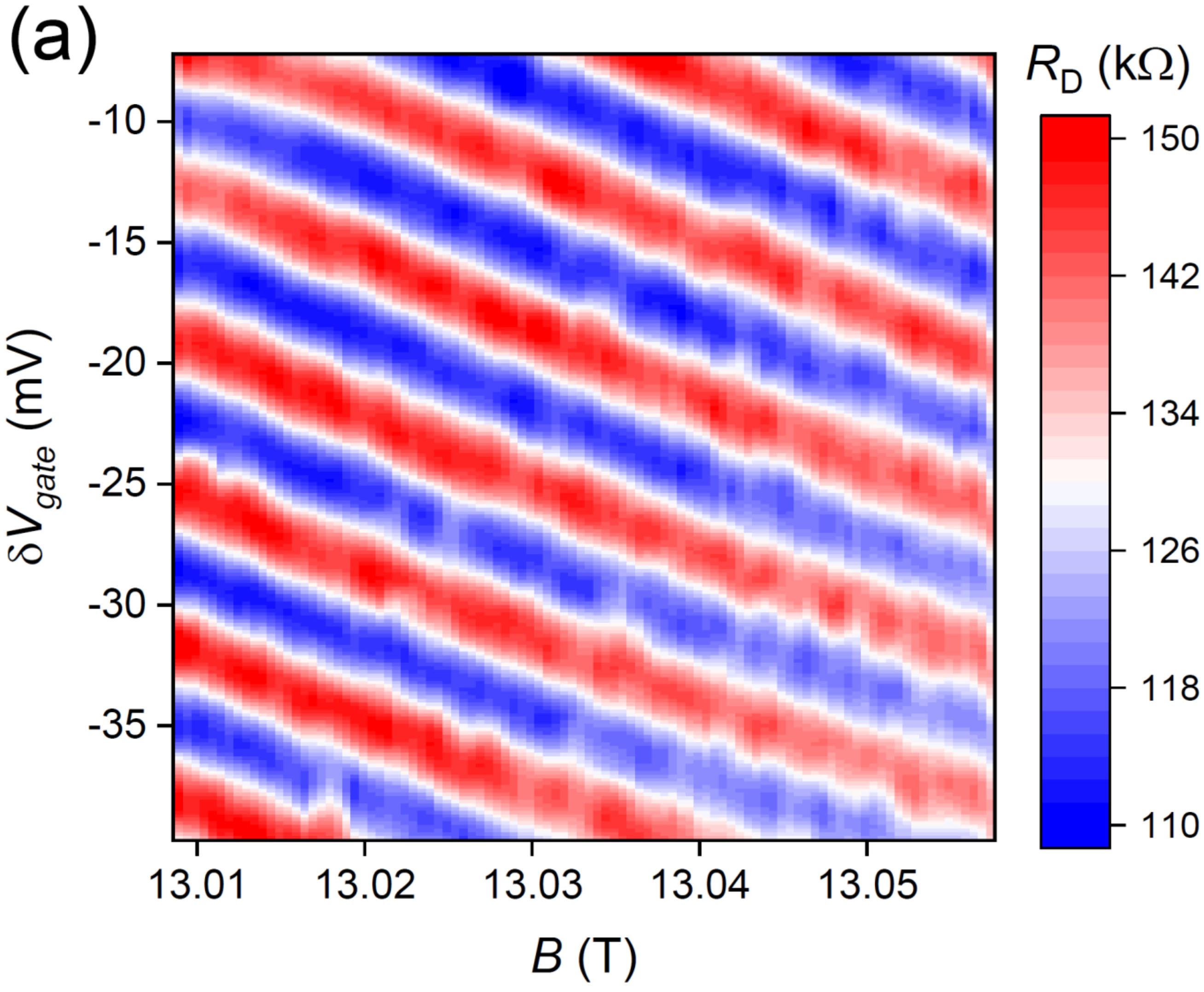
Single QPC sweep at $\nu = 1$





- Simulations indicate that SW structure results in a sharper confining potential at the edge of the gates
- Qualitatively, SW creates a “mask” so QW feels gate potential only in a sharply defined area
- QPCs exhibit much sharper conductance curves compared to standard structures

Aharonov-Bohm interference of e*=e/3 FQHE quasiparticles



$$\nu_{bulk} = \frac{1}{3}$$

$$\Delta B = 22.2 \text{ mT}$$

$$\Delta V_{gate} = 6.1 \text{ mV}$$

$$\frac{e^*}{e} = 0.29$$

ν=1/3

$$\frac{e^*}{e} = \frac{\Phi_0}{B \Delta V_g \frac{\partial A}{\partial V_g}}$$

nature physics
ARTICLES
https://doi.org/10.1038/s41567-019-0441-8

Aharonov–Bohm interference of fractional quantum Hall edge modes

J. Nakamura[1,2], S. Fallahi[1,2], H. Sahasrabudhe[1], R. Rahman[3], S. Liang[1,2], G. C. Gardner[2,4] and M. J. Manfra[1,2,3,4,5]*

Theoretical analysis: transition from incompressible to compressible droplet

B. Rosenow and A. Stern. *PRL* **124**, 106805(2020)

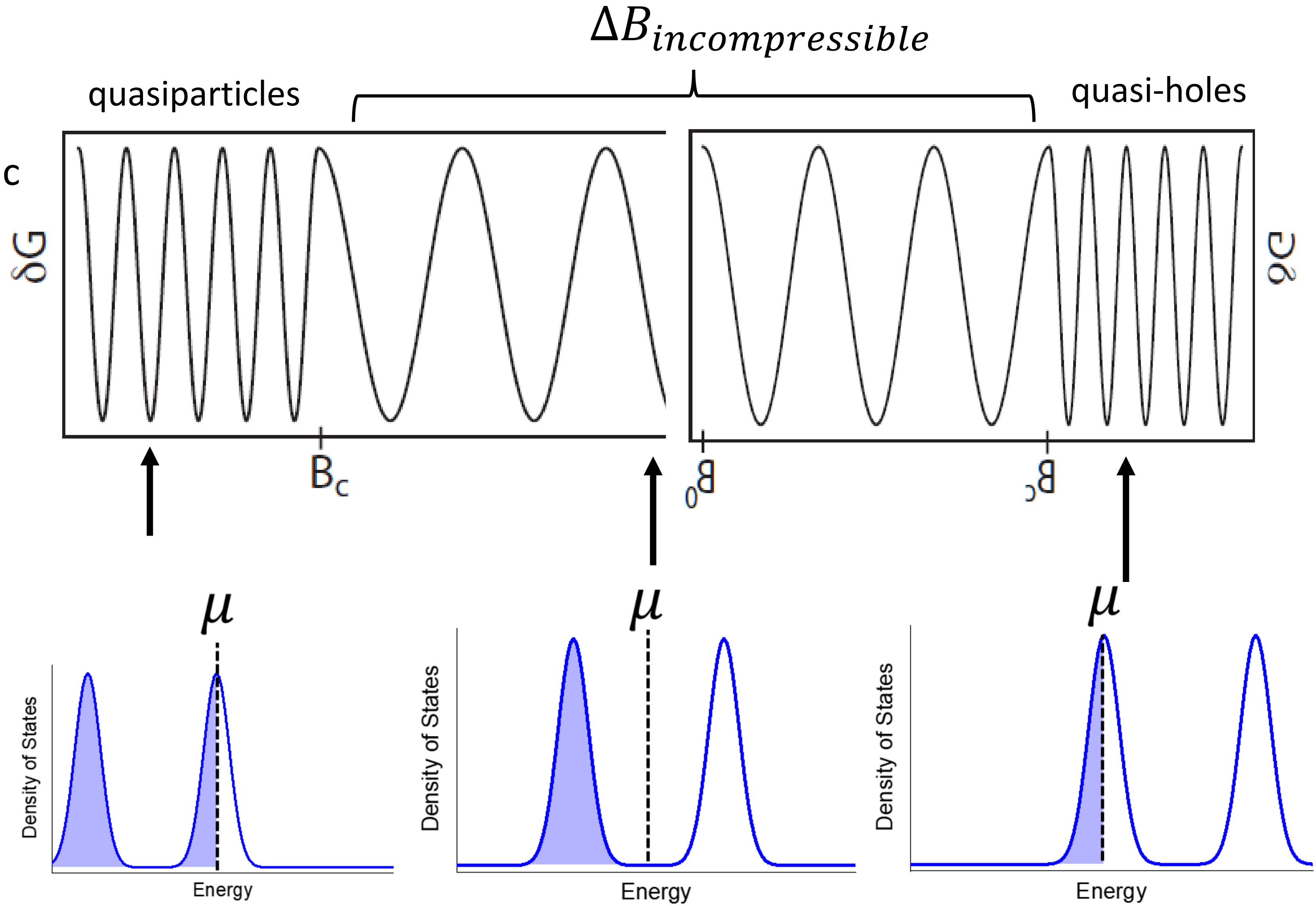
- Competition between energy cost to create quasiparticles Δ and electrostatic energy cost to keep ν fixed
- Predicted transition from AB (incompressible) with $3\Phi_0$ period to AB+qp creation with Φ_0 period (compressible bulk)

width in B with fixed ν where bulk is incompressible and $3\Phi_0$ oscillations:

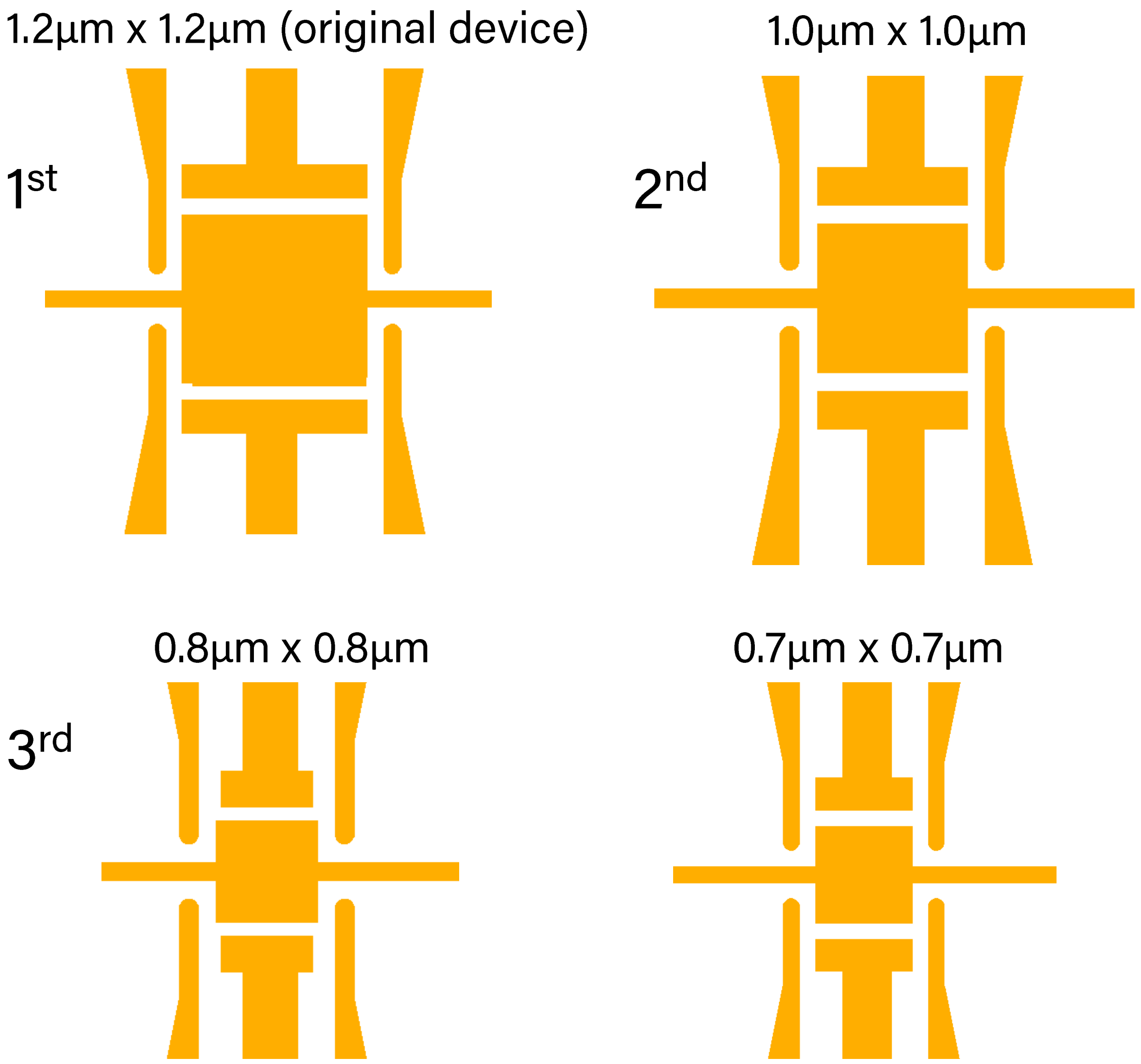
$$\Delta B_{incompressible} = \frac{\Delta \times \Phi_0}{\nu e^* \times \frac{e^2}{C}}$$

Δ = Energy gap of quantum Hall state

C = capacitance to screening layers (per unit area)



VARY DEVICE DIMENSIONS TO CHANGE ENERGY SCALES



- 1st device: AB at $\nu=1/3$
- 2nd device: anyonic phase slips
- 3rd device: analysis of compressible regime and coupling constants
- reduce 2DEG density

Observation of discrete phase slips at $\nu = 1/3$

- Primarily negative sloped constant-phase lines, but few discrete jumps in interference pattern

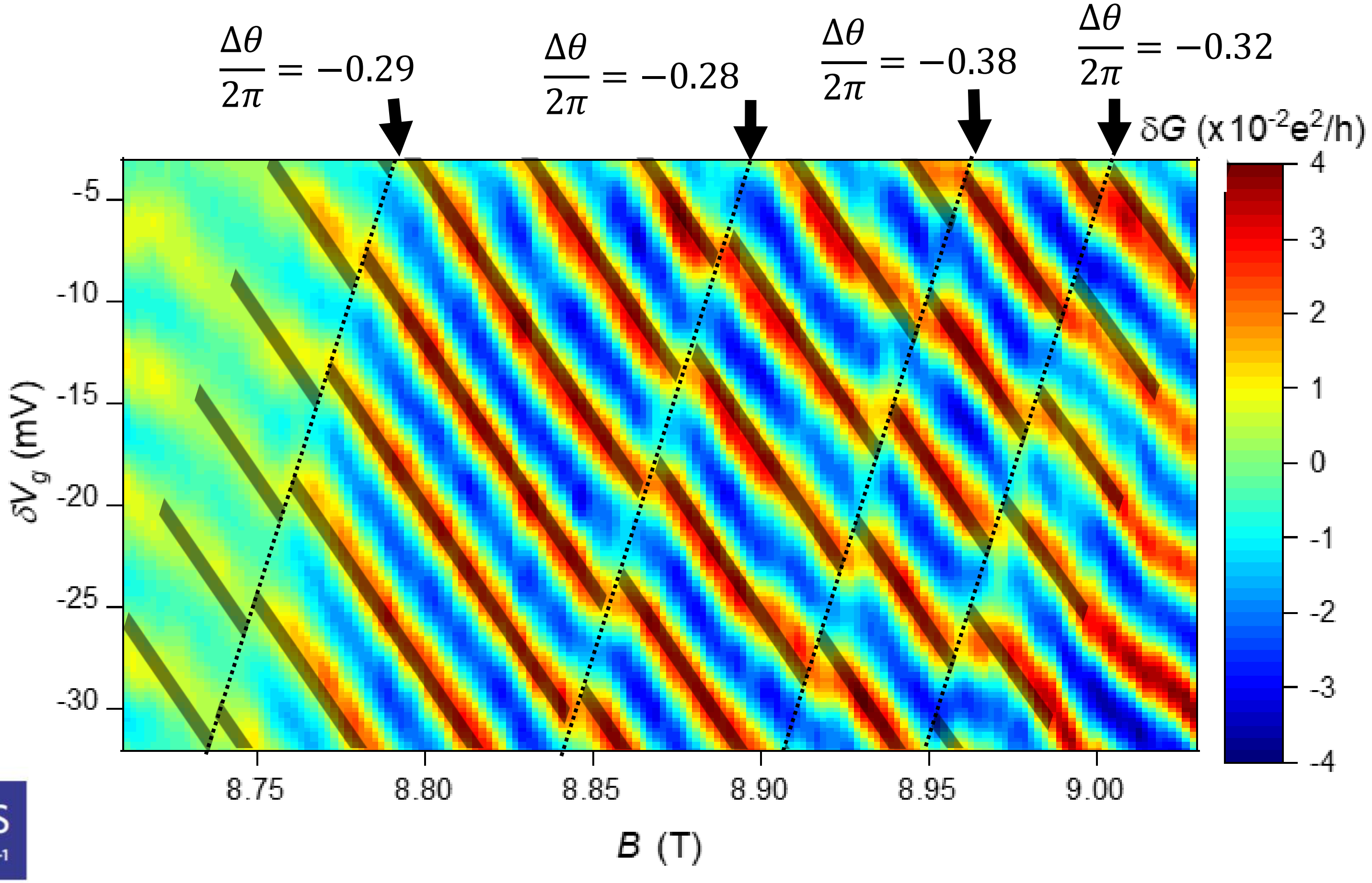
$$\theta = 2\pi \left(\frac{AB}{\Phi_0}\right)\frac{e^*}{e} + N_L\theta_{anyon}$$

- Both ΔB and ΔV_g indicate $e^* = \frac{1}{3}$

- Discrete jumps in phase: $\Delta\theta = -2\pi \times (0.31 \pm 0.04)$

  Theory: $\theta_{anyon} = \frac{2\pi}{3}$

- Negative sign consistent with removing QPs (or creating quasi-holes) with increasing B



nature physics

ARTICLES

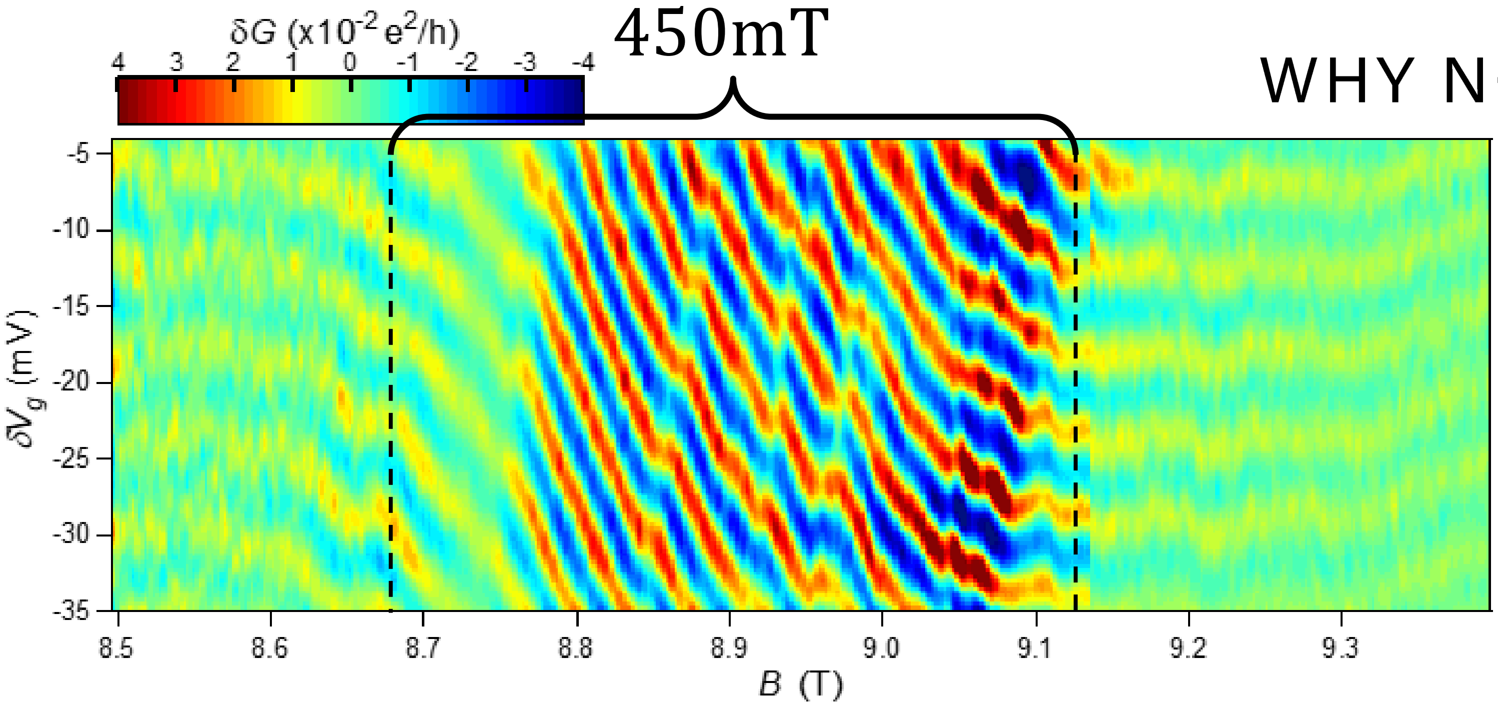
https://doi.org/10.1038/s41567-020-1019-1

Check for updates

Direct observation of anyonic braiding statistics

J. Nakamura[1,2], S. Liang[1,2], G. C. Gardner[2,3] and M. J. Manfra[1,2,3,4,5]

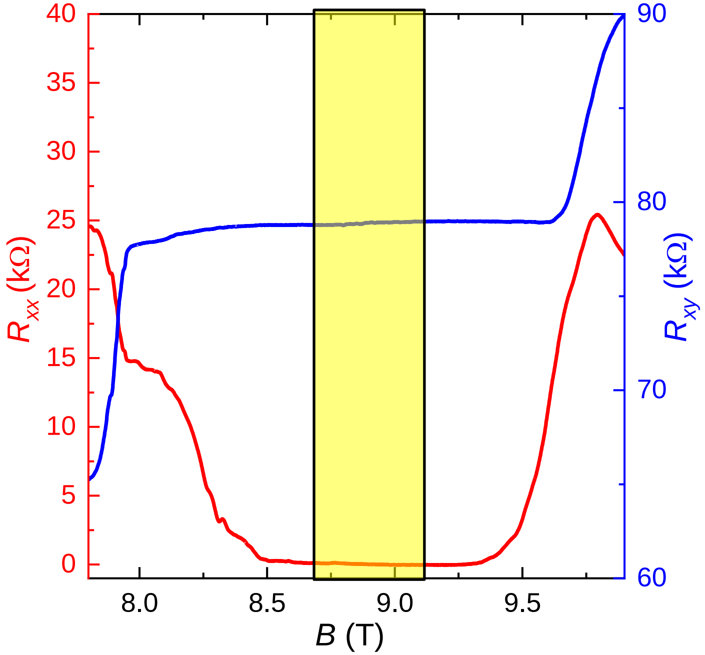
WHY NO Φ_0 PERIOD OSCILLATIONS?



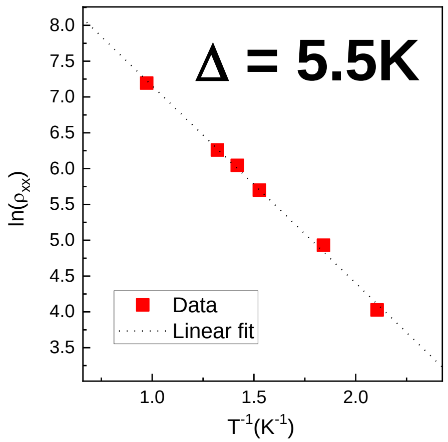
Rosenow & Stern predicted transition from $3\Phi_0$ AB period to Φ_0 period due to creation of quasiparticles (low field) or quasiholes (high field)

Change flux by Φ_0: $\Delta\theta_{AB} = \frac{2\pi}{3}$,

$$\Delta\theta_{anyon} = -\frac{2\pi}{3}$$



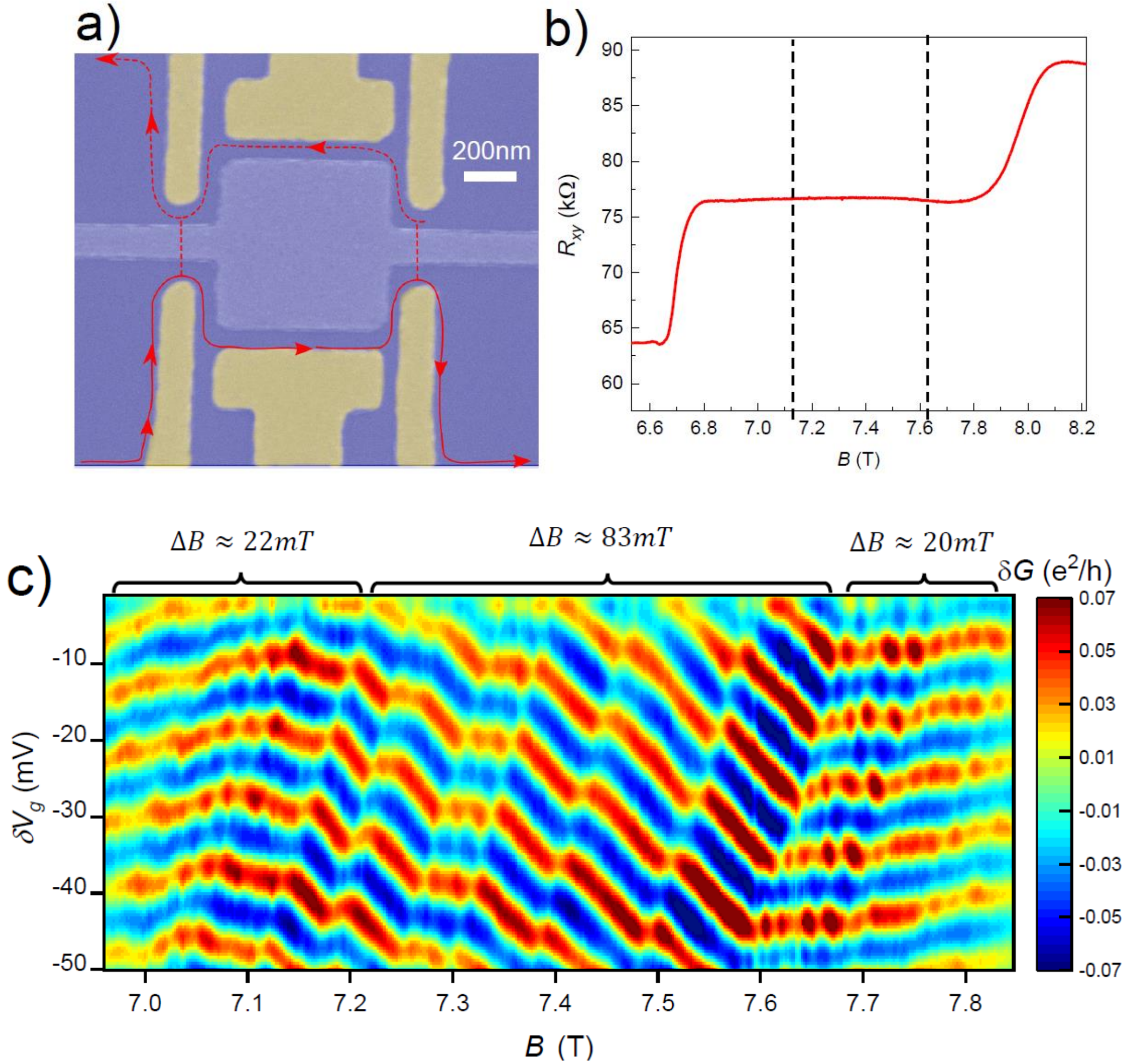
$$\frac{\Delta\times\Phi_0}{\nu e^*\times\frac{e^2}{C}} \approx \mathbf{530mT}$$



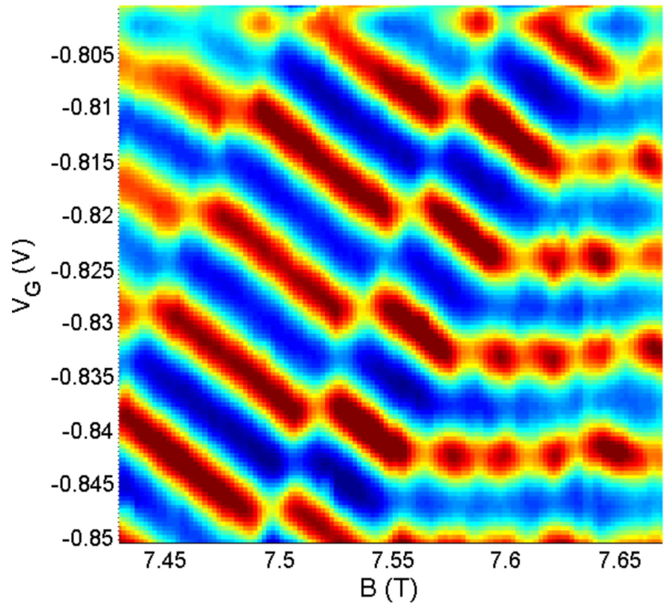
on average zero change in phase

While $\Delta\theta_{anyon}$ is discrete, quasiparticle number is likely thermally smeared – predicted temperature scale T ~ 4mK

Interference at $\nu = \frac{1}{3}$ in 800nm x 800nm Fabry-Perot interferometer



Extremely sharp phase slips &
Clear transition to compressible regime




nature COMMUNICATIONS

ARTICLE

https://doi.org/10.1038/s41467-022-27958-w OPEN

Impact of bulk-edge coupling on observation of anyonic braiding statistics in quantum Hall interferometers

J. Nakamura[1,2], S. Liang[1,2], G. C. Gardner[2,3] & M. J. Manfra[1,2,3,4,5]

Effect of bulk-edge coupling on observation of anyonic phase at $\nu = 1/3$

- Several discrete jumps in phase
- Average phase jump somewhat smaller previous device:

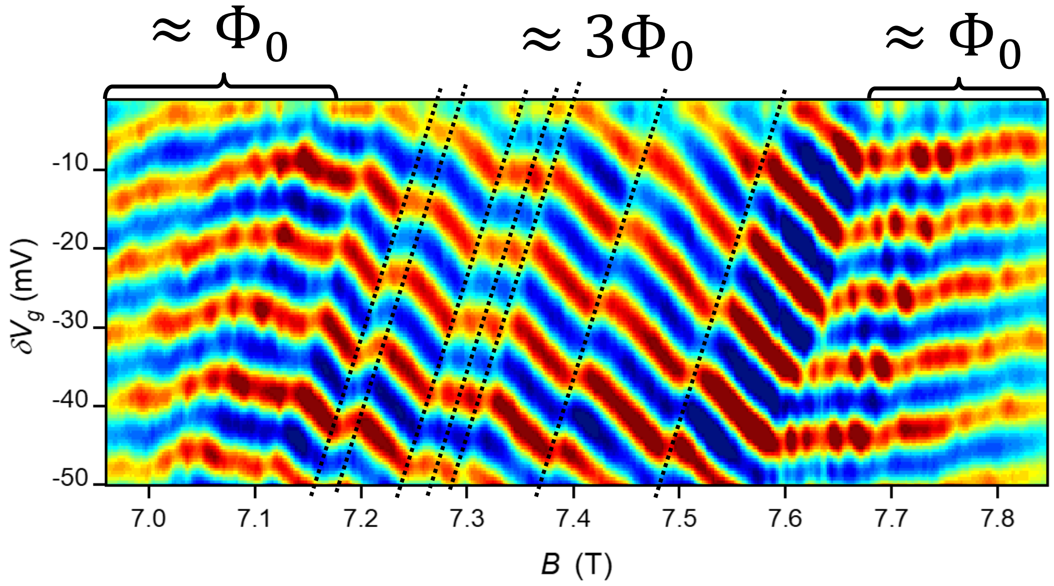
$$\frac{\overline{\Delta\theta}}{2\pi} = -0.24$$

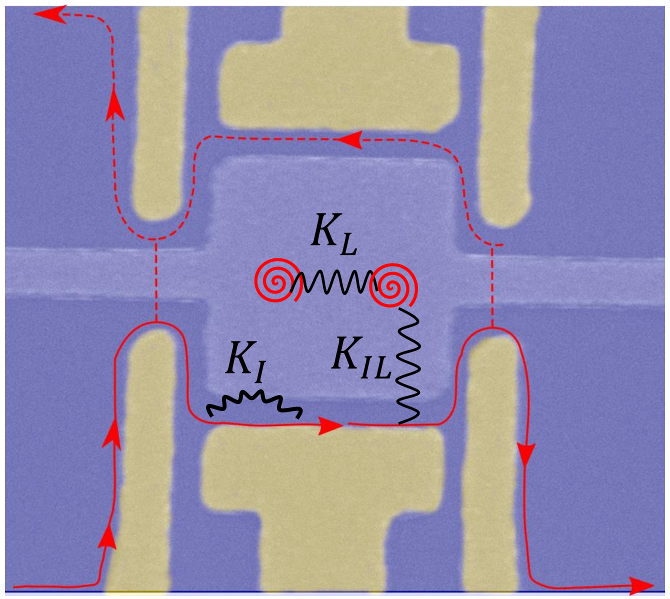
- Discrepancy explained by bulk edge coupling :

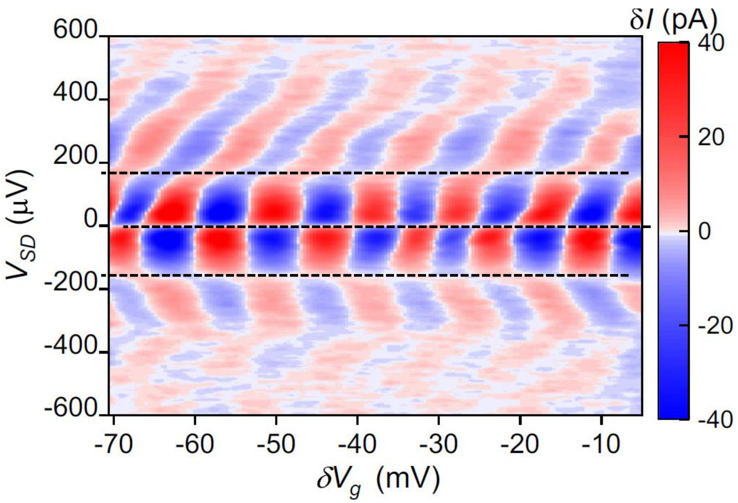
$$\Delta\theta = -\theta_{anyon} + 2\pi\left(\frac{K_{IL}}{K_I}\right)\frac{e^{*2}}{\Delta\nu}$$

$$\frac{\theta_{anyon}}{2\pi} = -\frac{\overline{\Delta\theta}}{2\pi} + \frac{1}{3}\frac{K_{IL}}{K_I} \approx \mathbf{0.33}$$

C. W. von Keyserlingk, S. H. Simon, B. Rosenow, ***PRL*** 115, 126807 (2015)





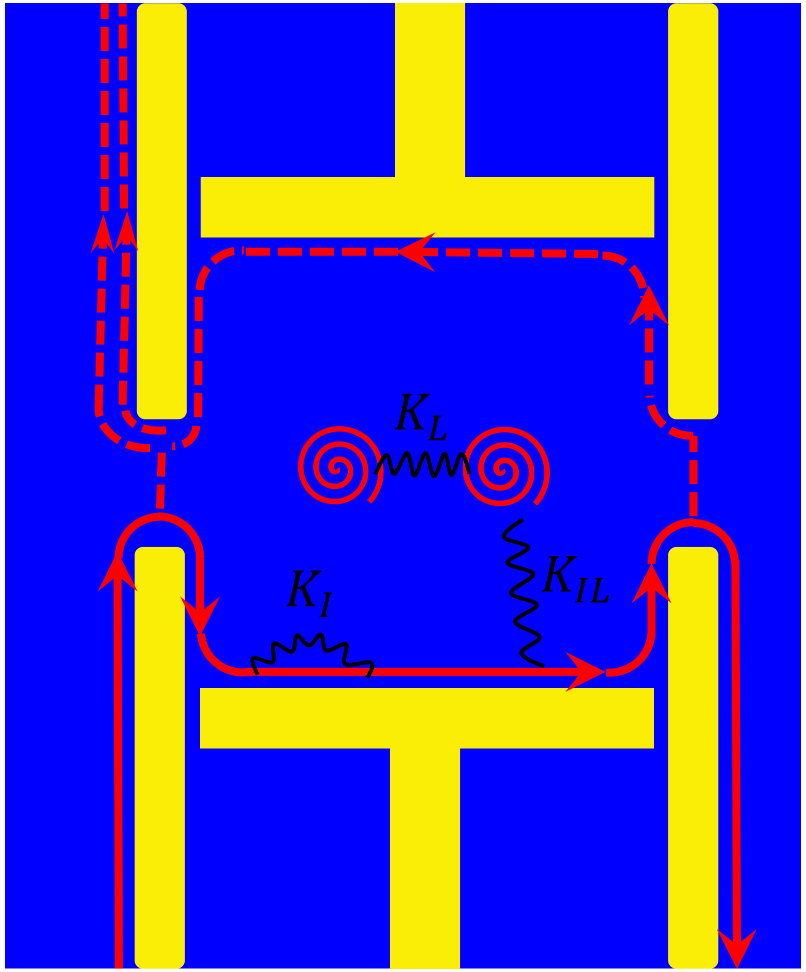


$K_I \approx 269\ \mu eV$

$K_{IL} \approx 72\ \mu eV$

$$\frac{\boldsymbol{K_{IL}}}{\boldsymbol{K_I}} \approx \mathbf{0.27}$$

ANALYSIS OF INTERFEROMETER COUPLING CONSTANTS



- Extensive theory on quantum Hall interferometers
  B. I. Halperin, A. Stern, I Neder, and B. Rosenow. PRB (2011)

- Area can change due to Coulomb interaction with charge in bulk. Electrostatic energy function:

$$E = \frac{K_I}{2}\delta n_I^2 + K_{IL}\delta n_I \delta n_L + \frac{K_L}{2}\delta n_L^2$$

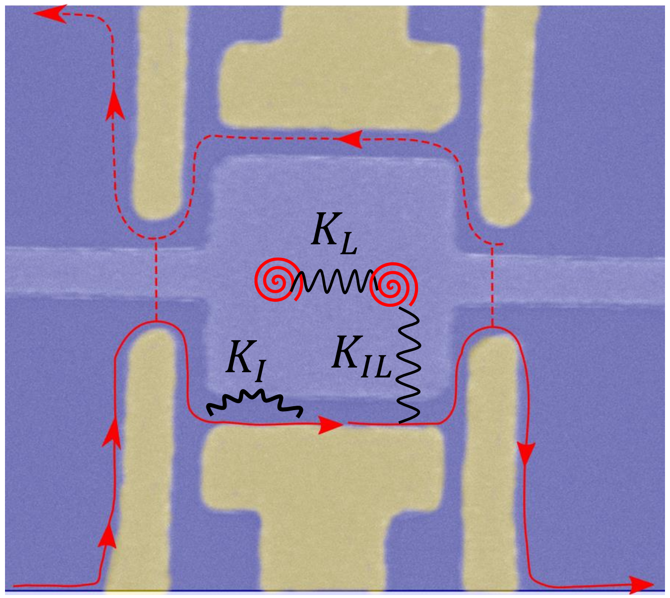
K_I: Edge stiffness: energy cost to vary the area of the edge state
K_L: Interaction of localized charges with each other
K_{IL}: Parameterizes bulk-edge coupling

- "modified" equation for interferometer phase:

$$\frac{\theta}{2\pi} = e^* \frac{\overline{A}B}{\Phi_0} - \frac{K_{IL}}{K_I}\frac{e^*}{\Delta\nu}\left(e^* N_L + \nu_{in}\frac{\overline{A}B}{\Phi_0} - \overline{q}\right) + N_L \frac{\theta_a}{2\pi}$$

Extract electrostatic coupling parameters at $\nu = 1$

Small Interferometer



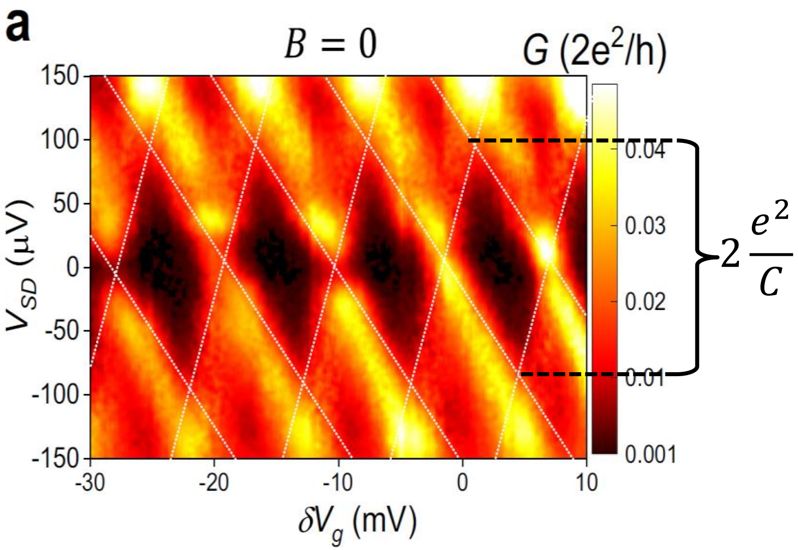
Estimate coupling constants by breaking energy into interacting part and single-particle energy:

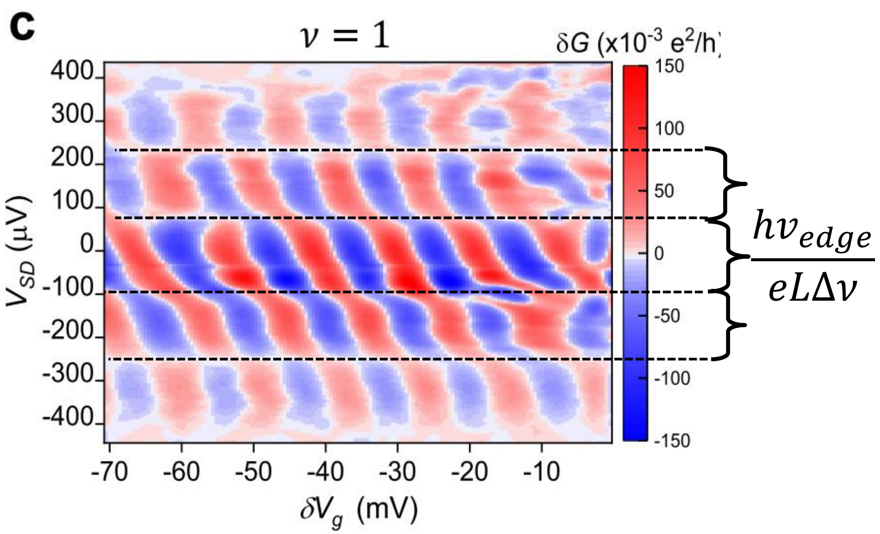
$$E = E_{\text{sp}} + E_{\text{int}} \qquad E_{sp} = \frac{\delta n_I^2}{2}\frac{hv_{edge}}{L\Delta\nu}$$

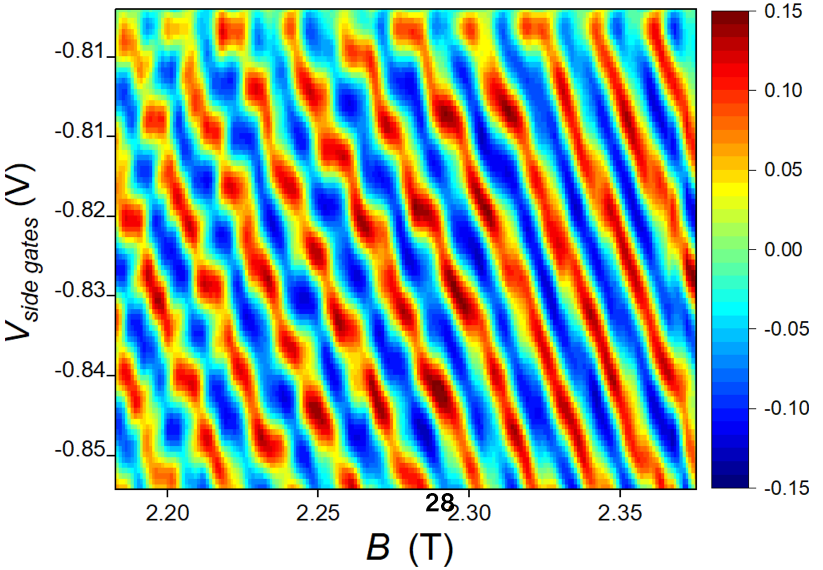
$$E_{\text{int}} = \frac{\delta q_{\text{total}}^2}{2C} = \frac{e^2}{C}\left(\frac{\delta n_I^2}{2} + \frac{\delta n_L^2}{2} + \delta n_L \delta n_I\right)$$

- Estimate E_{int} from B = 0 Coulomb diamonds
- Estimate E_{sp} from finite bias edge velocity measurement

$$\frac{\boldsymbol{K_{IL}}}{\boldsymbol{K_I}} \approx \mathbf{0.31}$$



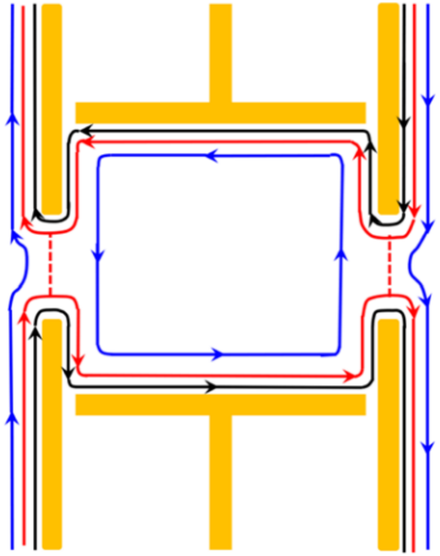


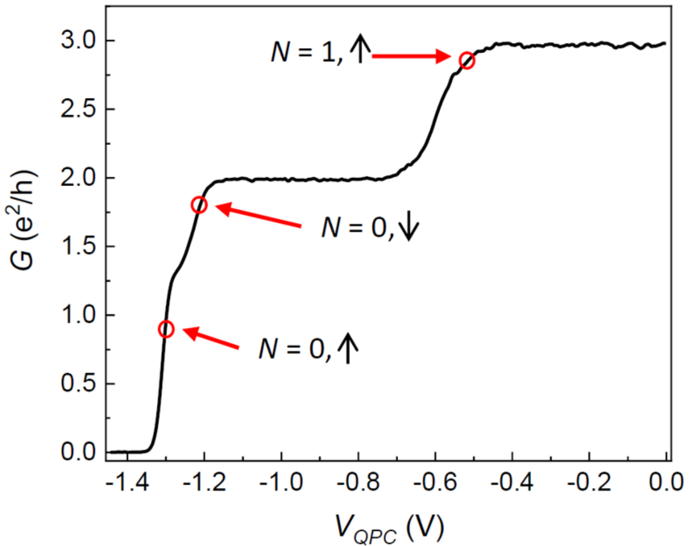


DIFFERENT BULK-EDGE COUPLING AT $\nu = 3$

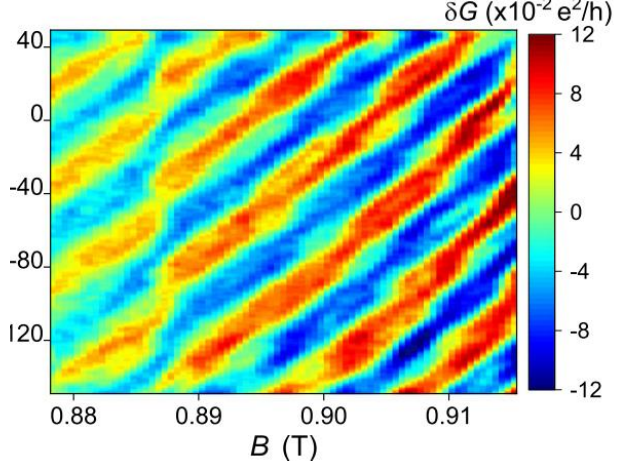
- Three edge states with different edge stiffnesses K_I, can be interfered independently
- Innermost edge: $\frac{K_{IL}}{K_I} = 0.62$, Coulomb dominated!
- Next outer edge: $\frac{K_{IL}}{K_I} = 0.35$, AB regime
- Outer edges have higher velocity and greater K_I
- Outermost edge: period-halving, edge-edge interaction relevant.

G. Frigeri, D. Scherer, and B. Rosenow. Sub-periods and apparent pairing in integer quantum Hall interferometers. EPL 126, 67007 (2019)

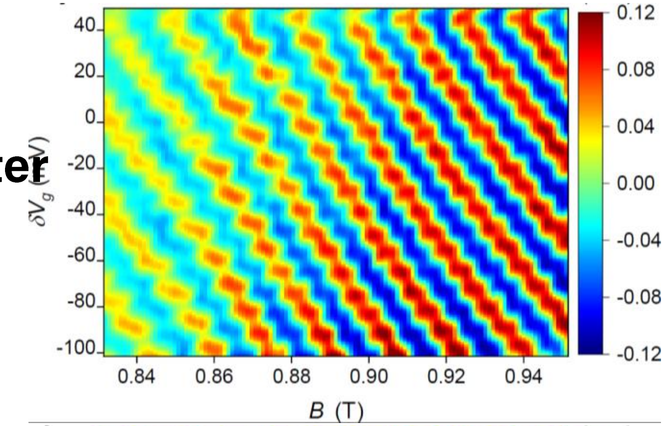




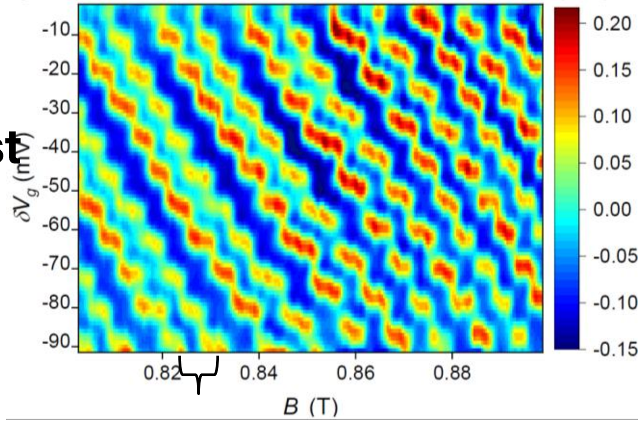
Innermost



Next-outer



Outermost



$\frac{\Phi_0}{2}$

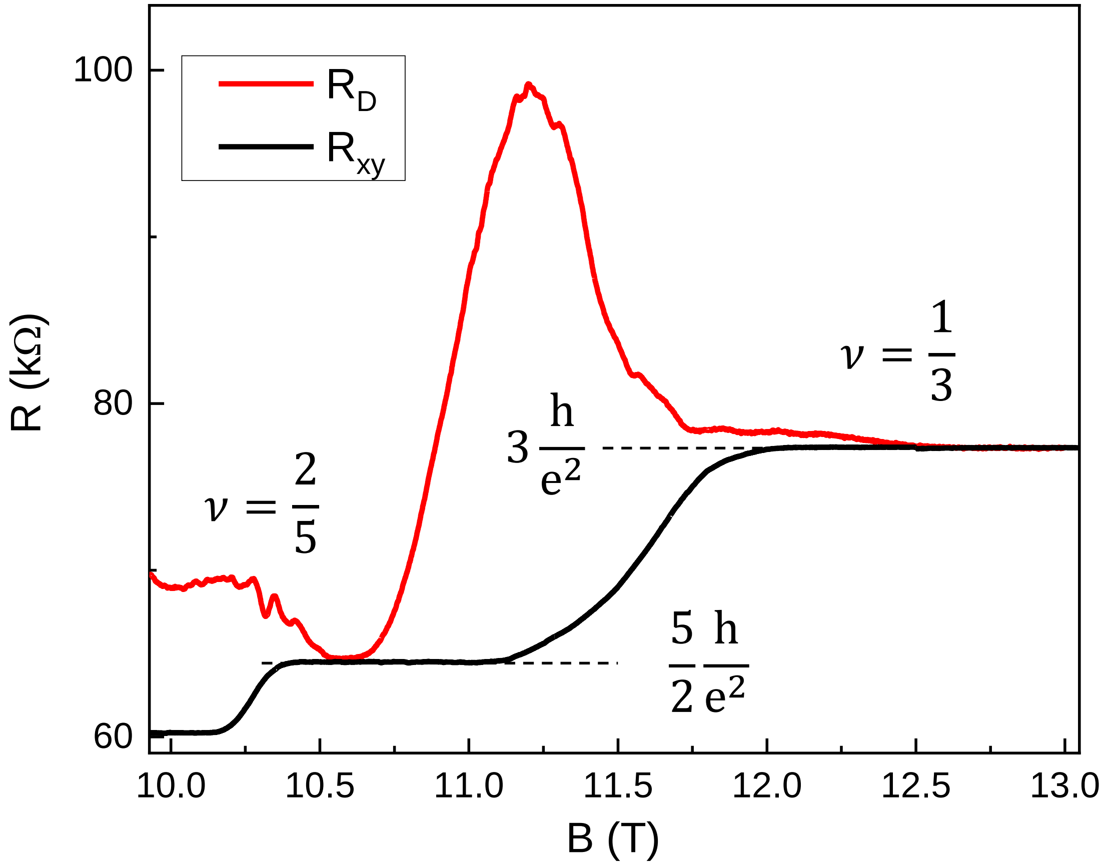
Can we extend the technique to more complicate fractions?

Robustness of quantum Hall interferometry

D. E. Feldman [1,2] and Bertrand I. Halperin [3]
[1] *Department of Physics, Brown University, Providence, Rhode Island 02912, USA*
[2] *Brown Theoretical Physics Center, Brown University, Providence, Rhode Island 02912, USA*
[3] *Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*

(Received 4 March 2022; accepted 20 April 2022; published 28 April 2022)

R_{XY} AND R_D ACROSS DEVICE WITH QPCS JUST DEPLETED

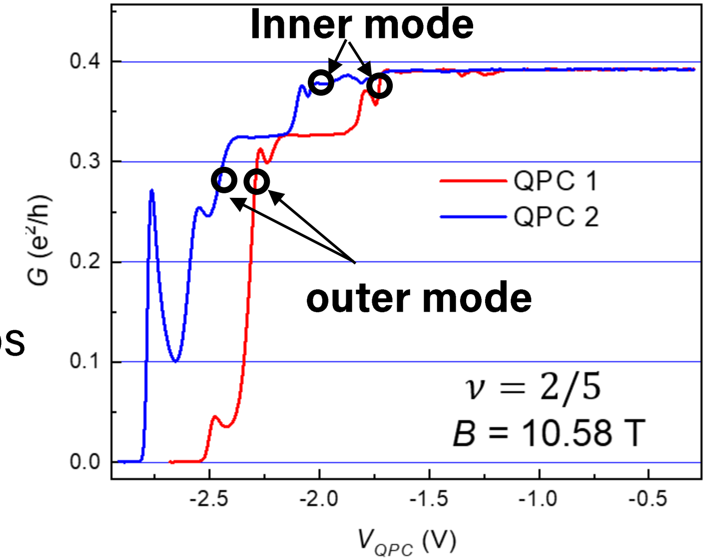


INTERFERENCE AT 2/5

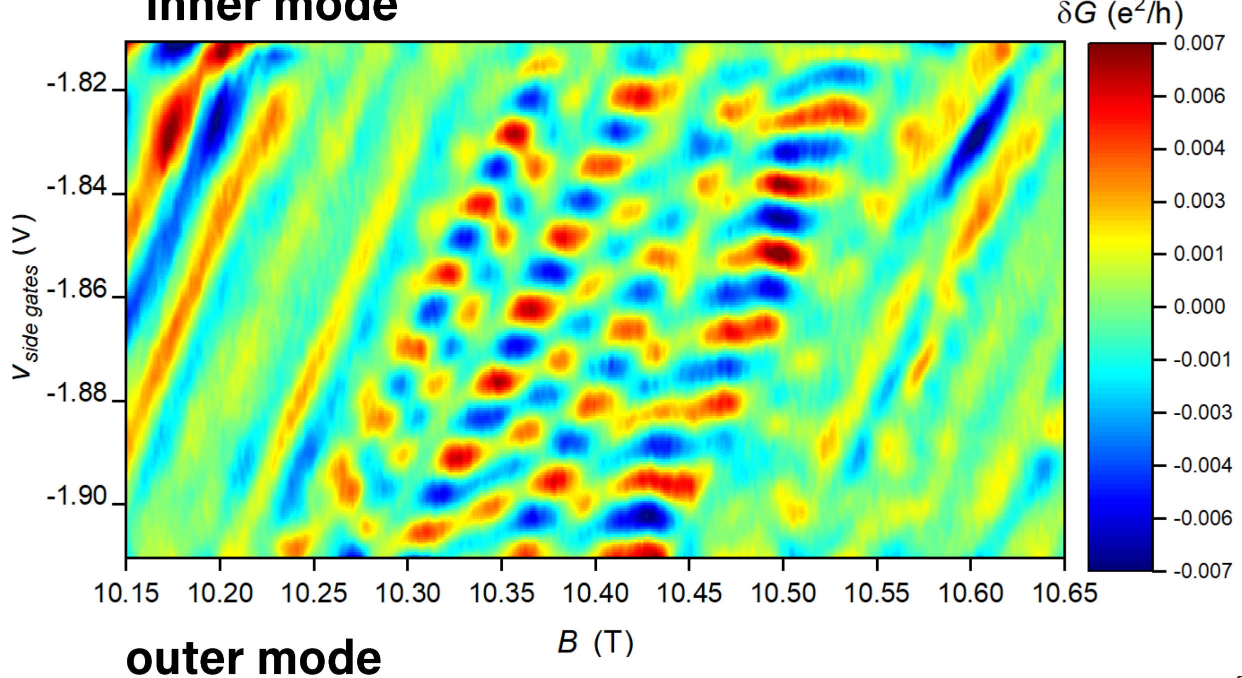
- 1um x 1um interferometer with screening well heterostructure, n ~ 1x10[11] cm-2, effective area $A \approx 0.36\ \mu m^2$ from $\nu = 1$ period
- QPCs show a clear primary plateau and intermediate plateau, suggesting two separate edge states
- Interference vs. B and V_{SG} shows very different behavior when QPCs are tuned to interfere each mode
- Inner mode has large V_{sg} period and large phase jumps, outer mode looks similar to $\nu = 1/3$

Single QPC sweeps and approximate operating points

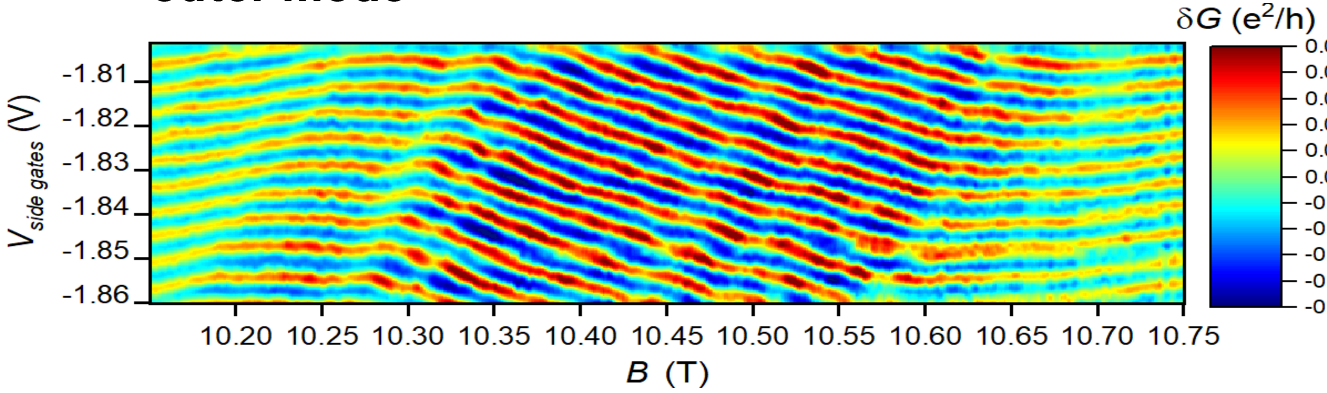
Single QPC sweeps



inner mode



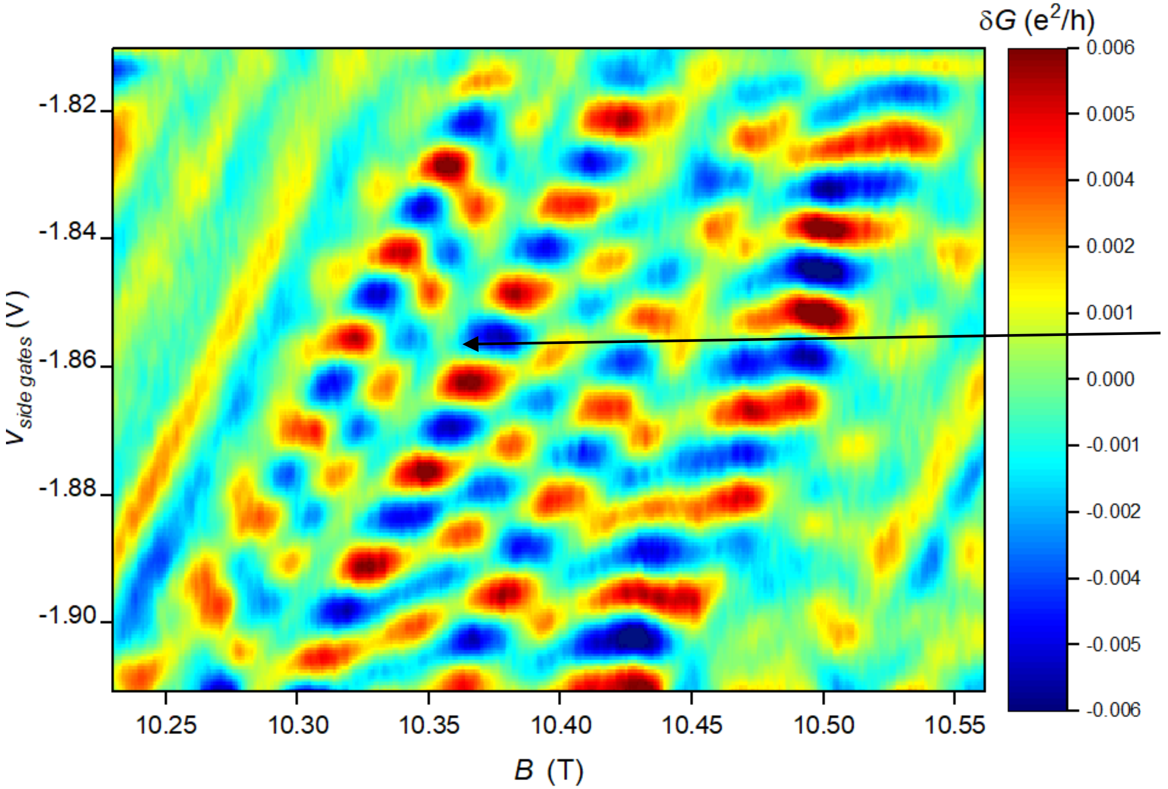
outer mode

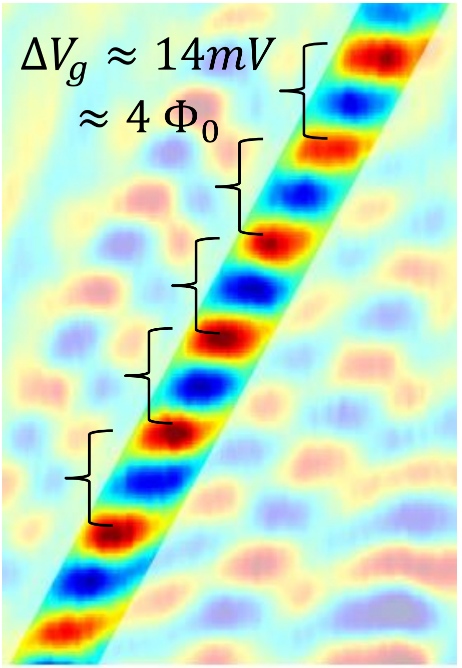


(the plots here are sized so that the x and y scales are approximately the same, although the color scale is different by more than a factor of 10)

INTERFERENCE OF INNER MODE AT 2/5

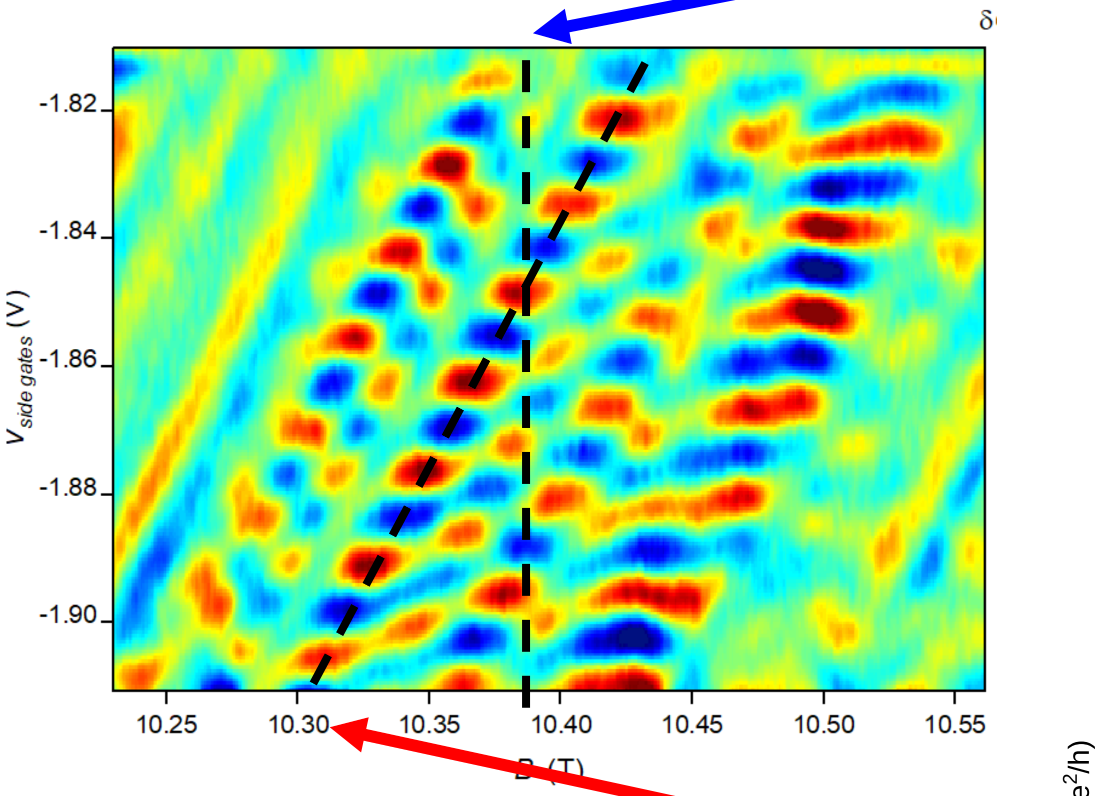
- Naïve expectation: $\theta = \frac{2\pi e^* AB}{\Phi_0} + N_{qp}\theta_a$ with $e^* = \frac{1}{5}$ and $\theta_a = 2\pi \times \frac{-2}{5}$
- Observed: oscillations w/ large V_{sg} period and weak B dependence interrupted by discrete jumps
- In regions between discrete jumps, period is ~14mV, implying ~4 flux quanta per oscillation, e* ~ 0.25 (neglecting bulk-edge coupling effects)
- Discrete jumps have slope ~0.75 mV/mT, consistent with transitions in localized QP number
- Spacing of discrete jumps is nonuniform, with average value ~25mT or ~2.5 Φ_0, suggesting that these oscillations occur in an incompressible region



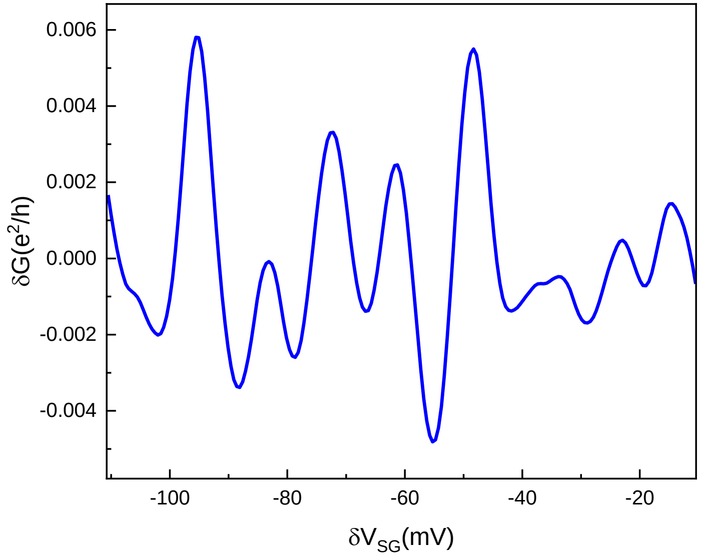


Line cuts (vertical vs diagonal)

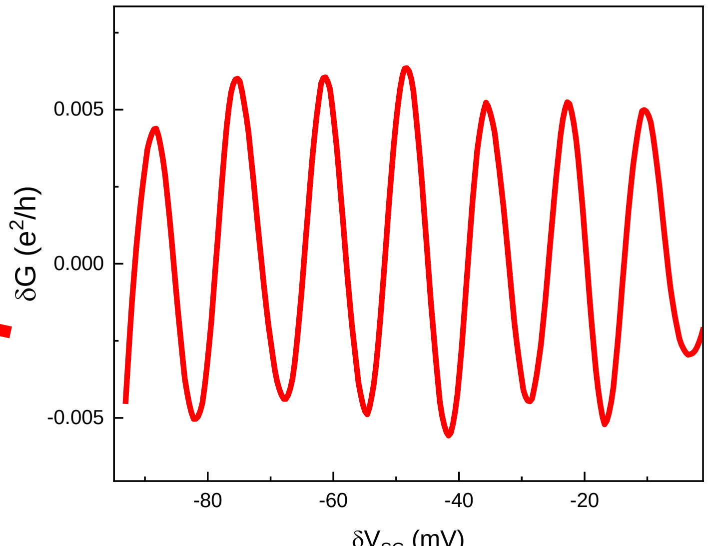
- Vertical line cuts of G vs. V_{SG} show non-sinusoidal behavior due to unevenly spaced quasiparticle transitions
- Diagonal line cuts parallel to discrete jumps show sinusoidal oscillations since QP number is fixed, and only AB phase changes
- Contours of constant quasiparticle number have slope ~0.75 mV/mT, close to the value expected based on the device area and lever arms ($\frac{\delta V_{SG}}{\delta B} = \frac{\nu A}{\Phi_0 \alpha_{bulk}} \approx \mathbf{0.6\ mV/mT}$).



Vertical line cut



Diagonal line cut



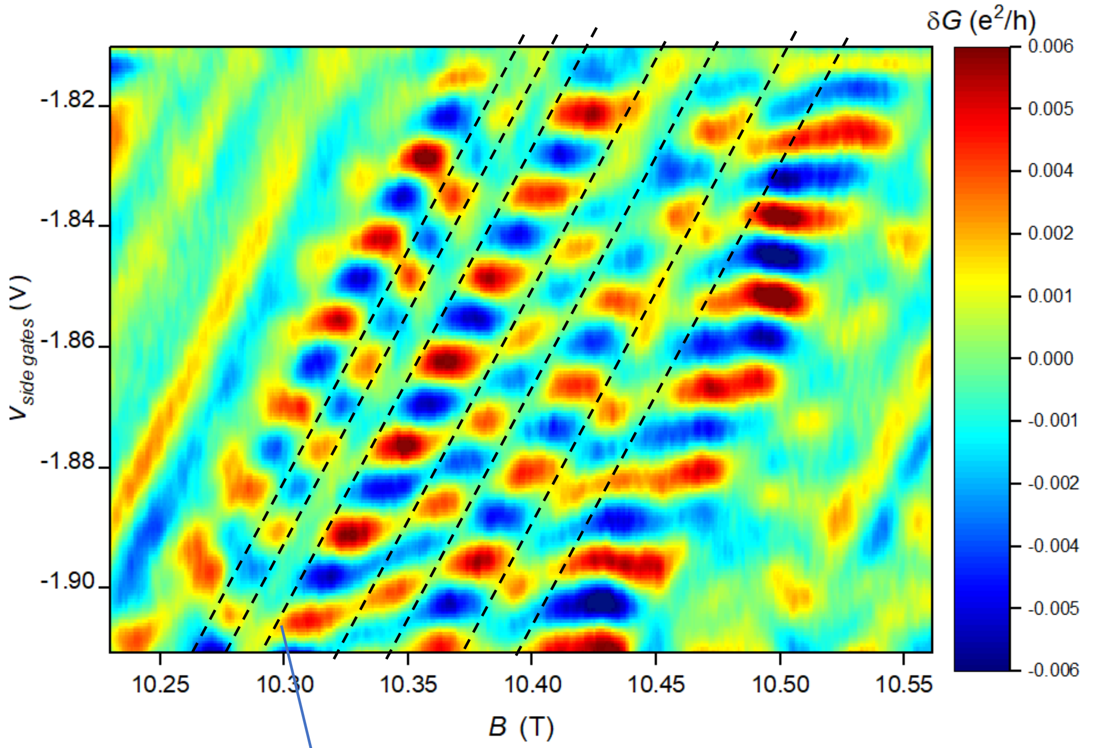
Analysis of discrete jumps for inner mode

- The discrete jumps have different values, with some positive and some negative, clustered around ~$\pm 0.5 \times 2\pi$
- The theoretical value for removing an e/5 QP would be $\Delta\theta = -\theta_a = +\frac{2}{5} \times 2\pi$, but this will be modified by bulk edge coupling:

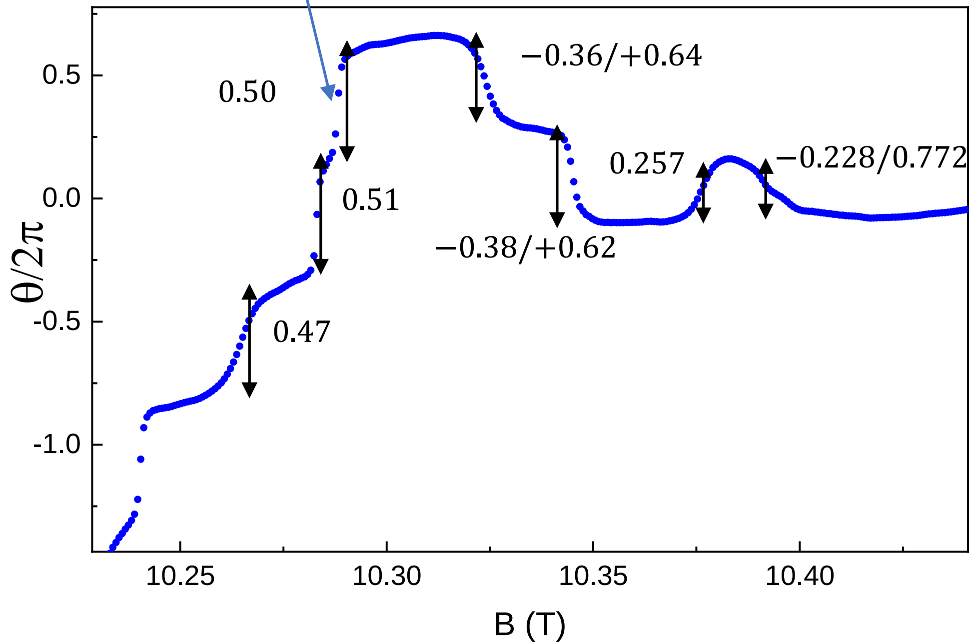
$$\frac{\Delta\theta}{2\pi} = -\theta_a + e^*_{int} e^*_{local} \frac{1}{\Delta\nu} \frac{K_{IL}}{K_I} = \frac{2}{5} + \frac{3}{5} \frac{K_{IL}}{K_I}$$

- Defining phase from 0 to 2π, the average is $\frac{\Delta\bar{\theta}}{2\pi} = 0.54$, consistent with a moderate degree of bulk edge coupling
- A bulk-edge coupling of this size is also consistent with the lines of constant phase being nearly flat as a function of B between the discrete jumps as well as the gate voltage period being slightly smaller than $5\Phi_0$

Using $\kappa = 0.2$ inferred from finite bias measurements (see following slides), $\boldsymbol{\theta_a = -\Delta\bar{\theta} + \frac{3}{5}\kappa = -0.43 \times 2\pi}$

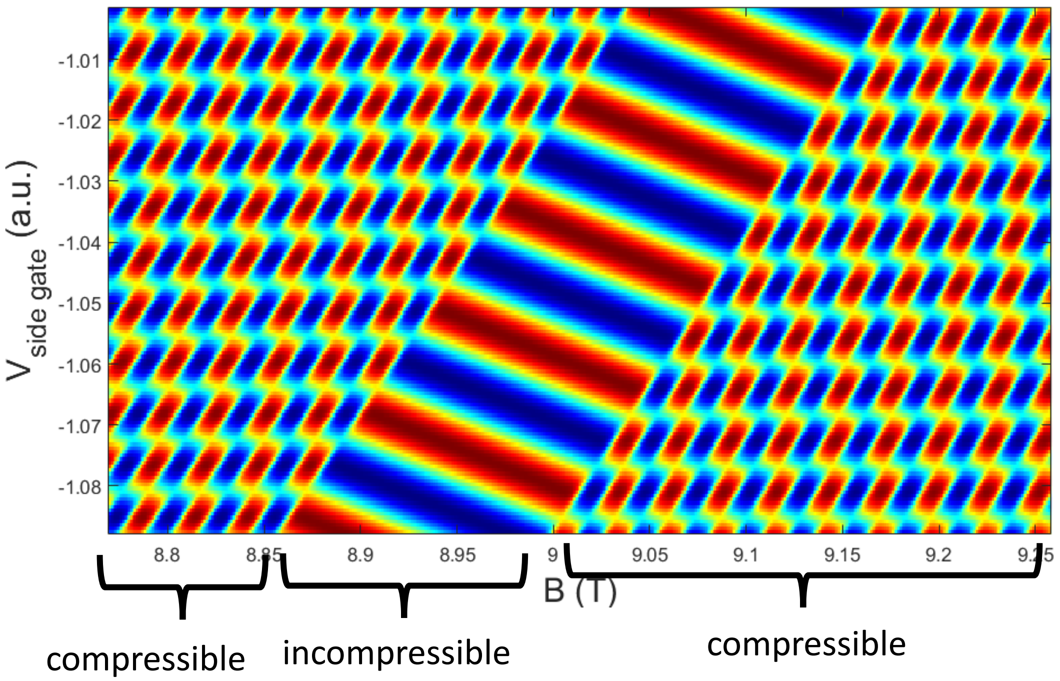


Phase extracted via FFT

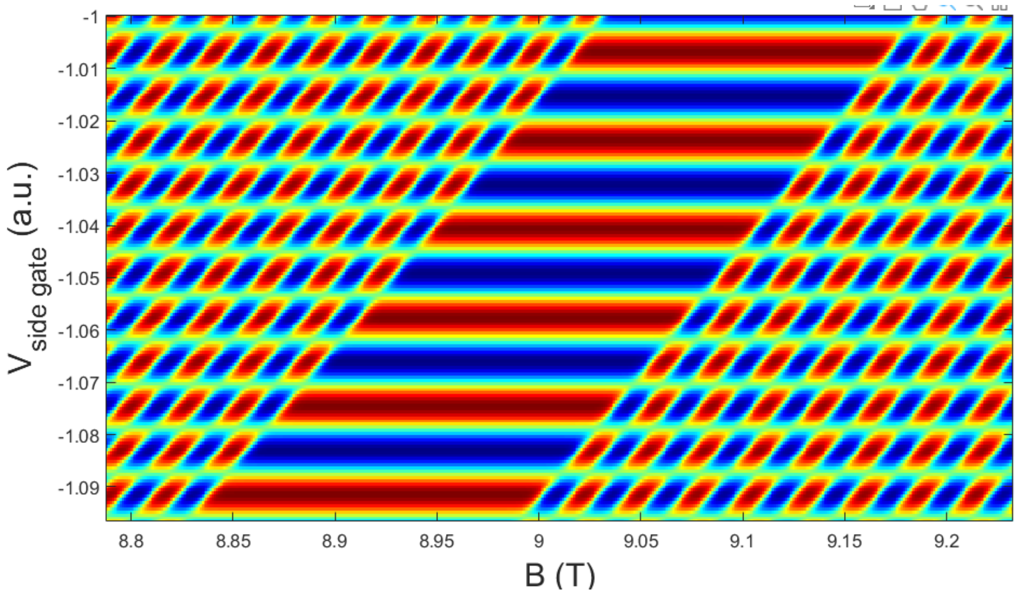


Simulations of inner mode interference at 2/5

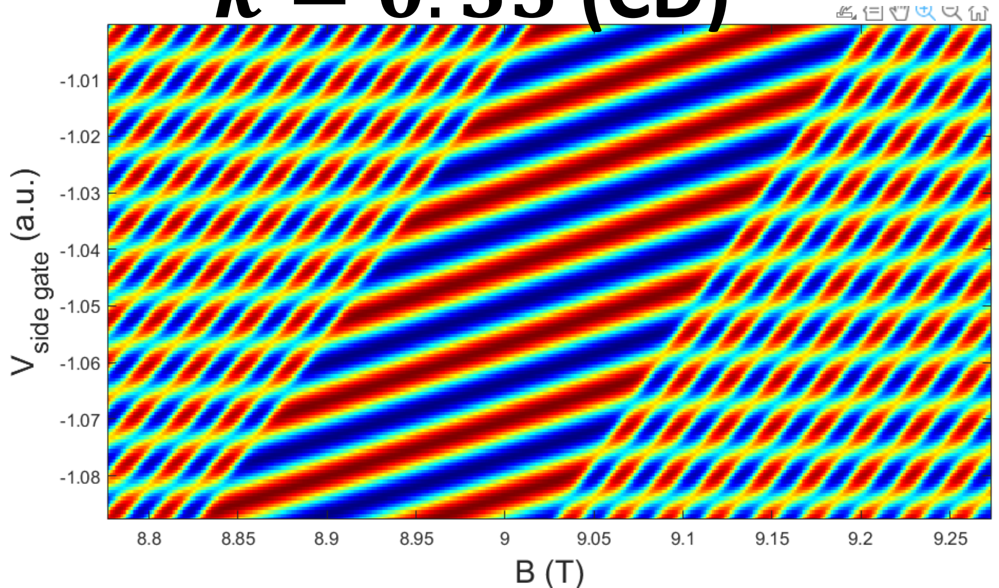
$\kappa = 0$ (pure AB)



$\kappa = 0.167$



$\kappa = 0.33$ (CD)

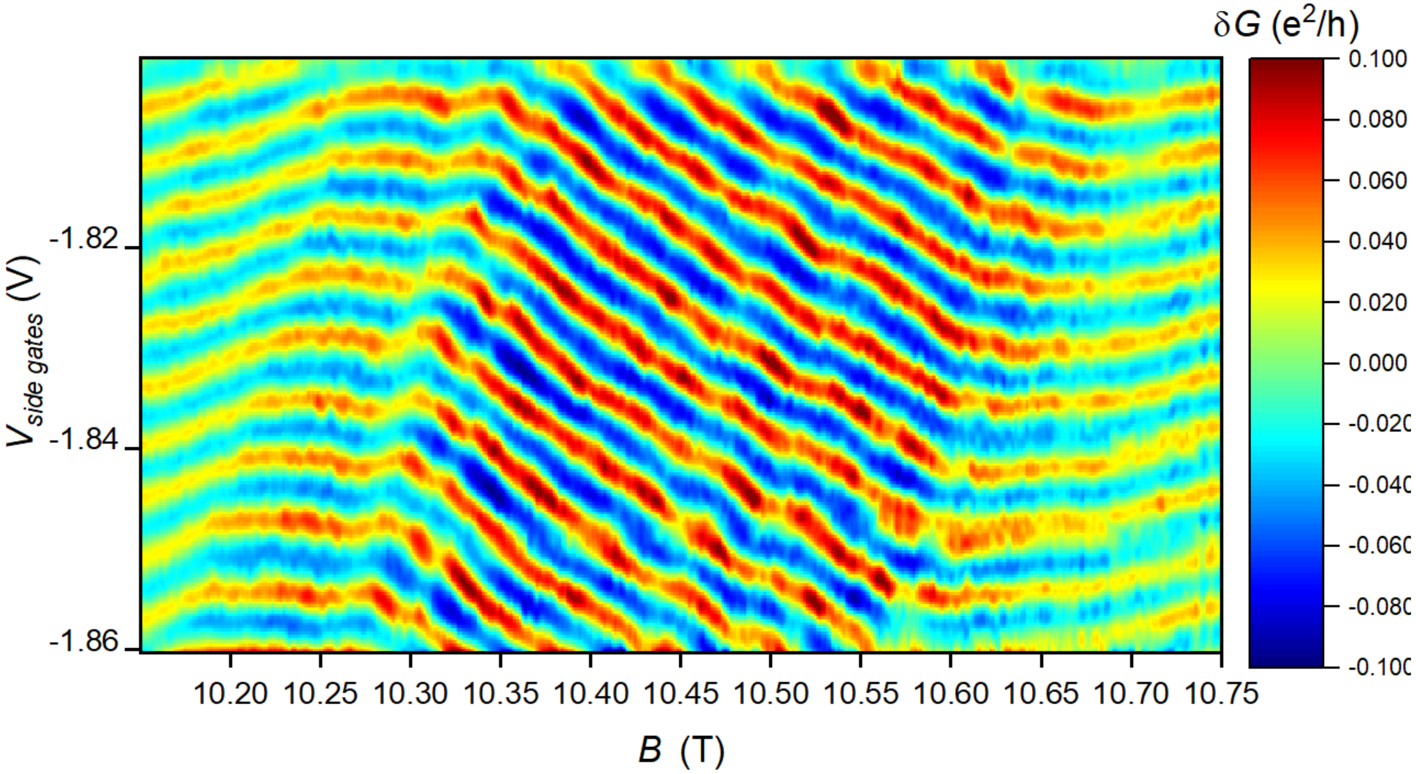


Qualitatively, our data resembles the $\kappa = 0.167$ case (close to the transition from AB to CD) with a mostly incompressible bulk, but several discrete jumps due to disorder. This value of κ is consistent with our finite bias measurements (assuming the model is right), with the average value of discrete jumps, and with the fact that between jumps the lines of constant phase are nearly flat.

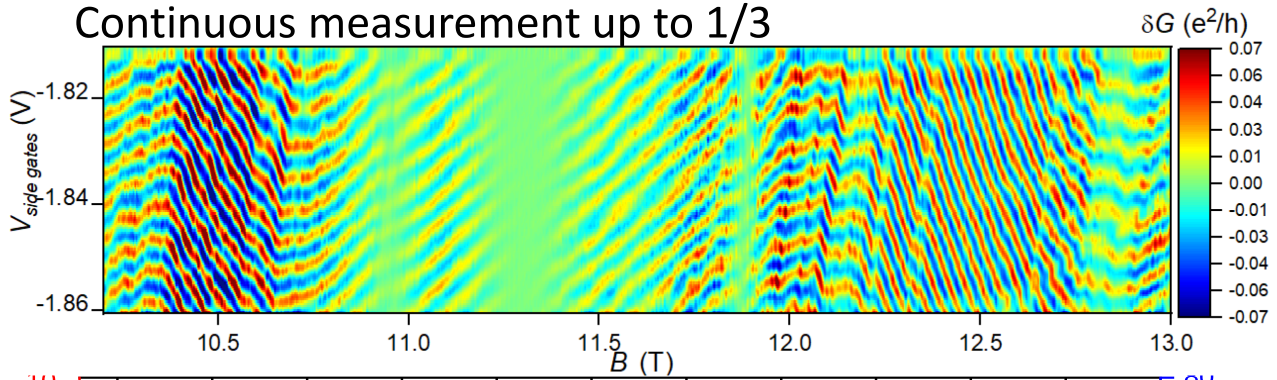
We don't see evidence of fully compressible regimes (where e/5 QPs are created with $\frac{\Phi_0}{2}$ period), but maybe the transitions are too strongly thermally smeared, or the bulk conductivity dephases too much when the DOS is high.

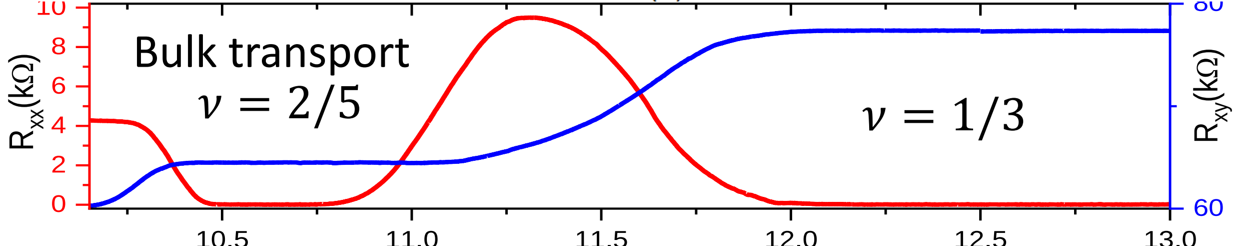
Interference of outer mode at 2/5

- Outer mode exhibits central region with negative slope consistent with (partial) incompressibility at 2/5, similar to 1/3.
- The range of magnetic field for the incompressible region is similar to the inner mode
- Some quasiparticles seem to be created in incompressible region, but discrete jumps not as clear (complicated charge re-arrangements between edge and e/5 QPs in localized 2/5 puddle)
- Outer mode interference is continuous up to $\nu = 1/3$, but greatly suppressed amplitude when bulk is conducting



Continuous measurement up to 1/3





Thank you!