

# MULTI-COMPONENT SUPERCONDUCTIVITY: CHIRAL, NEMATIC, AND CHARGE-4e SUPERCONDUCTING STATES

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Science

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## References for these lectures

- *Phenomenological superconductivity (review + intro):*

*Sigrist & Ueda, Rev. Mod. Phys. **63**, 239 (1991)*

*Sigrist, AIP Conf. Proc. **789**, 165 (2005)*

- *Vestigial orders (review):*

*RMF, Orth, & Schmalian, Annu. Rev. Condens. Matter Phys. **10**, 133 (2019)*

- *Recent results discussed in these lectures:*

*RMF & Fu, Phys. Rev. Lett. **127**, 047001 (2021)*

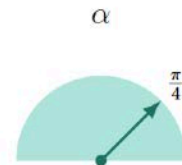
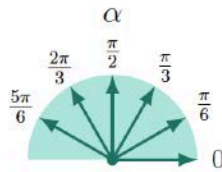
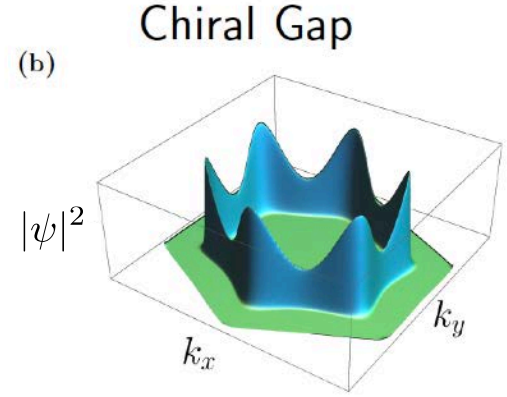
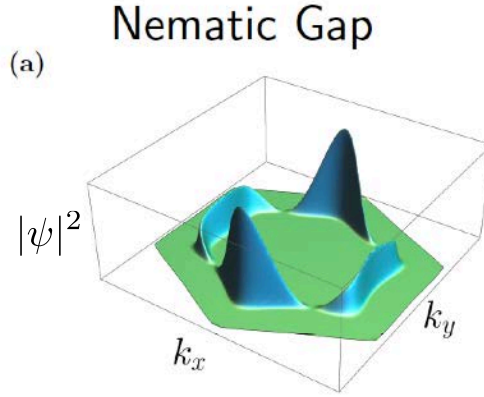
*Gali & RMF, Phys. Rev. B **106**, 094509 (2022)*

*Hecker, Willa, Schmalian, & RMF, Phys. Rev. B **107**, 224503 (2023)*

# Nematic and chiral superconductivity

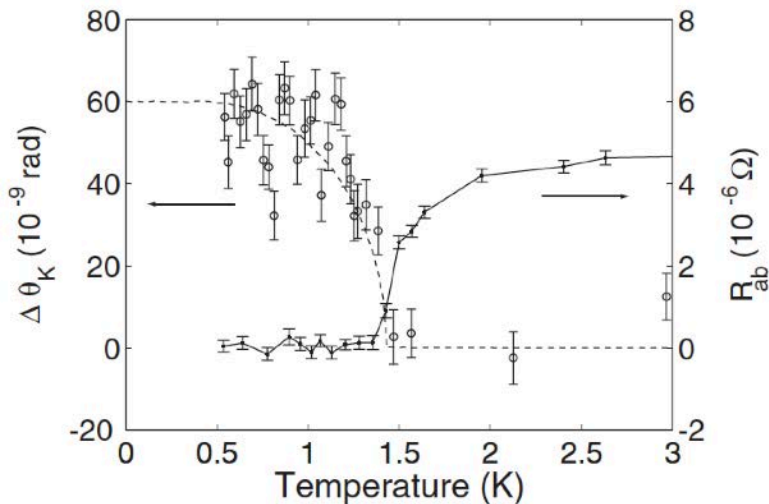
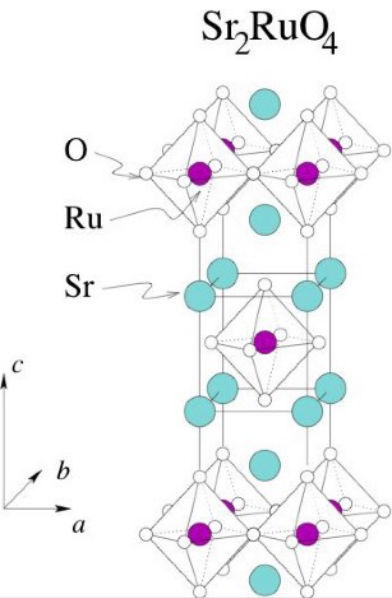
- Two-component order parameter  $(d_{x^2-y^2}, d_{xy})$

$$\psi = (\psi_1, \psi_2) = |\psi| e^{i\varphi} (\cos \alpha, e^{i\beta} \sin \alpha)$$

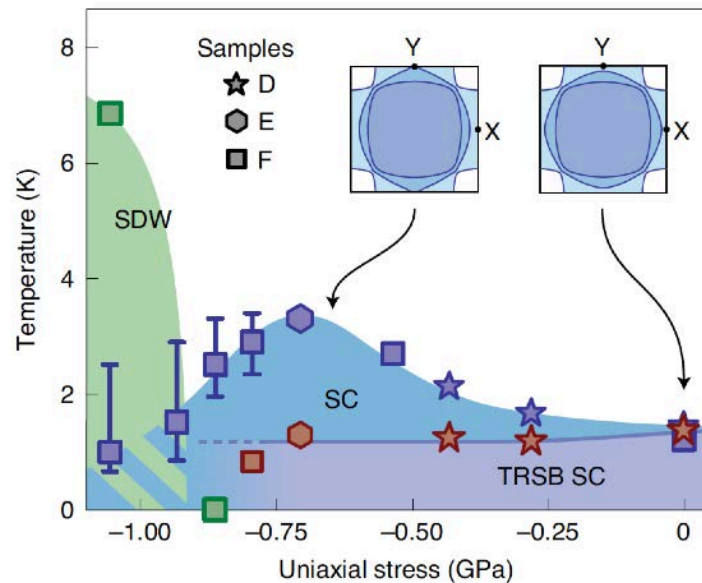


# Experimental candidates: chiral SC

- $\text{Sr}_2\text{RuO}_4$  (tetragonal): time-reversal symmetry-breaking (TRSB) observed below  $T_c$  via Kerr and  $\mu\text{SR}$ . Tunable by uniaxial stress.



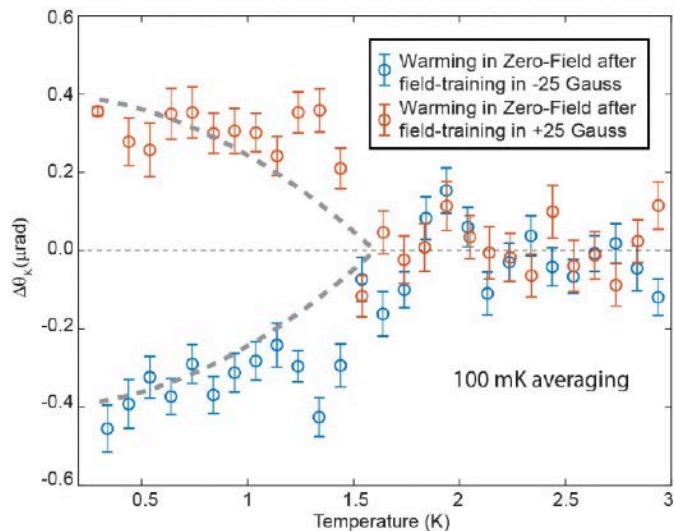
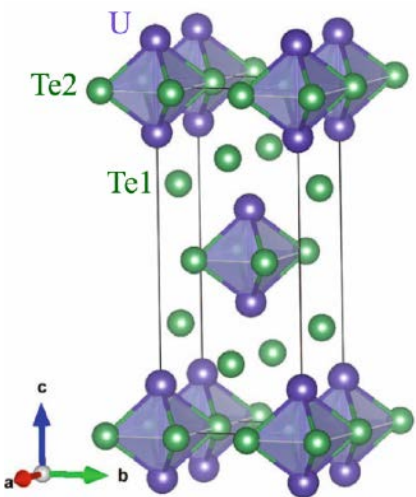
*Xia et al, PRL (2006)*



*Grinenko et al, Nat Phys (2021)*

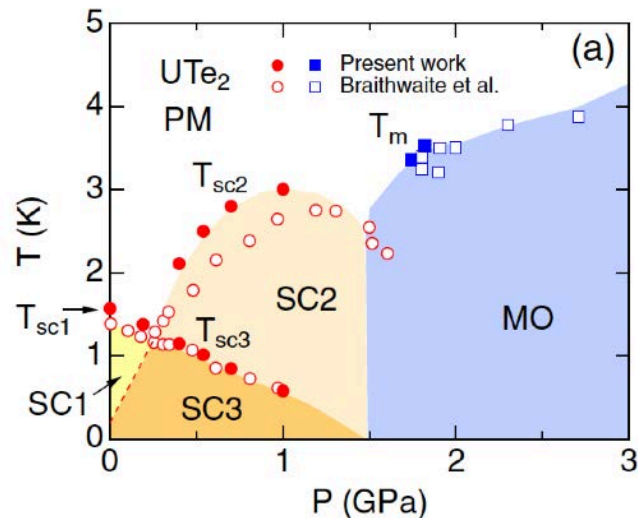
# Experimental candidates: chiral SC

- $\text{UTe}_2$  (orthorhombic): TRSB seen by Kerr (under debate). Thermodynamic evidence for two SC transitions as pressure increases.



*Hayes et al, Science (2021)*

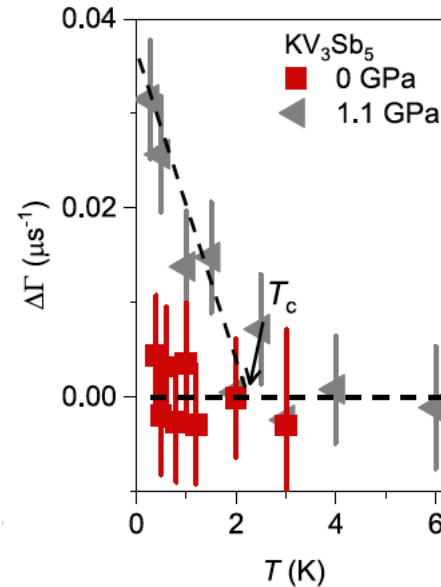
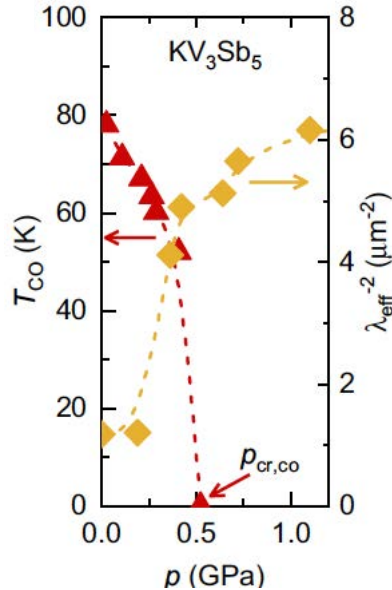
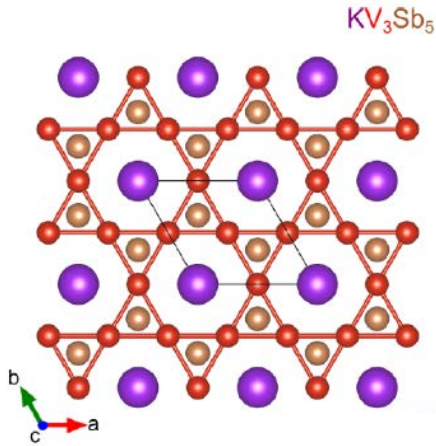
*see also: Ajeesh et al, PRX (2023)*



*Aoki et al, JPSP (2020)*

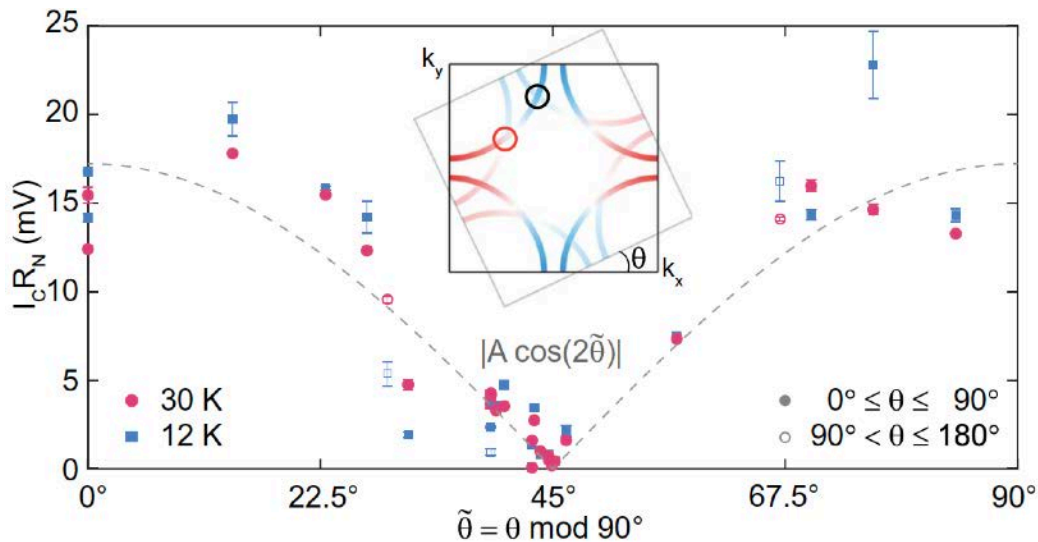
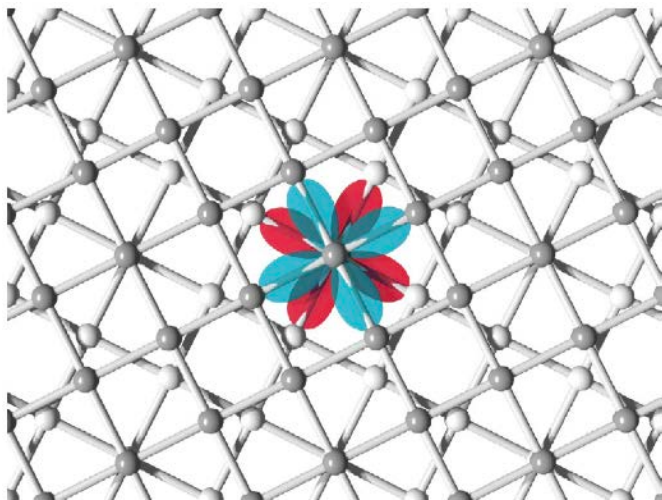
# Experimental candidates: chiral SC

- $\text{KV}_3\text{Sb}_5$  under pressure (hexagonal): TRSB seen by  $\mu\text{SR}$ .



# Experimental candidates: chiral SC

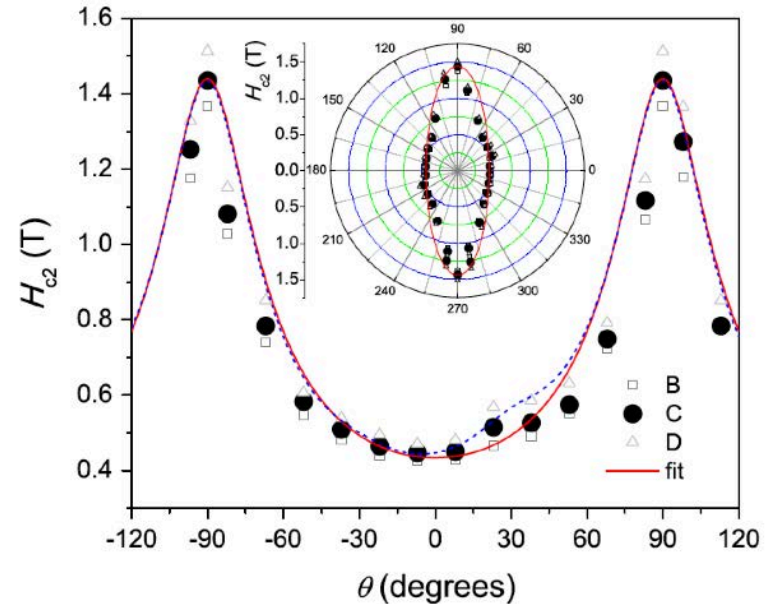
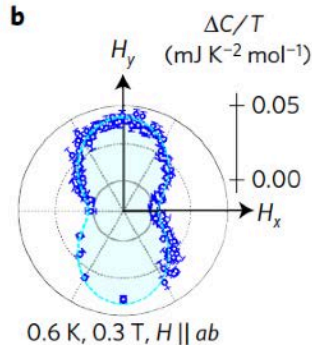
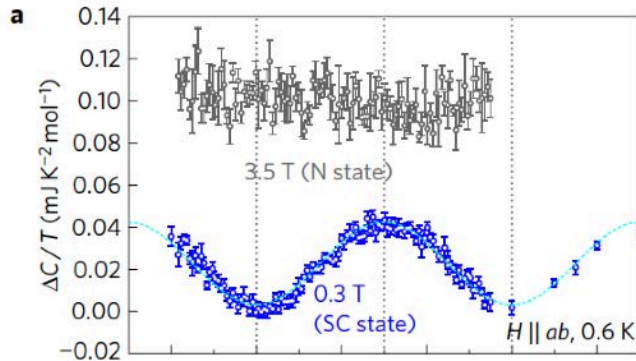
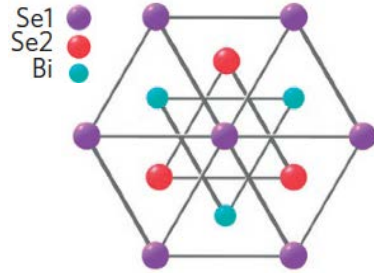
- Twisted  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  (tetragonal): possible TRSB observed.



*Zhao et al, Science (2023)*

# Experimental candidates: nematic SC

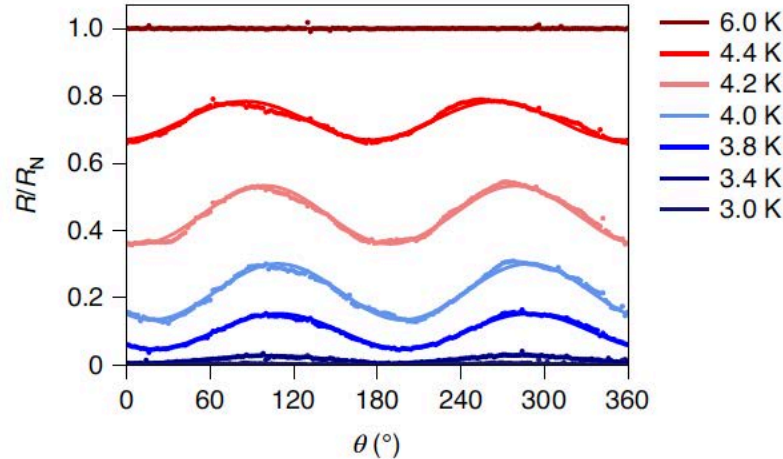
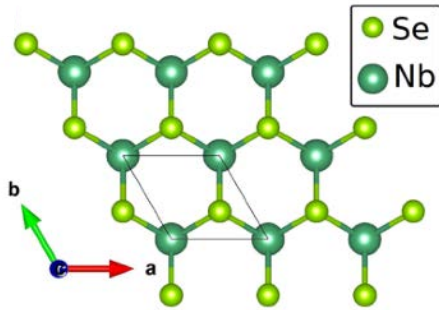
- Doped  $\text{Bi}_2\text{Se}_3$  (trigonal): three-fold rotational symmetry breaking observed via  $H_{c2}$ , angle-dependent specific heat, and other probes.



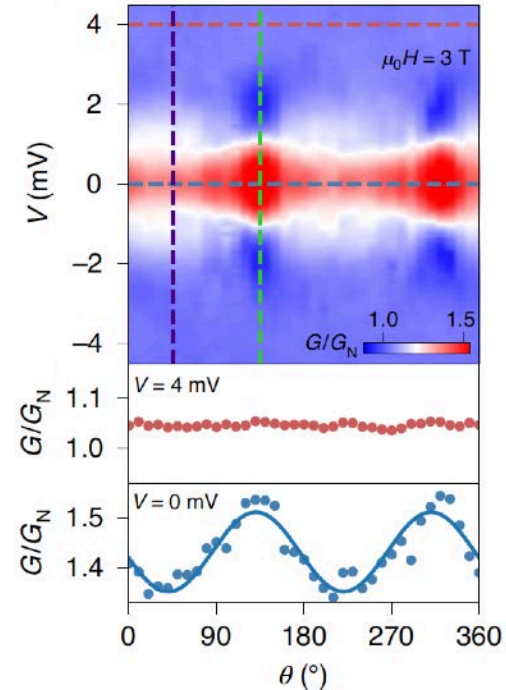


# Experimental candidates: nematic SC

- few-layer NbSe<sub>2</sub> (hexagonal): three-fold rotational symmetry breaking observed via magneto-resistance,  $H_{c2}$  and tunneling data.

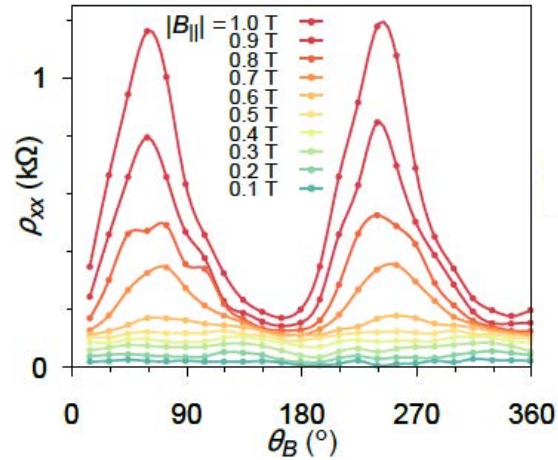
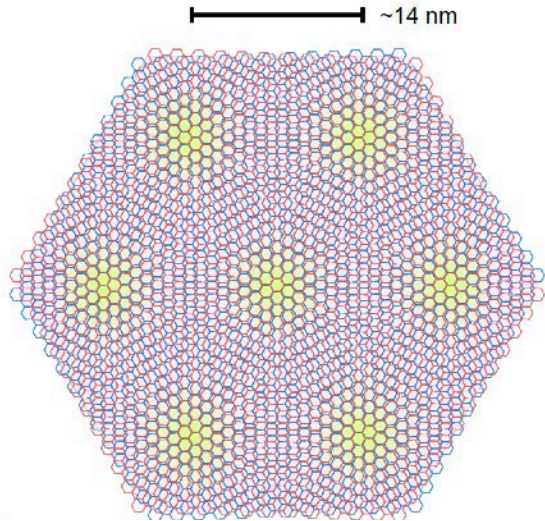


*Hamill et al, Nat Phys (2021)*  
*Cho et al, PRL (2022)*



# Experimental candidates: nematic SC

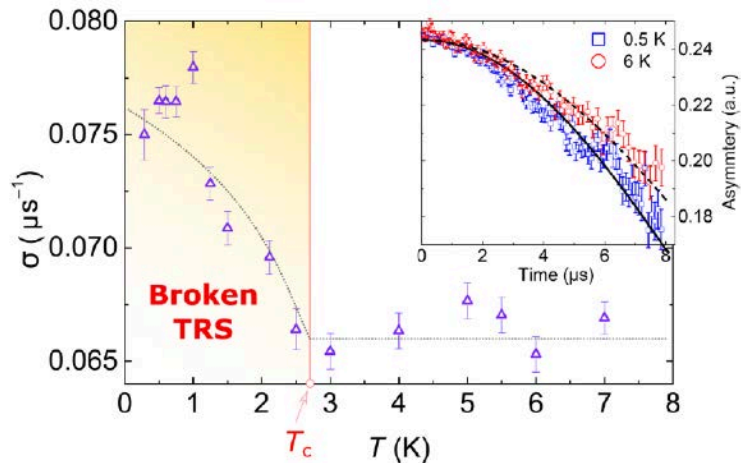
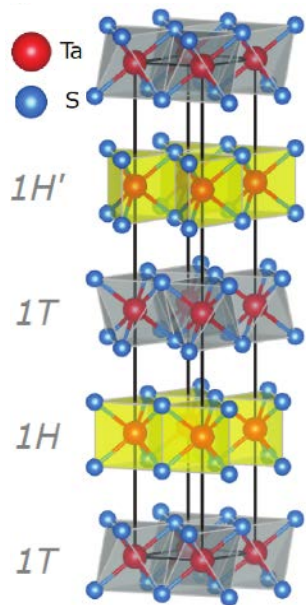
- Twisted bilayer graphene (hexagonal): three-fold rotational symmetry breaking observed in  $H_{c2}$  measurements.



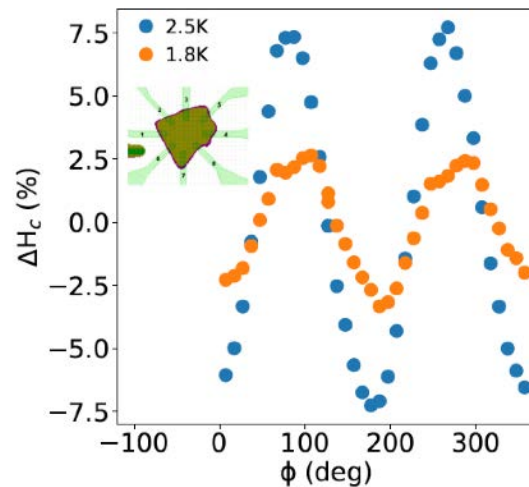
*Cao et al, Science (2021)*

# Experimental candidates: nematic + chiral SC

- 4Hb-TaS<sub>2</sub> (hexagonal): both three-fold rotational symmetry breaking and TRSB are observed (H<sub>c2</sub> and  $\mu$ SR).



Ribak et al, Sci Adv (2020)

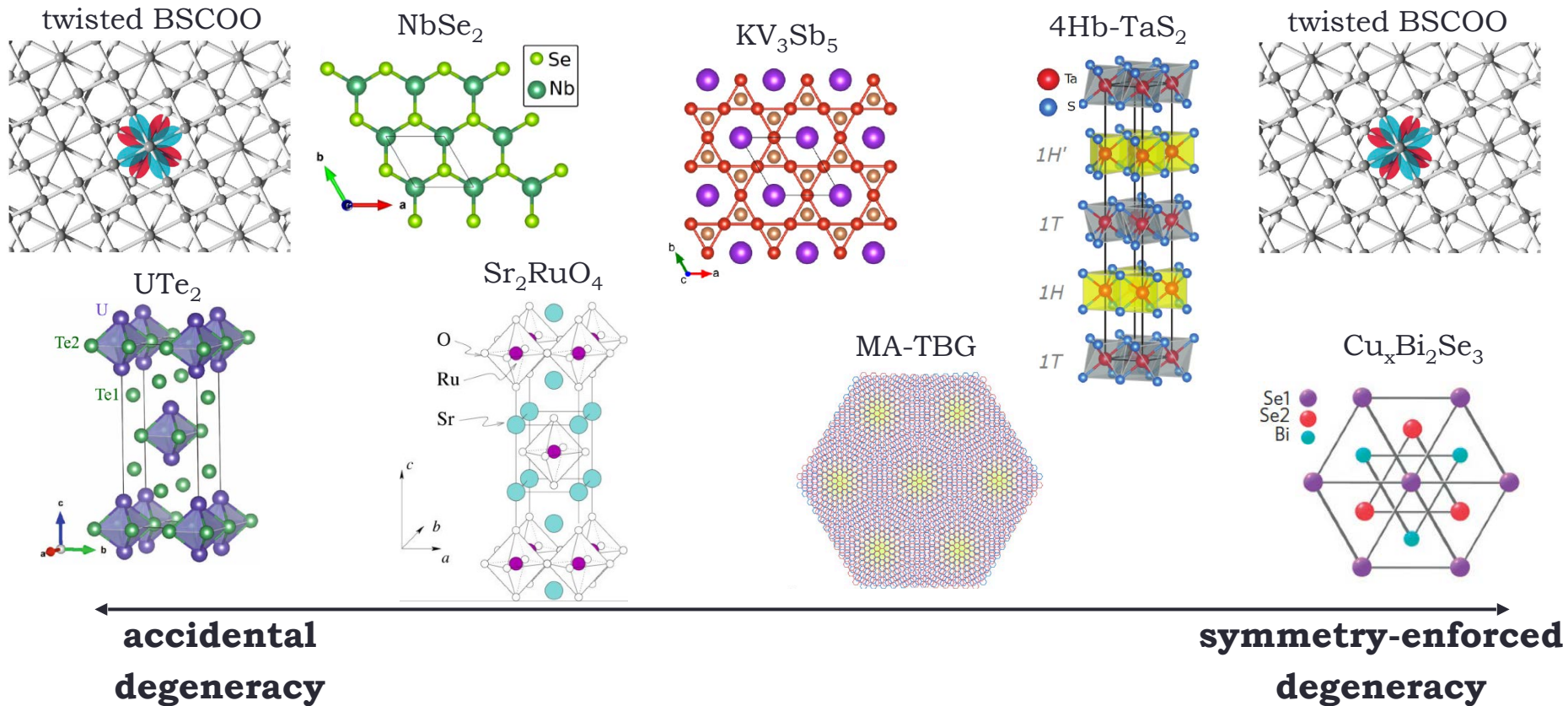


Silber et al, arxiv (2022)

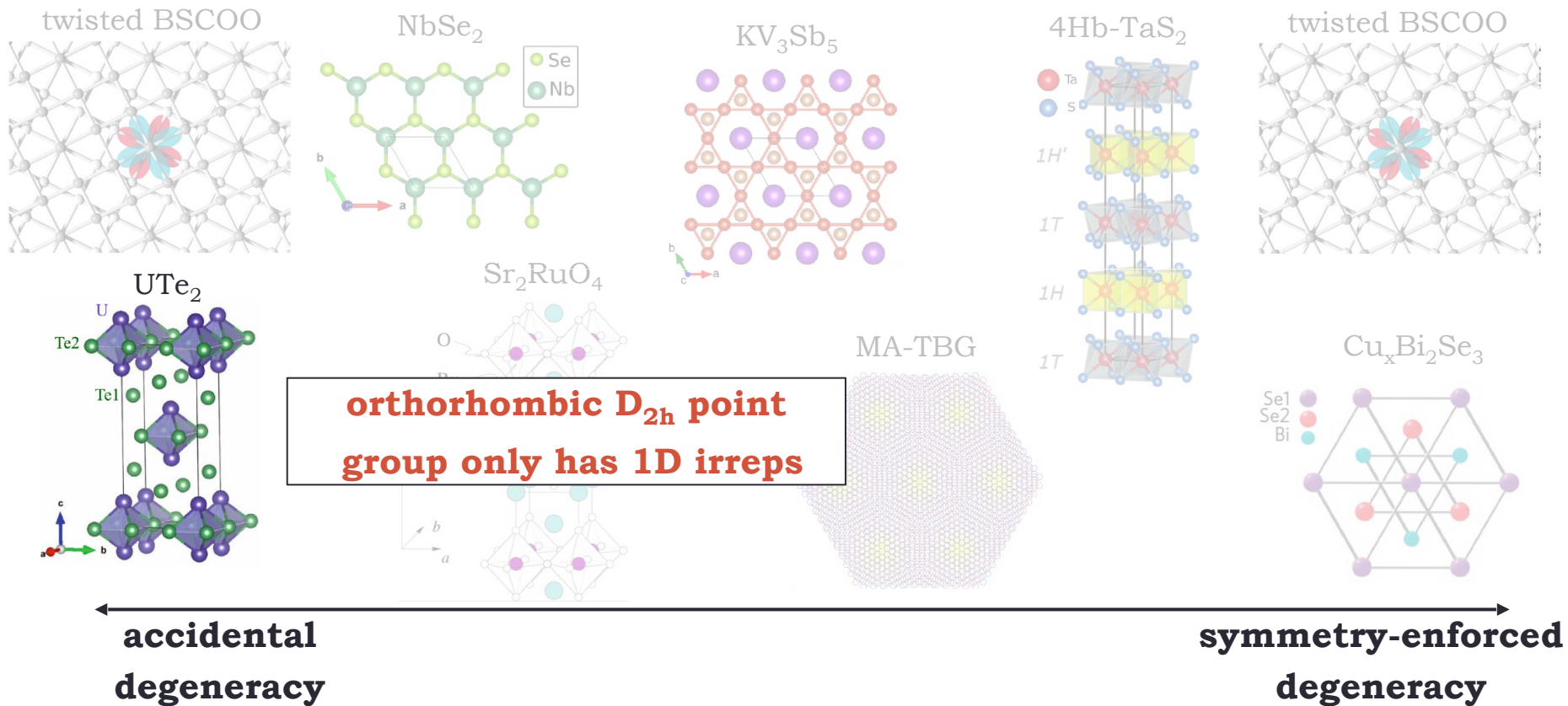
## Experimental candidates: challenges

- Are the observed effects extrinsic or intrinsic?
  - Particularly important in the case of nematic SC due to the ubiquitous presence of random strain.
- Why experiments usually observe a single nematic or chiral domain?
- Do these examples of multi-component superconductivity arise from accidental or symmetry-enforced degeneracies?

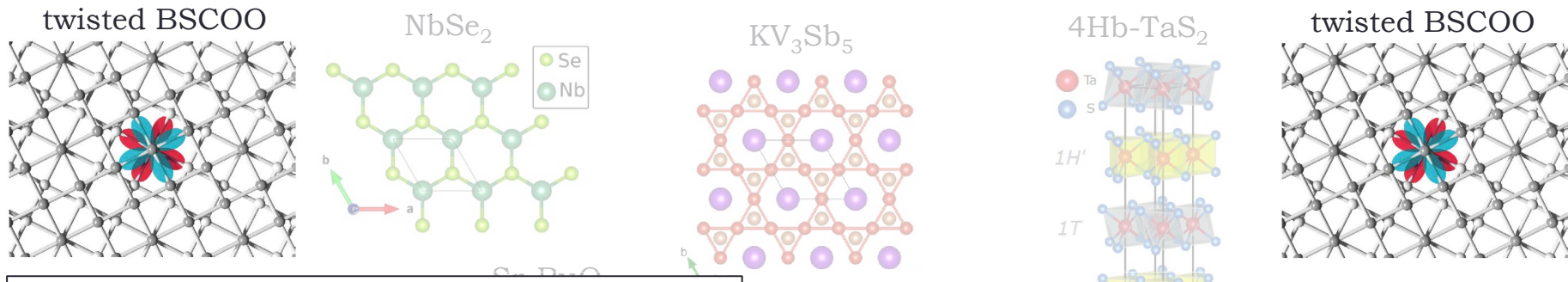
- Where to look for *symmetry-enforced* multi-component superconductivity?
- Materials described by non-Abelian point groups, since they have multi-dimensional irreducible representations: cubic > hexagonal > tetragonal.



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**$d_{xy}$  and  $d_{x^2-y^2}$  gaps belong to different 1D irreps of the crystallographic point group  $D_{4h}$**

**$d_{xy}$  and  $d_{x^2-y^2}$  gaps belong to the same 2D irrep of the non-crystallographic point group  $D_{4d}$**

*see Haenel et al, PRB (2022)*

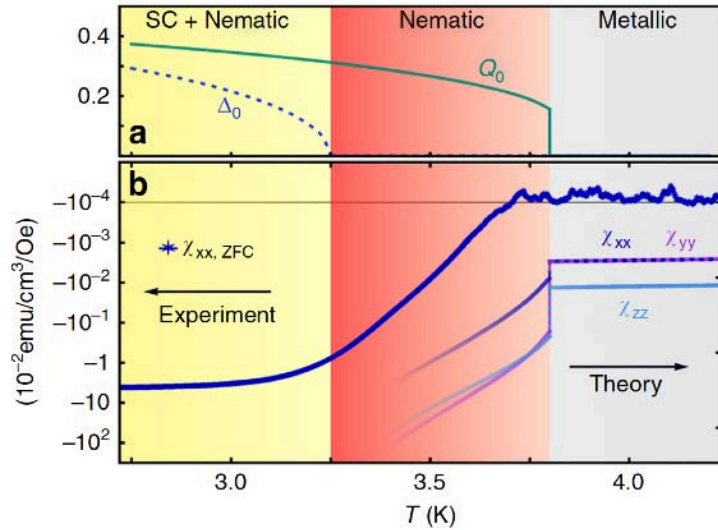


**← accidental degeneracy**

**symmetry-enforced degeneracy →**

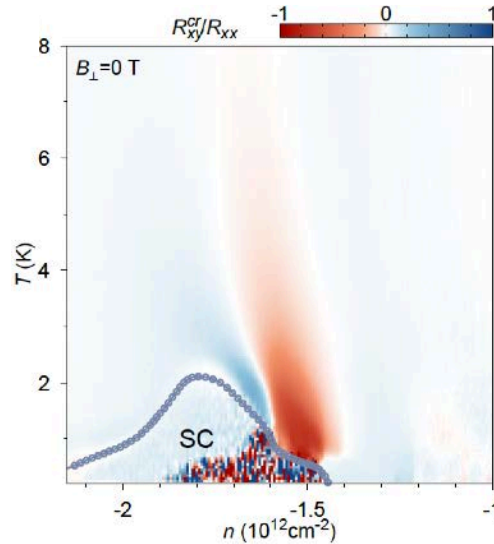
# Vestigial superconducting phases: candidates

$\text{Cu}_x\text{Bi}_2\text{Se}_3$



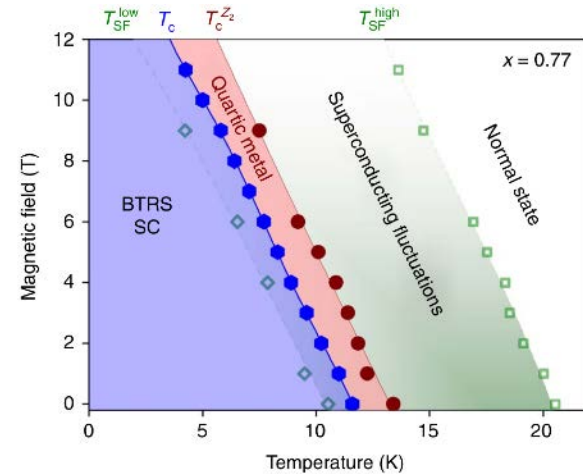
Cho et al, Nat Comm (2020)

MA-TBG



Cao et al, Science (2021)

$\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$

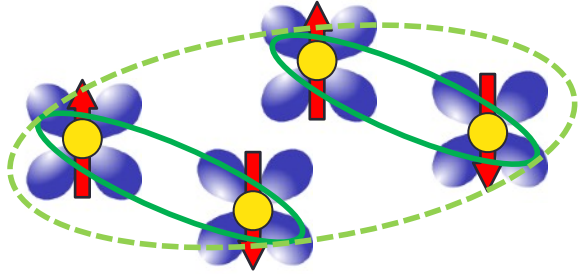


Grinenko et al, Nat Phys (2021)



# Charge-4e superconductivity: theory

- Can electrons form quartets?



*a pair of pairs*

$$\Delta^{4e} \sim \langle \psi\psi\psi\psi \rangle$$

$$\Delta \rightarrow e^{i\theta} \Delta, \quad \Delta^{4e} \rightarrow e^{2i\theta} \Delta^{4e}$$

$$U(1) \rightarrow Z_2 : \{1, e^{i\pi}\}$$

**residual  $Z_2$  gauge symmetry**

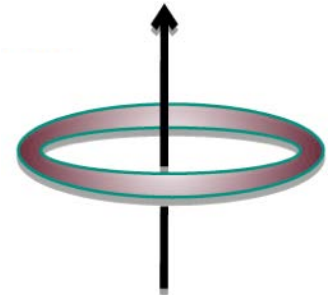
*Nozières & Saint James, J. Phys. (1982)*

*Korshunov, Zh. Eksp. Teor. Fiz (1985)*

- Fingerprint of charge-4e order: half flux-quantum.

*Berg et al, Nature Phys (2009)*

$$\frac{h}{4e} = \frac{1}{2} \Phi_0$$

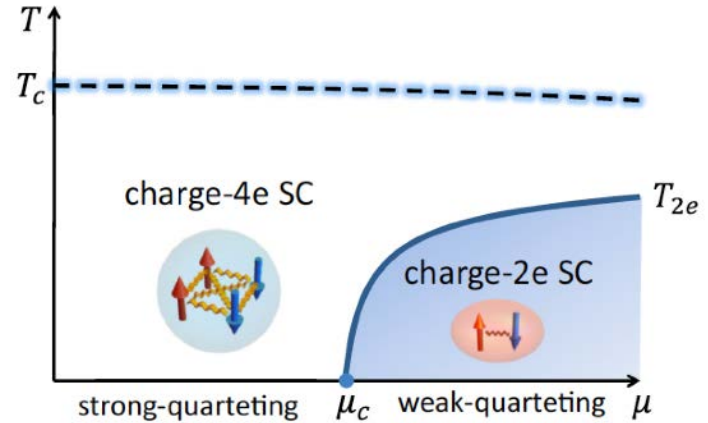
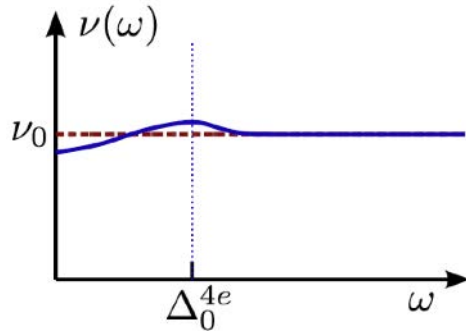


# Charge-4e superconductivity: theory

- Properties of the charge-4e superconducting state.
  - Challenging even at mean-field: charge-4e order parameter acts as a 4-fermion effective interaction.

*Jiang et al, PRB (2017)*

*Gnezdilov & Wang, PRB (2022)*



- Gapless state with small superfluid density.

***where to look for charge-4e SC?***

# Multi-component SC: vestigial orders

- To capture vestigial orders, we need a method that accounts for fluctuations and goes beyond mean-field: large-N, RG, variational...
- Here we use the **Gaussian variational approach**, as it allows us to compare different vestigial instabilities.

*Fisher & Berg, PRB (2016)*  
*Nie et al, PRB (2017)*

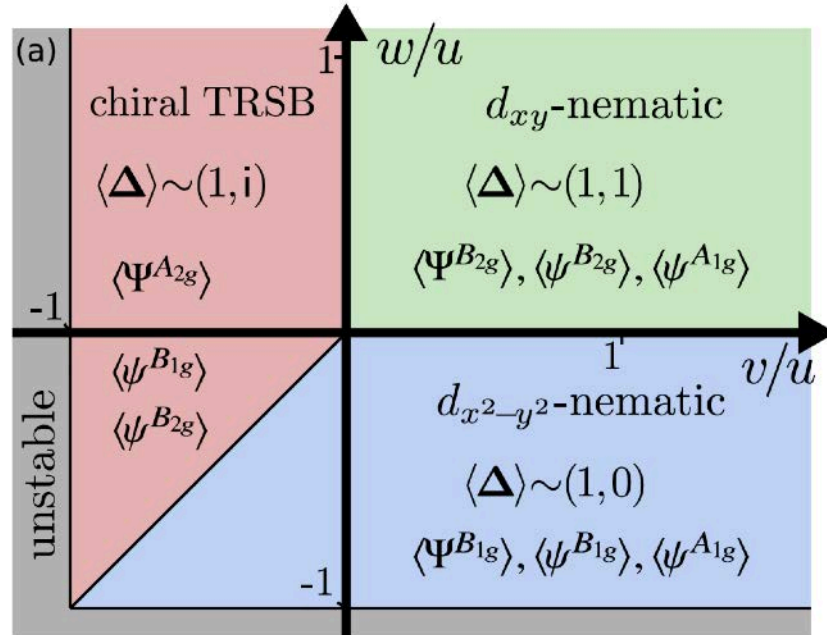
$$\text{Ansatz } \mathcal{S}_0 = \frac{1}{2} \hat{\Delta}^\dagger \begin{pmatrix} R_0 + \Phi^{B1g} & \Phi^{B2g} - i\Phi^{A2g} & \phi^{A1g} + \phi^{B1g} & \phi^{B2g} \\ & R_0 - \Phi^{B1g} & \phi^{B2g} & \phi^{A1g} - \phi^{B1g} \\ & & R_0 + \Phi^{B1g} & \Phi^{B2g} + i\Phi^{A2g} \\ \text{H.c.} & & & R_0 - \Phi^{B1g} \end{pmatrix} \hat{\Delta} + \mathcal{S}_{\text{grad}}$$

$$\text{Variational free energy } F_v = F_0 + T \langle \mathcal{S} - \mathcal{S}_0 \rangle_0 \leq F$$

Gaussian integrals

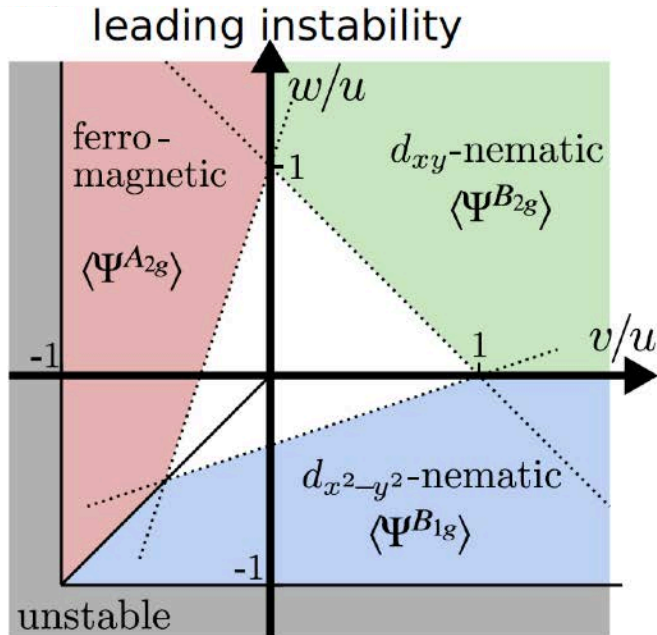
# Multi-component SC: tetragonal lattice

- Each SC ground state is compatible with three non-zero bilinears: one real-valued and two complex-valued.



# Multi-component SC: tetragonal lattice

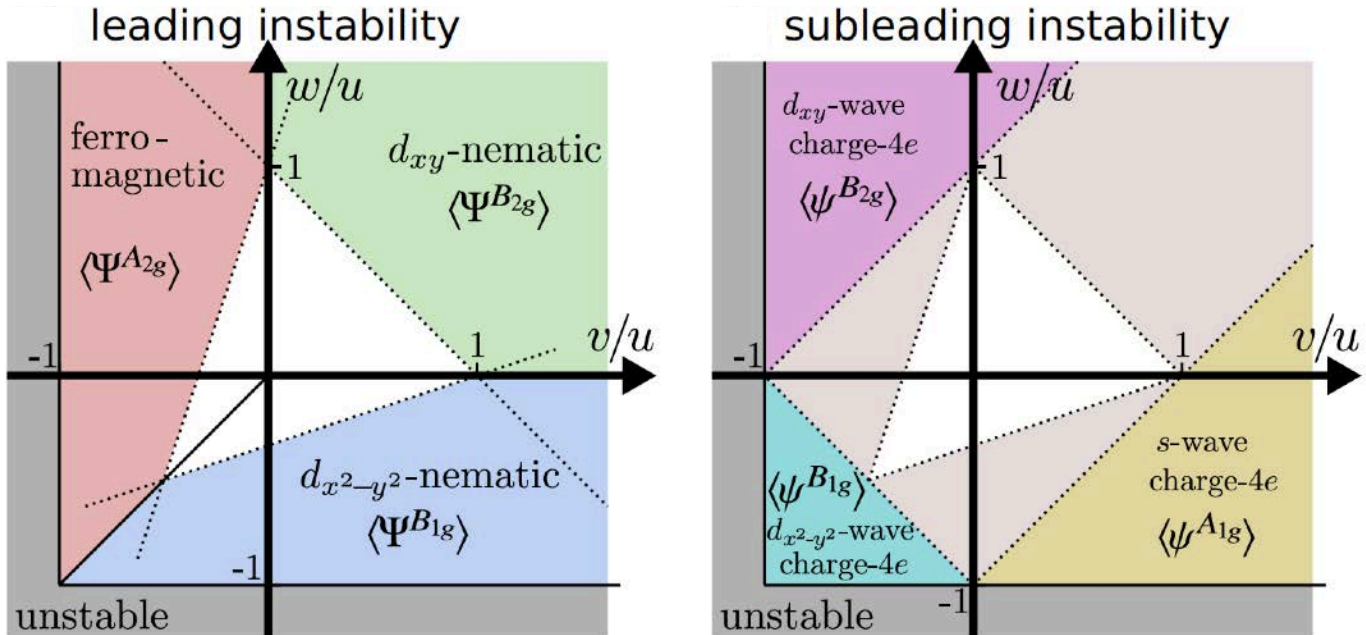
- Leading vestigial orders are the real-valued ones (magnetic or nematic).



see also Fisher & Berg, PRB (2016)

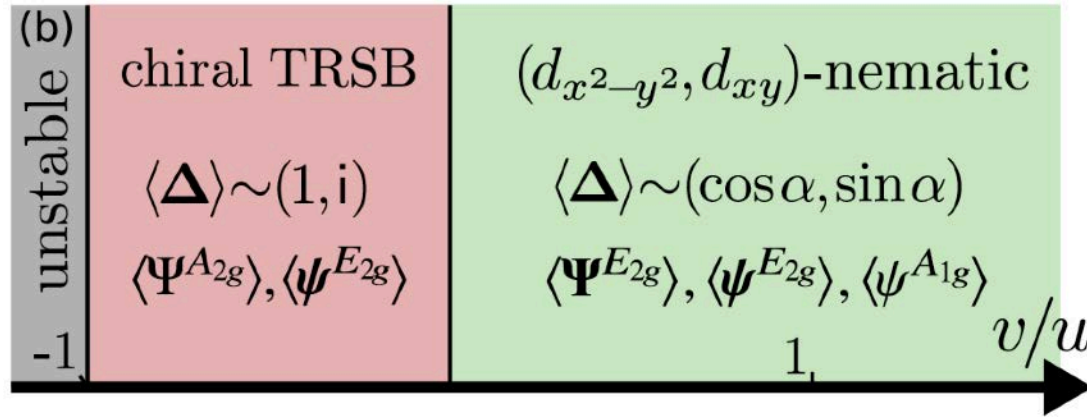
# Multi-component SC: tetragonal lattice

- Leading vestigial orders are the real-valued ones (magnetic or nematic).
- Charge-4e channels are attractive in wide regions of the phase diagrams, but always sub-leading. No indication of coexistence.



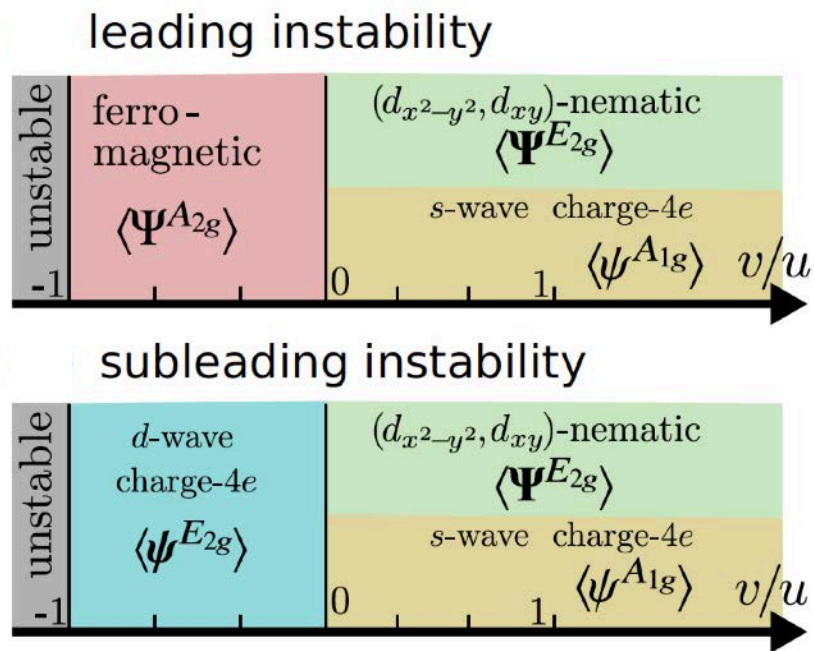
# Multi-component SC: hexagonal lattice

- Mean-field phase diagram: chiral and one type of nematic SC state.



# Multi-component SC: hexagonal lattice

- Leading vestigial orders are the real-valued ones (magnetic or nematic), but there is a degeneracy between the nematic and s-wave charge-4e instabilities.





# Vestigial order: numerical results

- Monte Carlo simulations of the  $O(2) \times O(2)$  Ginzburg-Landau model show split Ising (vestigial) and Kosterlitz-Thouless (SC) transitions.

$$\mathcal{H}_\phi = -J \sum_{\langle xy \rangle, i} \vec{\phi}_{i,x} \cdot \vec{\phi}_{i,y} + \sum_{i,x} [\phi_{i,x}^2 + U(\phi_{i,x}^2 - 1)^2] + 2(U + D) \sum_x \phi_{1,x}^2 \phi_{2,x}^2$$

