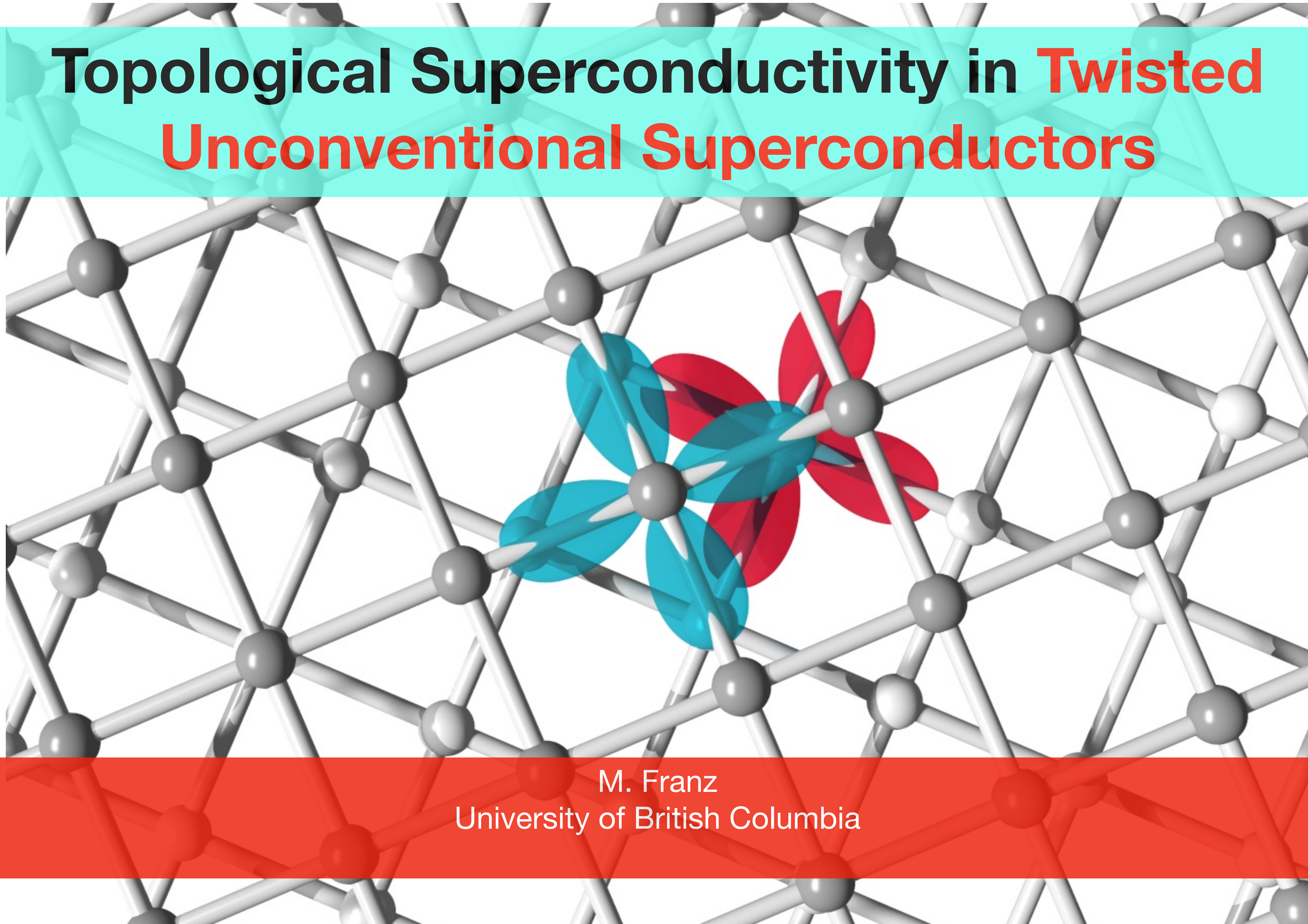


Topological Superconductivity in **Twisted** **Unconventional Superconductors**



M. Franz
University of British Columbia

People involved

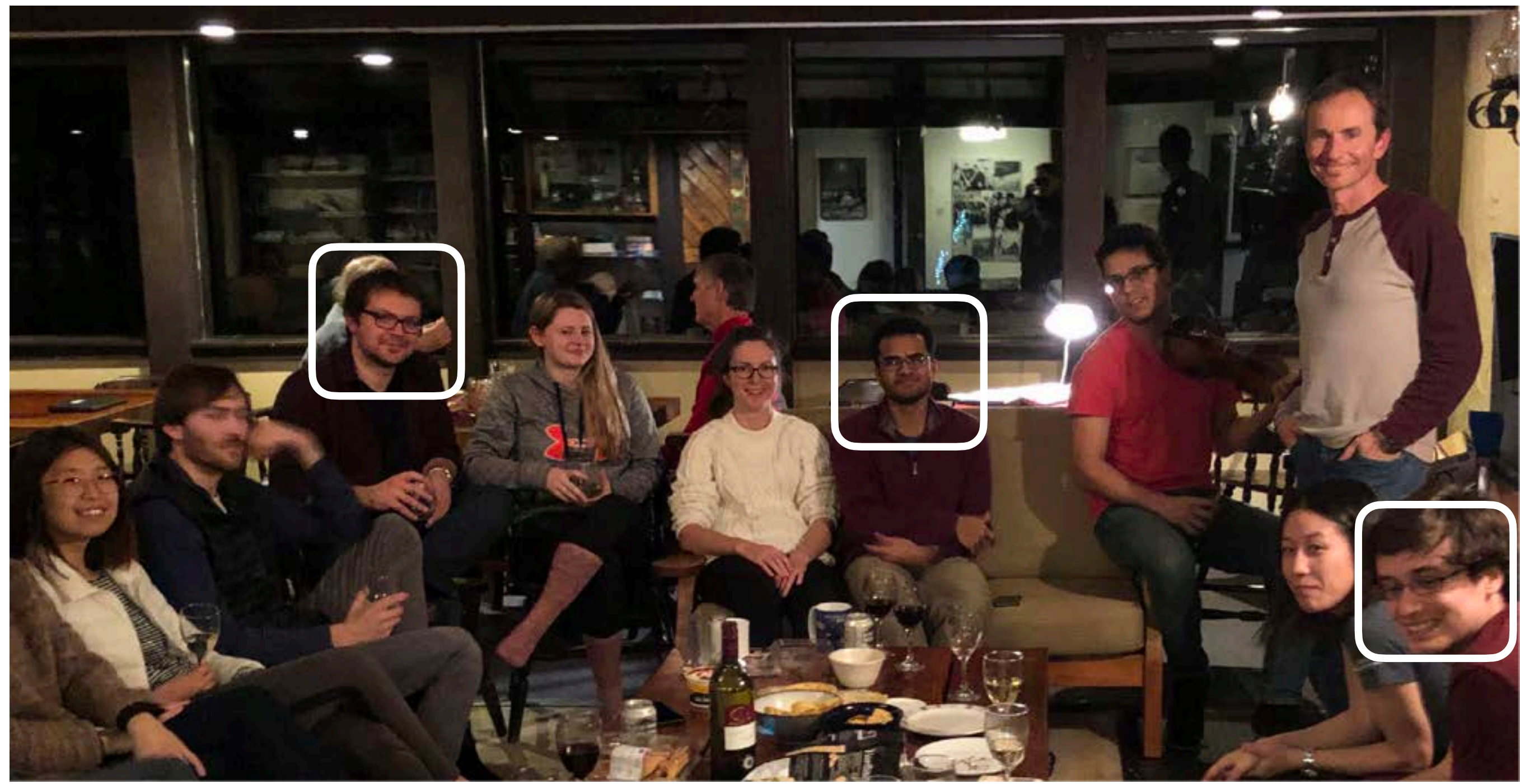
Theory

- Marcel Franz (UBC & QMI)
- Oguz Can (Franz group)
- Tarun Tummuru (Affleck & Franz group)
- Ryan Day (Damascelli group)
- Ilya Elfimov (QMI)
- Andrea Damascelli (QMI)

- Rafael Haenel (Franz group)
- Etienne Lantagne-Hurtubise (Caltech)
- Stephan Plugge (Leiden)
- Xiao-Xiao Zhang (MPI/QMI)
- Catherine Kallin (McMaster)
- Stephan Plugge (QMI/Leiden)
- Jed Pixley, Pavel Volkov (Rutgers)

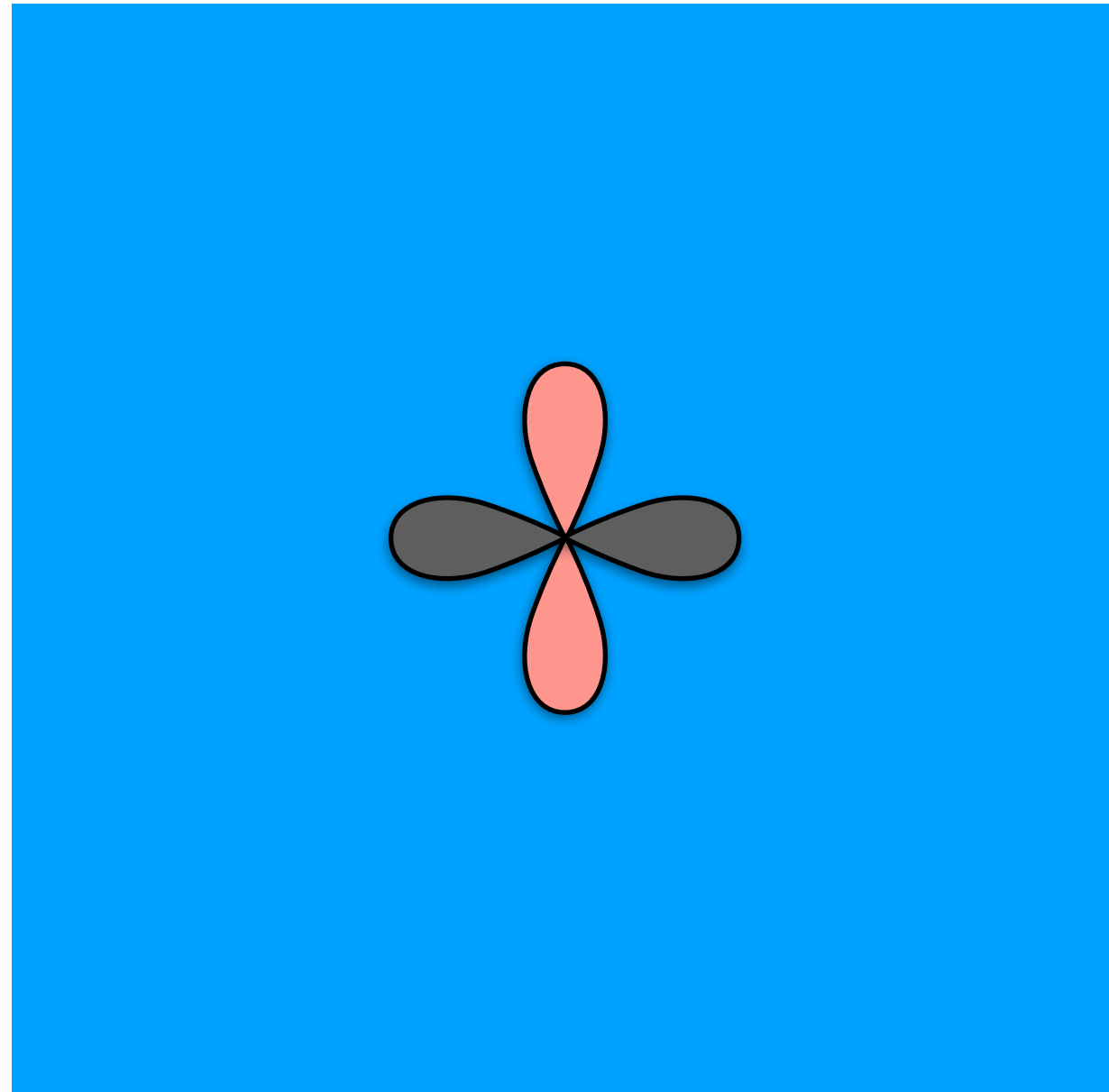
Experiment

- Ziliang Ye
- Yunhuan Xiao (Ye group)
- Yevgeny Ostroumov (QMI)
- Doug Bonn (QMI)
- Philip Kim, Frank Zhao (Harvard)



Nature Physics 17, 519 (2021); PRB Lett. 103, L100501 (2021); Phys. Rev. Lett. 127, 157001 (2021)

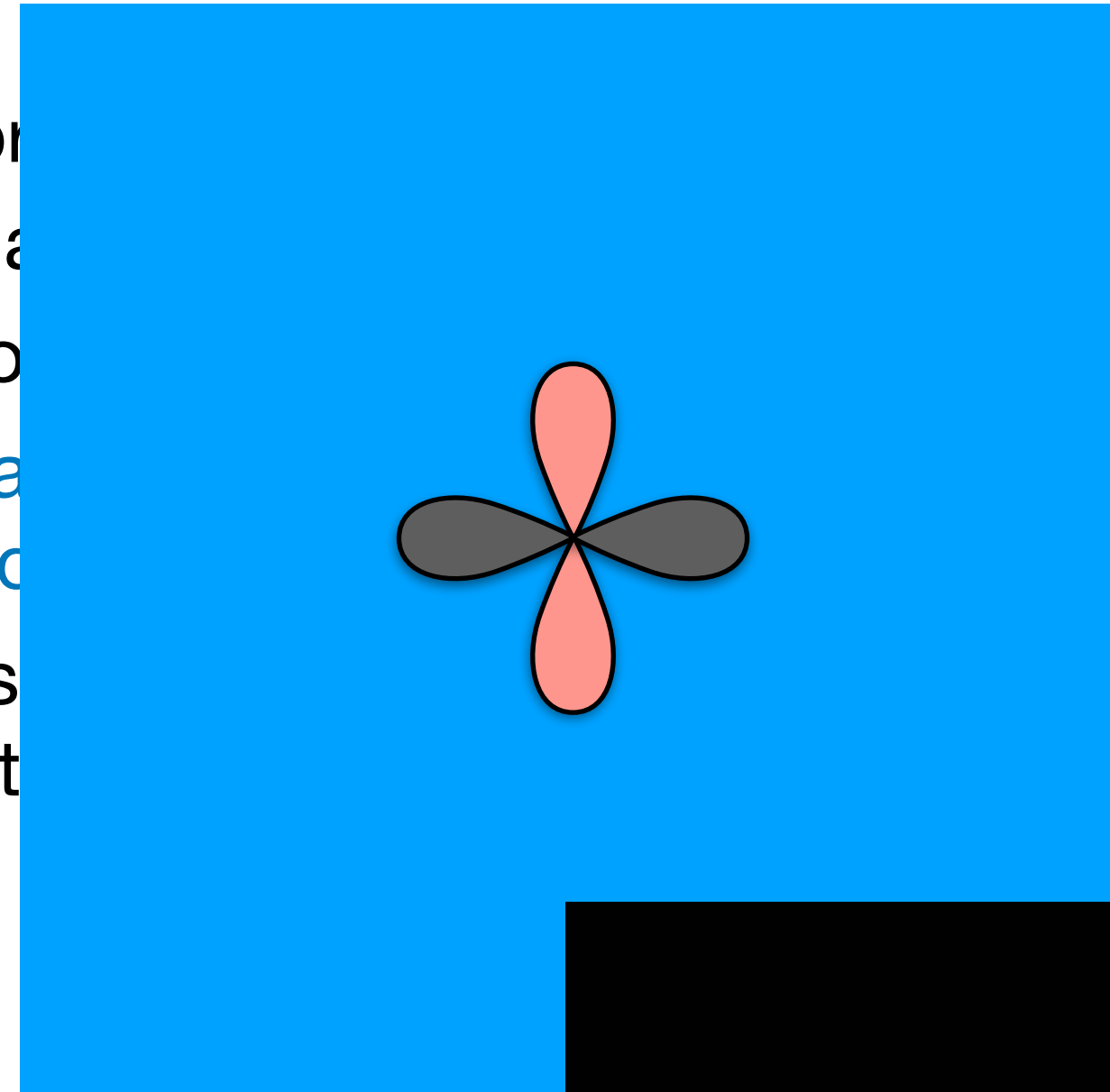
The idea: Engineer a high- T_c cuprate bilayer into a topological superconductor



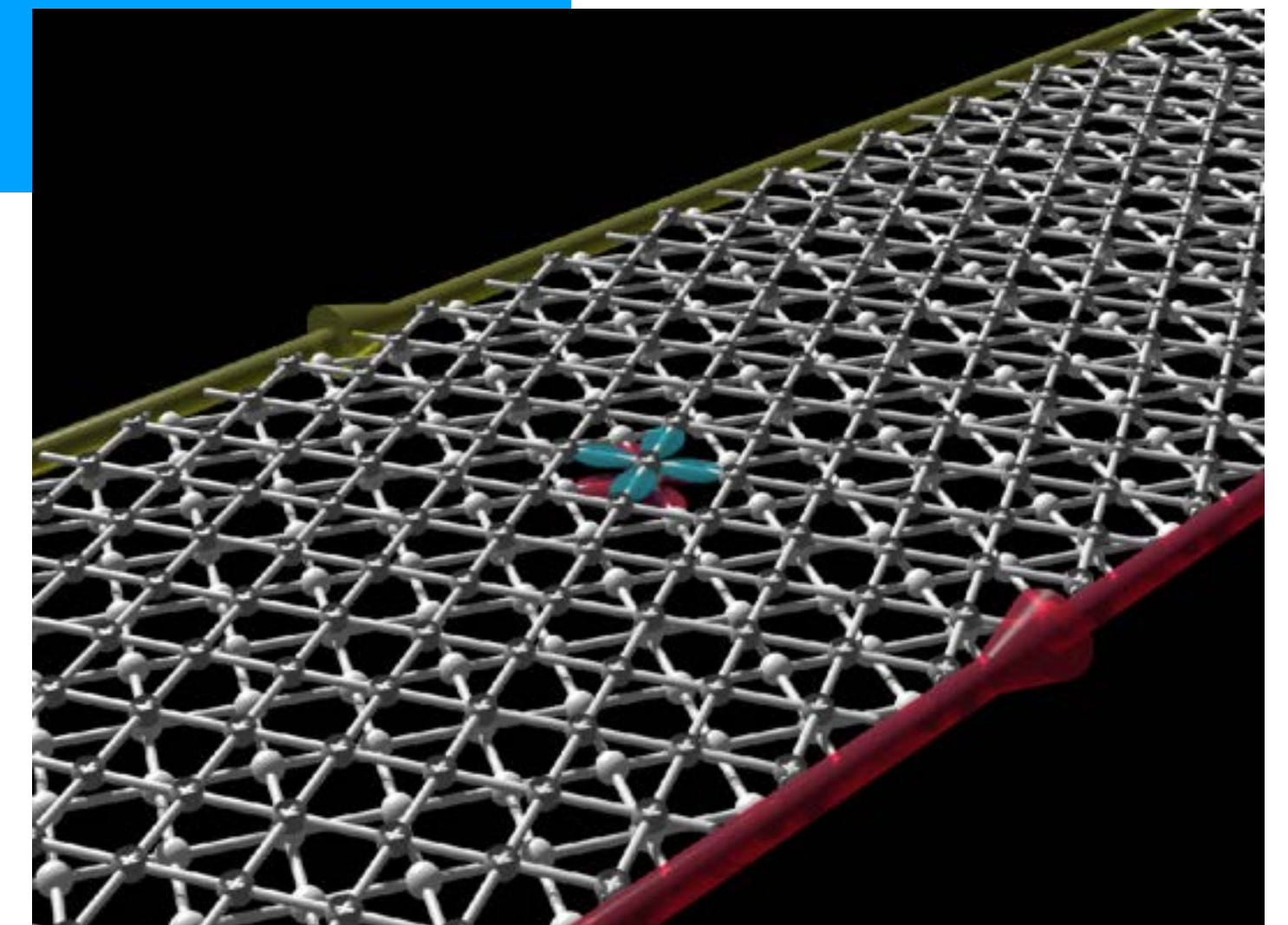
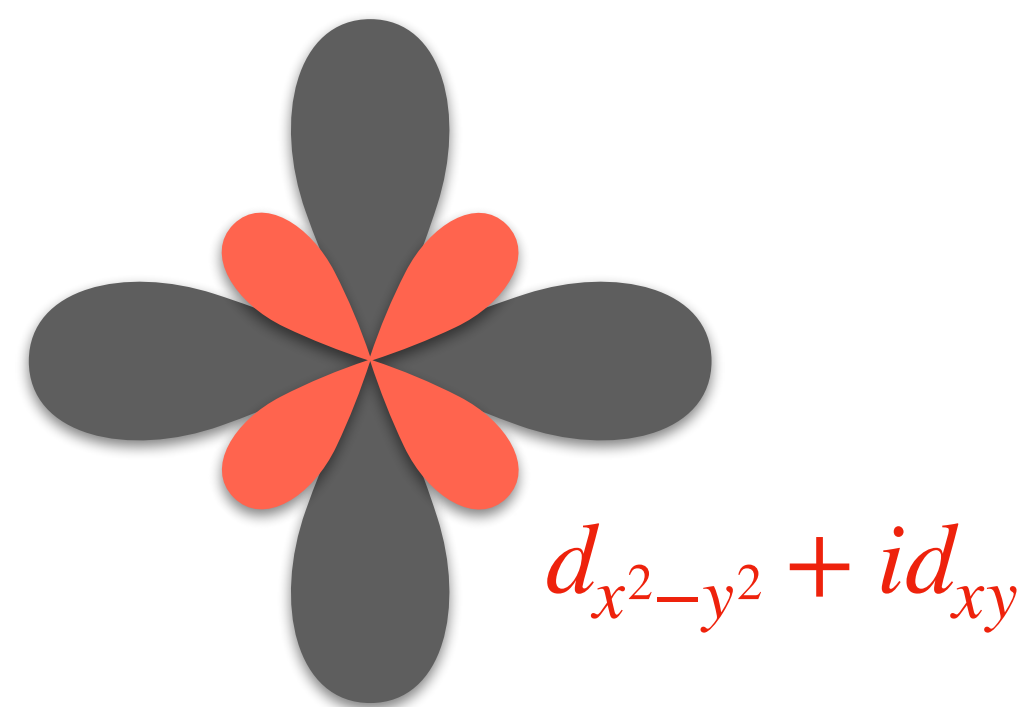
Monolayer cuprate, e.g. $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$:
 $d_{x^2-y^2}$ superconductor

Nature Physics 17, 519 (2021)

- In the proposed setup has a superconducting phase
- This is a chiral edge state
- Exhibits time-reversal symmetry



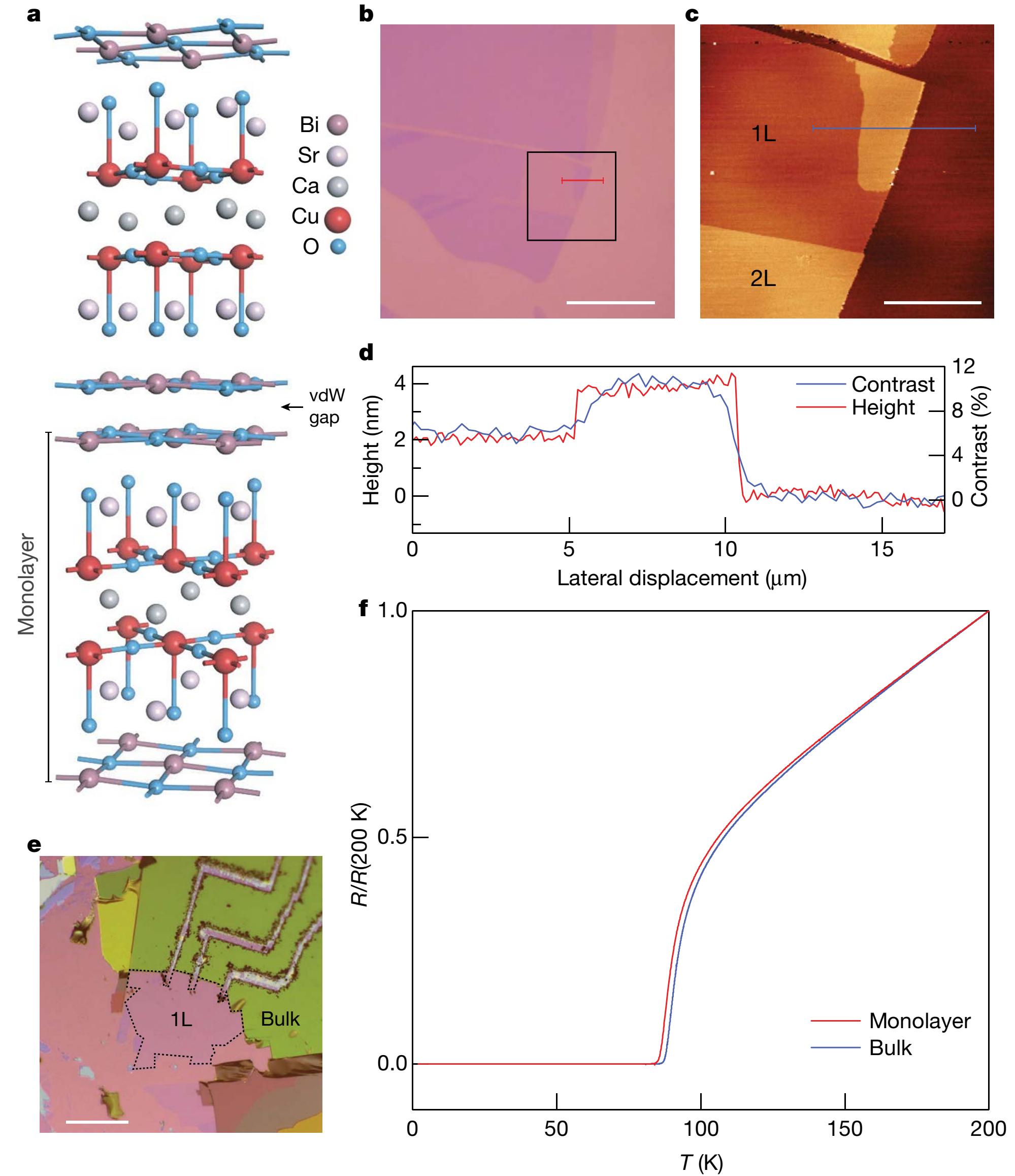
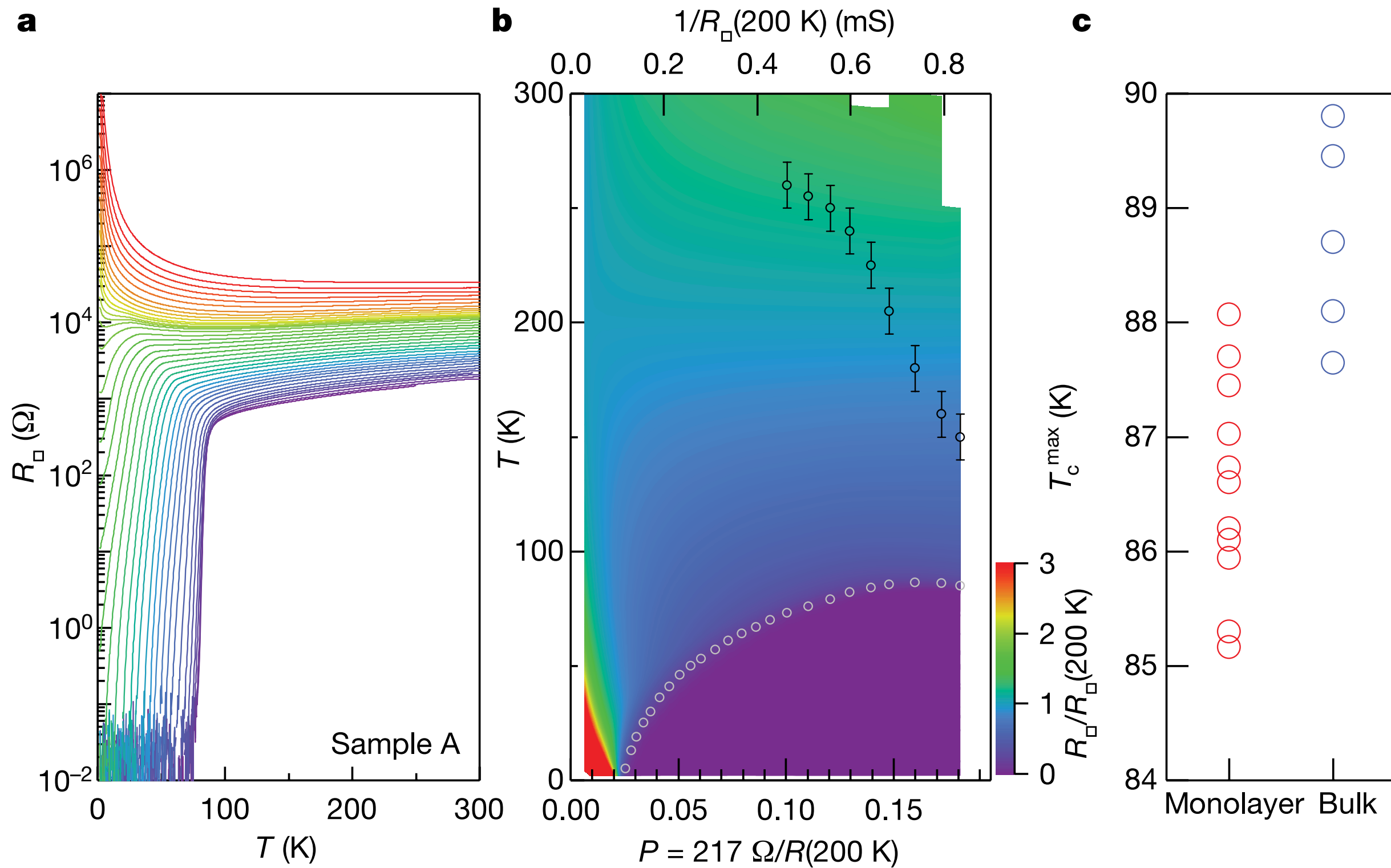
Engineering this
 $d_{x^2-y^2} + id_{xy}$
 is topologically protected
 chiral edge state



High-temperature superconductivity in monolayer $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$

<https://doi.org/10.1038/s41586-019-1718-x> Yijun Yu^{1,2,3,7}, Liguo Ma^{1,2,3,7*}, Peng Cai^{1,2,3,7}, Ruidan Zhong⁴, Cun Ye^{1,2,3}, Jian Shen^{1,2,3}, G. D. Gu⁴, Xian Hui Chen^{3,5,6*} & Yuanbo Zhang^{1,2,3*}

Received: 2 April 2019



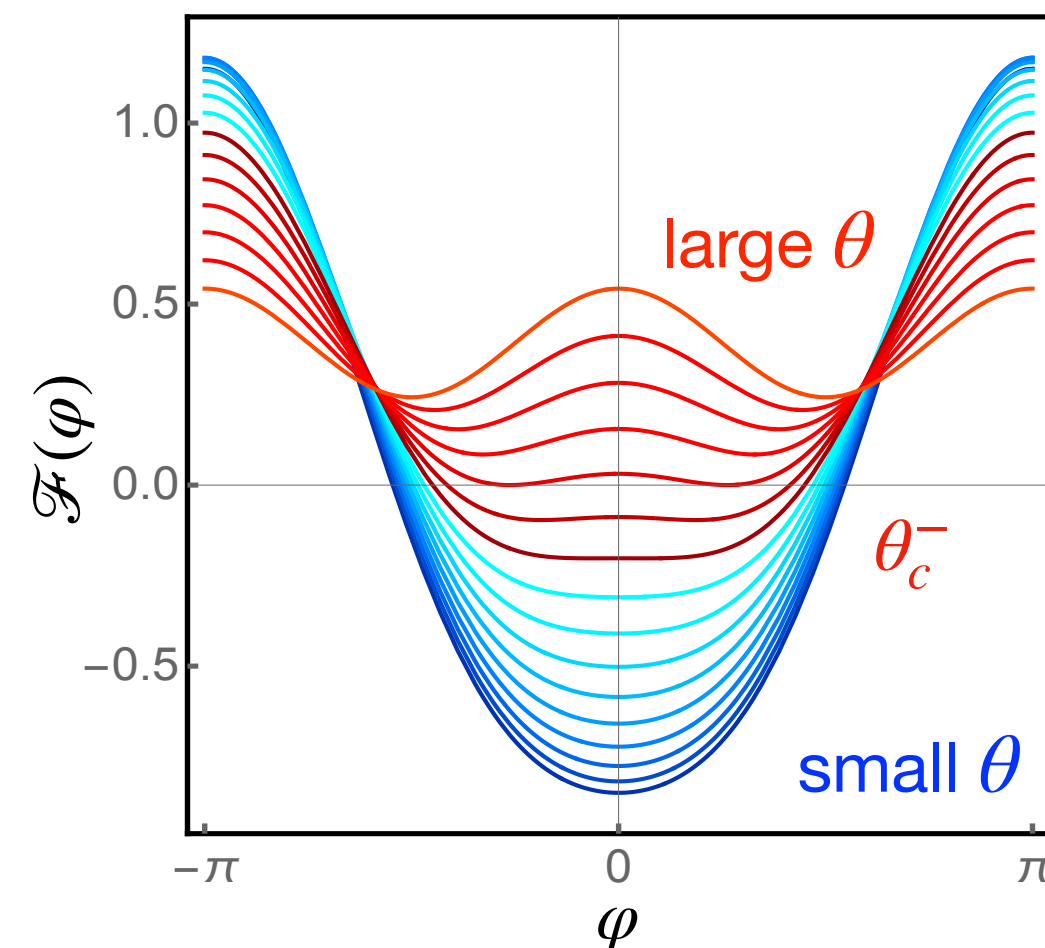
1. Ginzburg-Landau theory for twisted d -wave bilayers

$$\mathcal{F}[\psi_1, \psi_2] = f_0[\psi_1] + f_0[\psi_2] + A |\psi_1|^2 |\psi_2|^2 + B(\psi_1 \psi_2^* + \text{c.c.}) + C(\psi_1^2 \psi_2^{*2} + \text{c.c.})$$

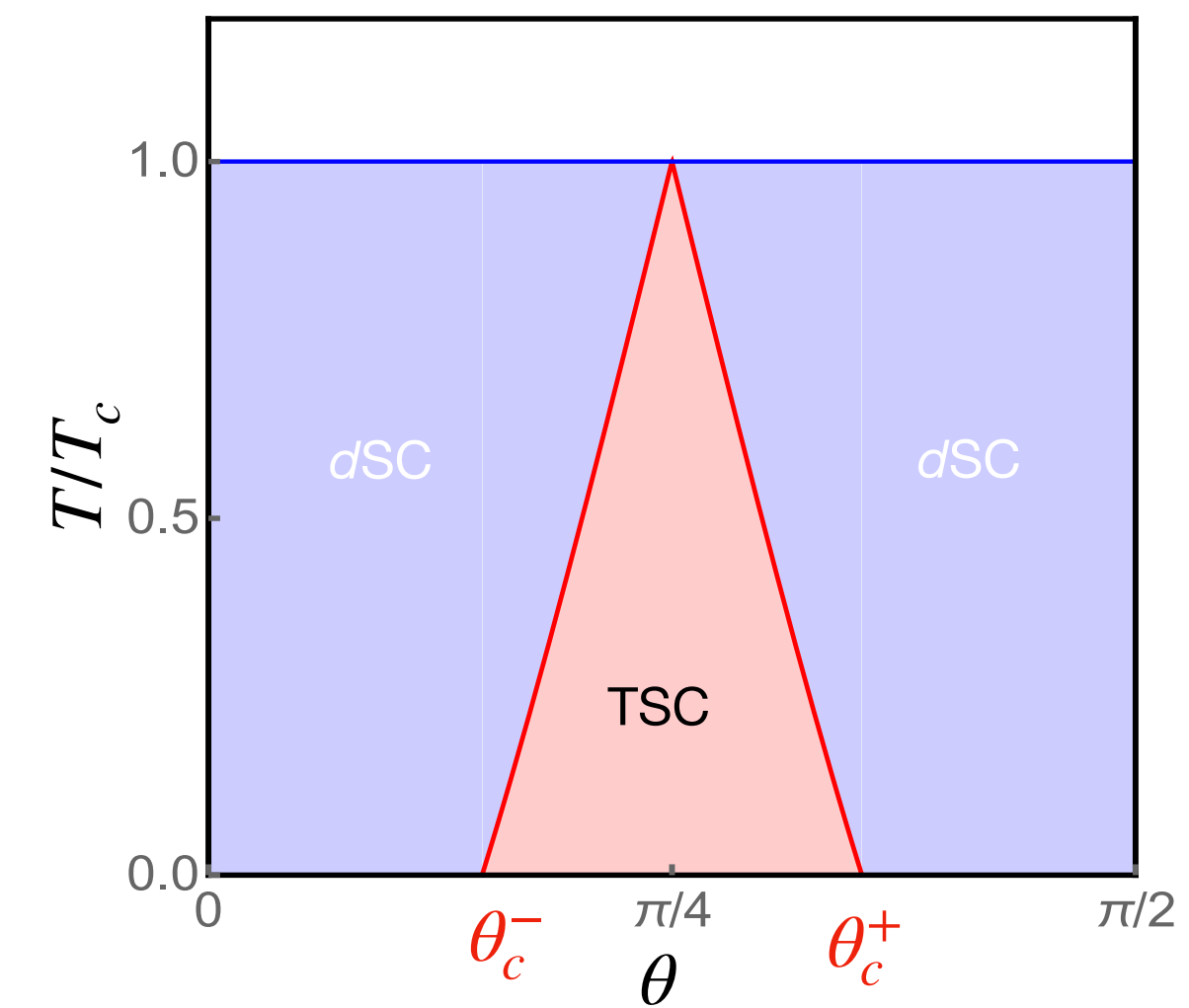
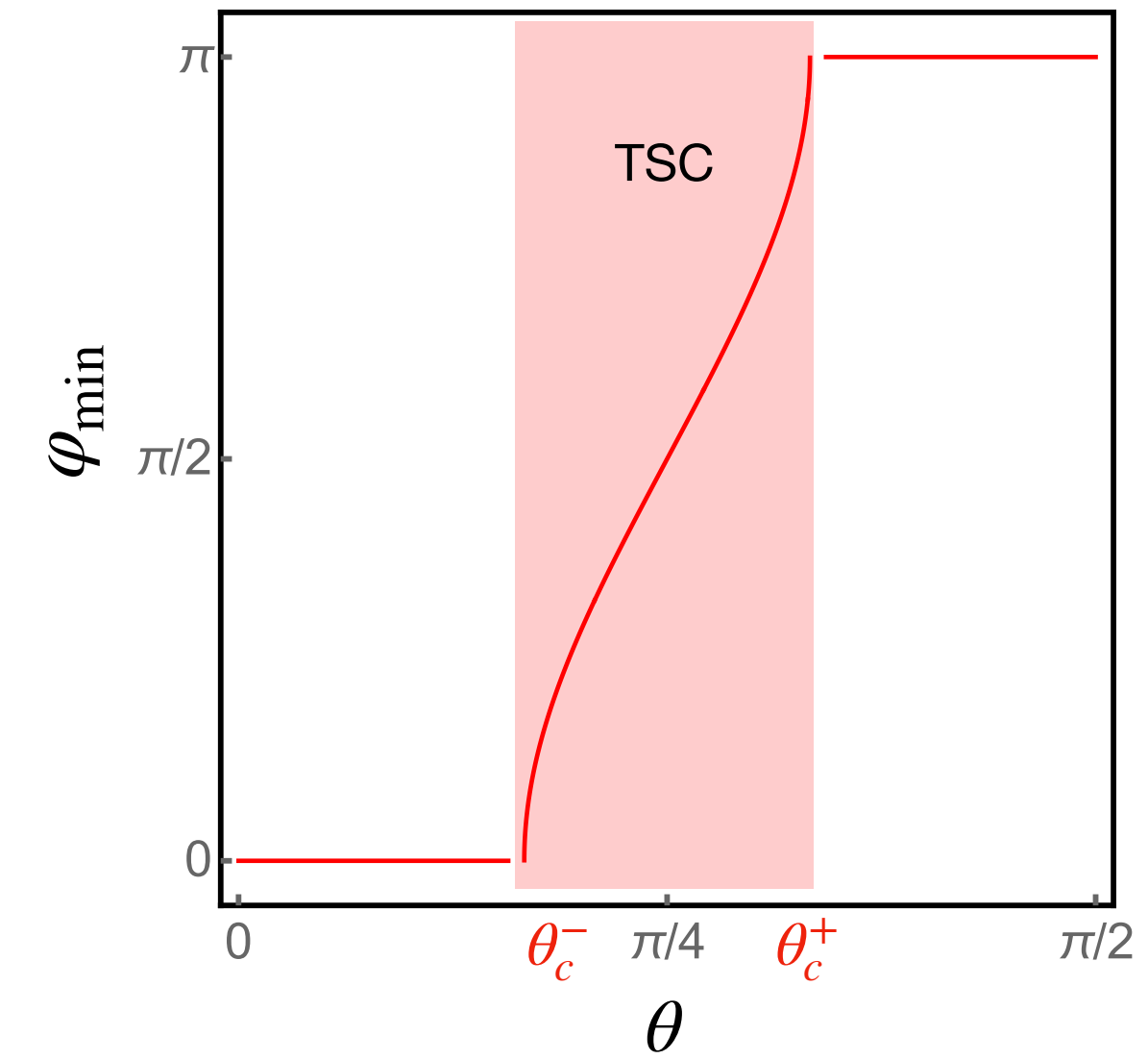
d -wave symmetry dictates $B = -B_0 \cos(2\theta)$

Assuming $\psi_1 = \psi$, $\psi_2 = \psi e^{i\varphi}$ we obtain free energy as a function of the phase

$$\mathcal{F}(\varphi) = \mathcal{F}_0 + 2B_0\psi^2 \left[-\cos(2\theta)\cos\varphi + \mathcal{K} \cos(2\varphi) \right]$$



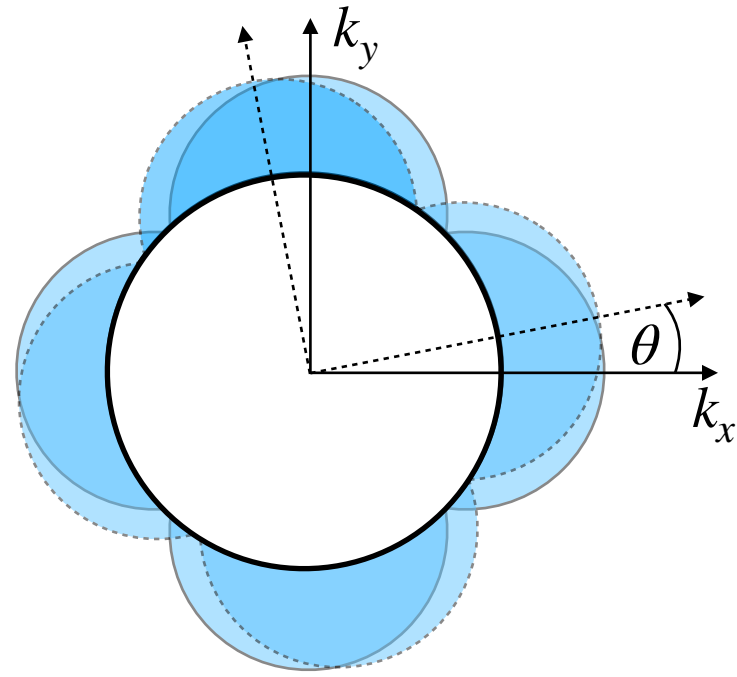
$$\mathcal{K} = C\psi^2/B_0$$



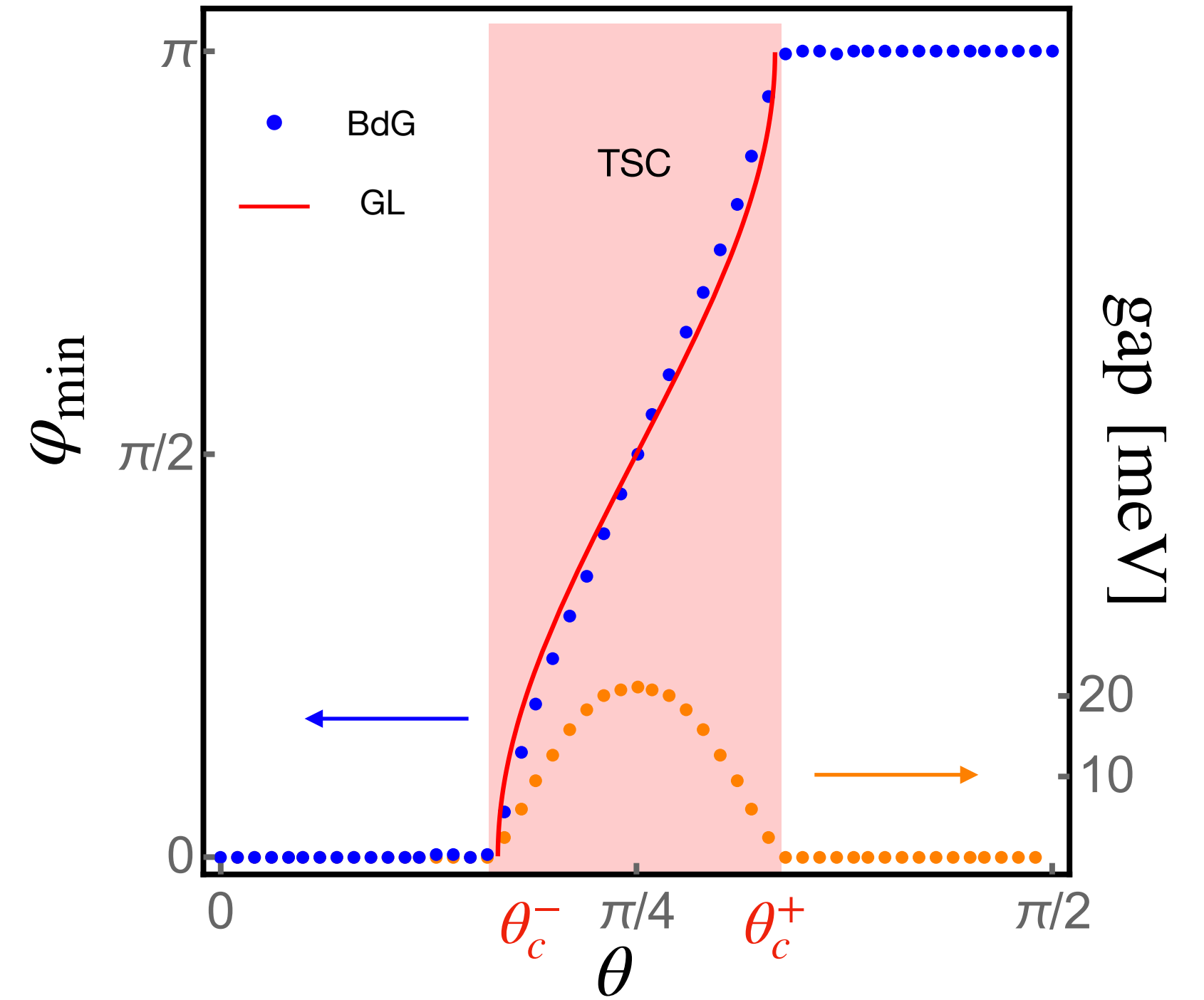
$$\tilde{T}_c(\theta) = T_c \left(1 - \frac{|\cos 2\theta|}{4\mathcal{K}_0} \right), \quad \theta_c^- < \theta < \theta_c^+$$

2. Microscopic theory - Continuum Bogoliubov-de Gennes

$$\mathcal{H} = \sum_{\mathbf{k}\sigma a} \xi_{\mathbf{k}a} c_{\mathbf{k}\sigma a}^\dagger c_{\mathbf{k}\sigma a} + g \sum_{\mathbf{k}\sigma} \left(c_{\mathbf{k}\sigma 1}^\dagger c_{\mathbf{k}\sigma 2} + \text{h.c.} \right) + \sum_{\mathbf{k}a} \left(\Delta_{\mathbf{k}a} c_{\mathbf{k}\uparrow a}^\dagger c_{-\mathbf{k}\downarrow a}^\dagger + \text{h.c.} \right) - \sum_{\mathbf{k}a} \Delta_{\mathbf{k}a} \langle c_{\mathbf{k}\uparrow a}^\dagger c_{-\mathbf{k}\downarrow a}^\dagger \rangle.$$

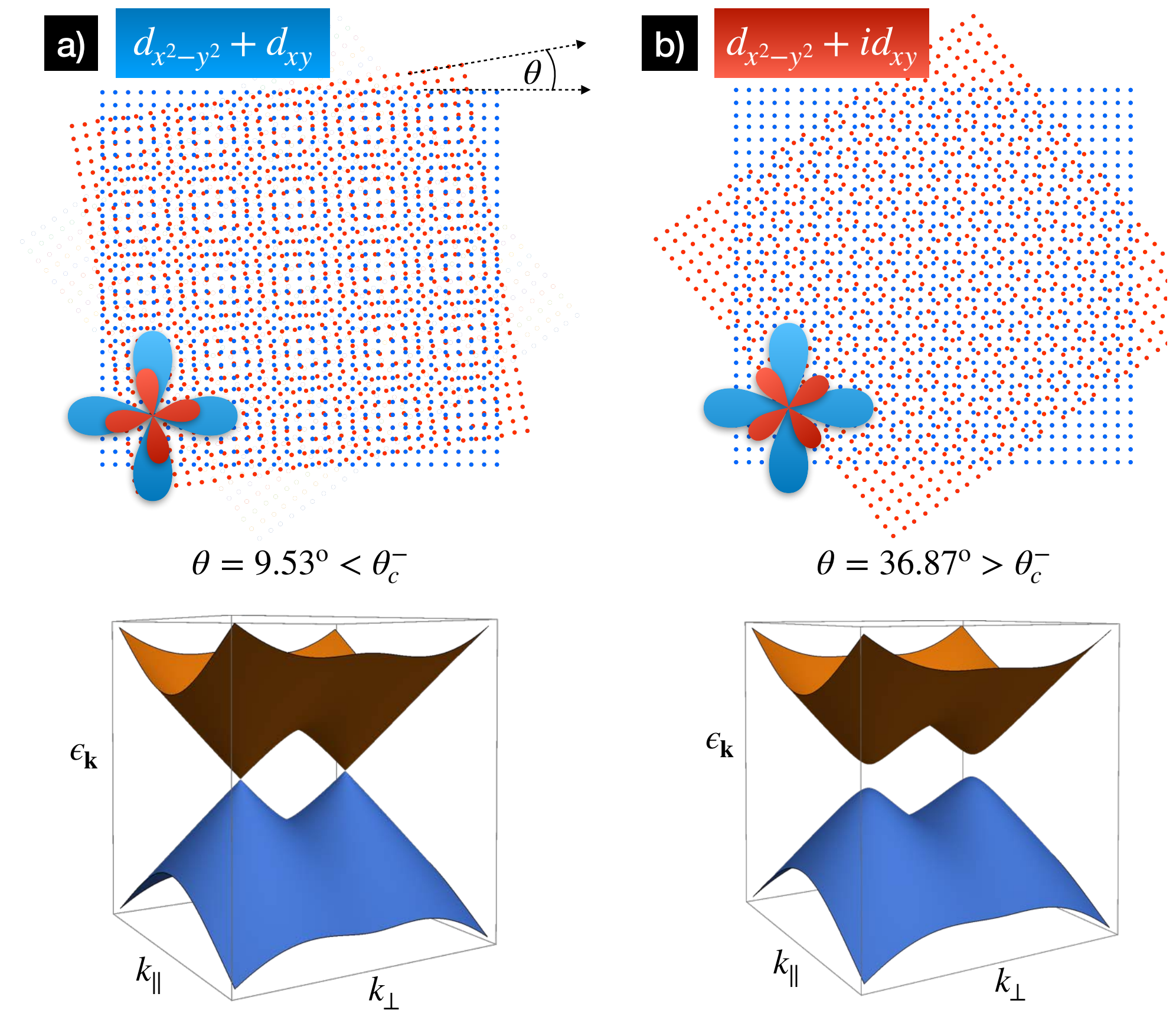
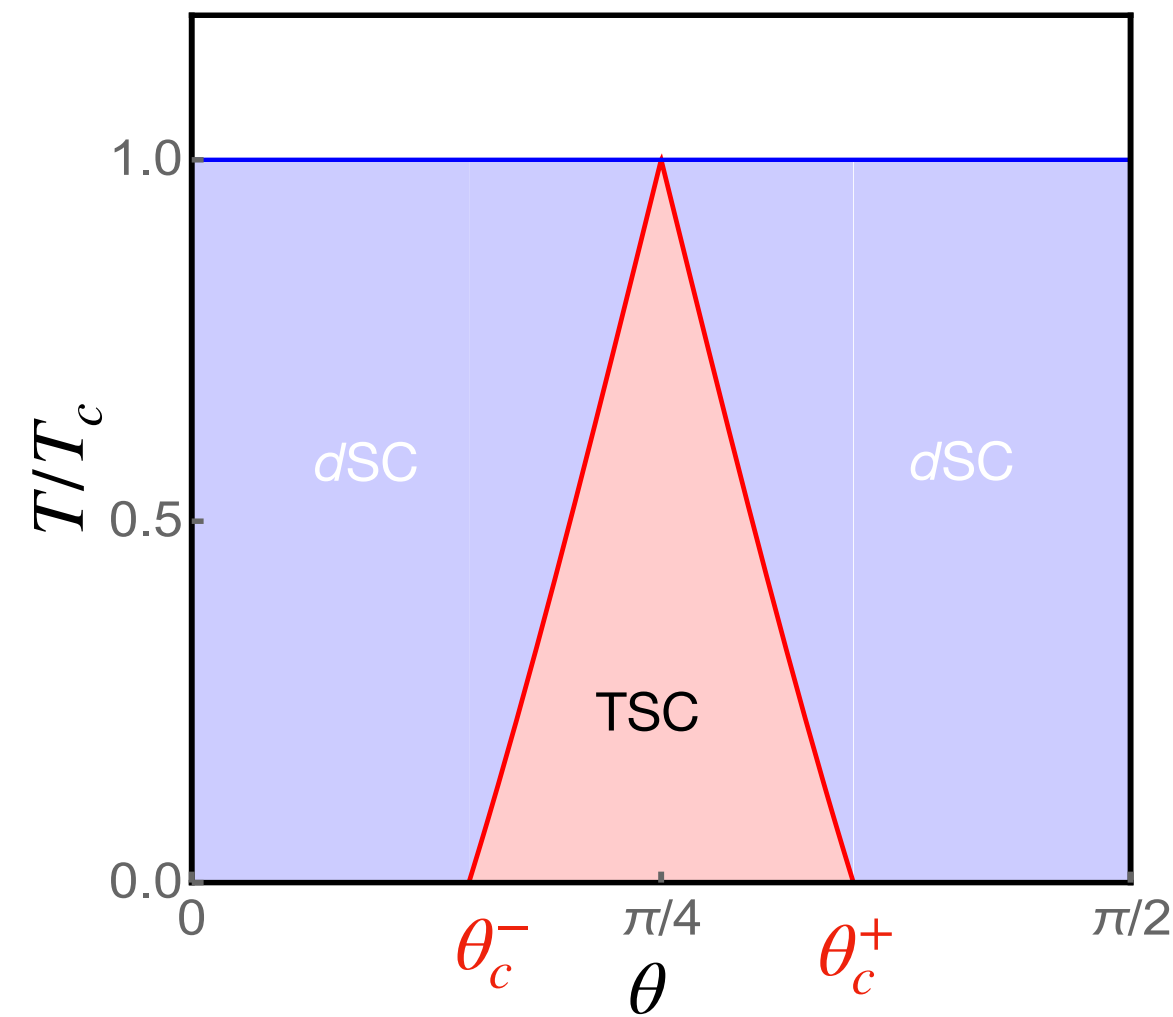


$$\mathcal{H} = \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^\dagger h_{\mathbf{k}} \Psi_{\mathbf{k}} + E_0 \quad h_{\mathbf{k}} = \begin{pmatrix} \xi_{\mathbf{k}} & \Delta_{\mathbf{k}1} & g & 0 \\ \Delta_{\mathbf{k}1}^* & -\xi_{\mathbf{k}} & 0 & -g \\ g & 0 & \xi_{\mathbf{k}} & \Delta_{\mathbf{k}2} \\ 0 & -g & \Delta_{\mathbf{k}2}^* & -\xi_{\mathbf{k}} \end{pmatrix}$$



3. Excitation spectra in the bilayer for $d_{x^2-y^2} + e^{i\phi}d_{xy}$ order parameter

parameter

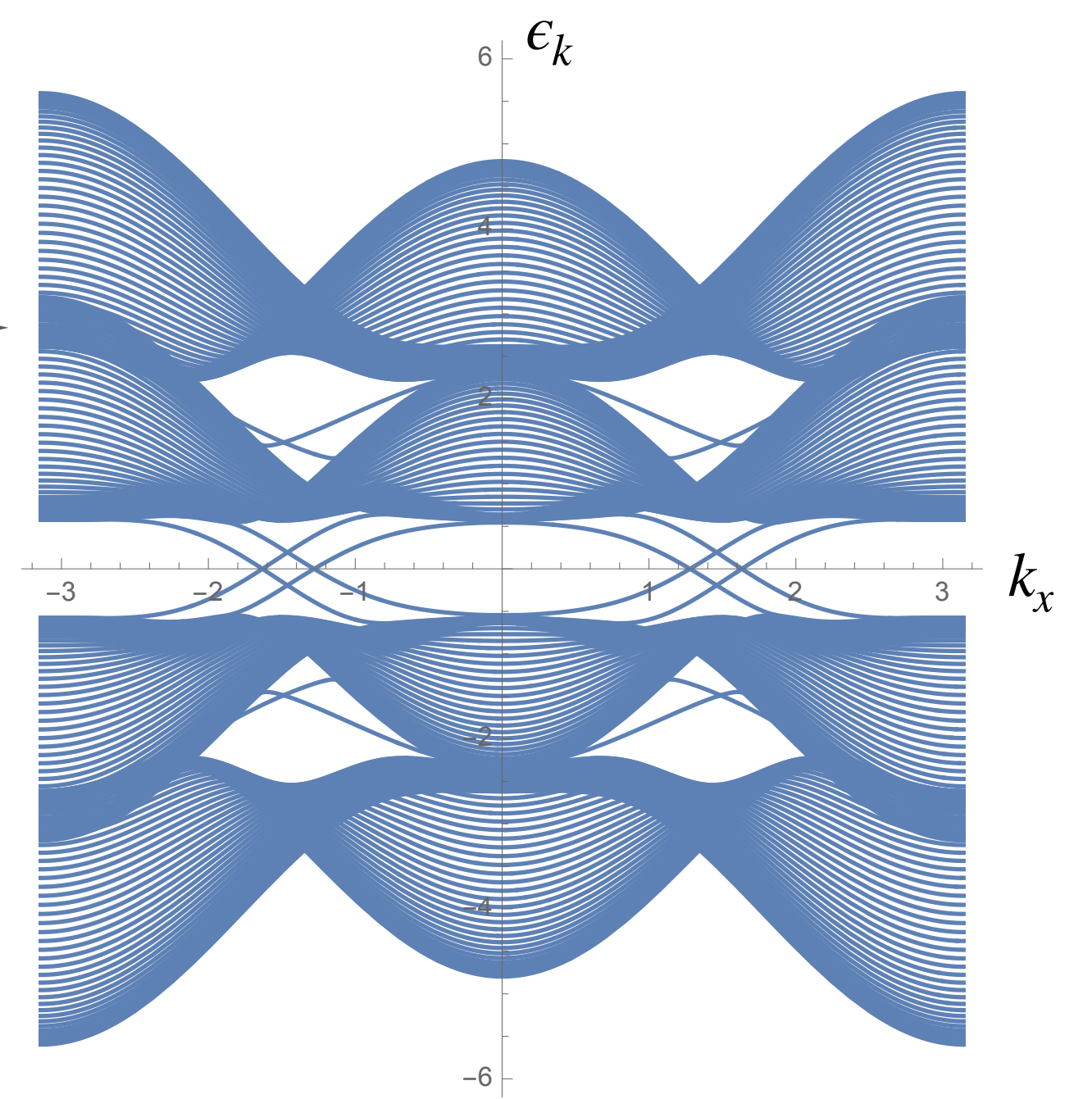
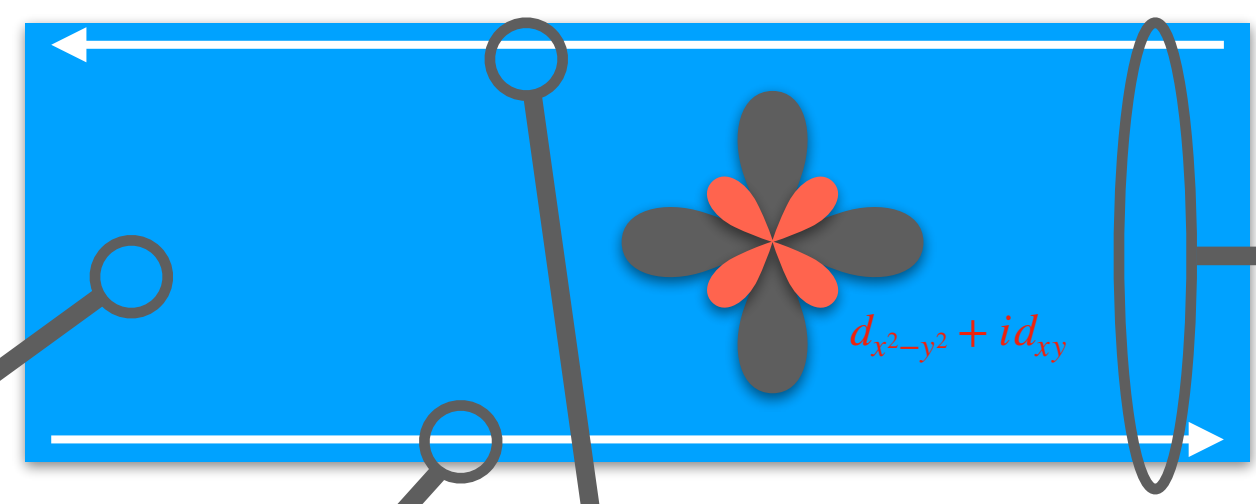


Time-reversal broken phase for any $\varphi \neq 0, \pi$

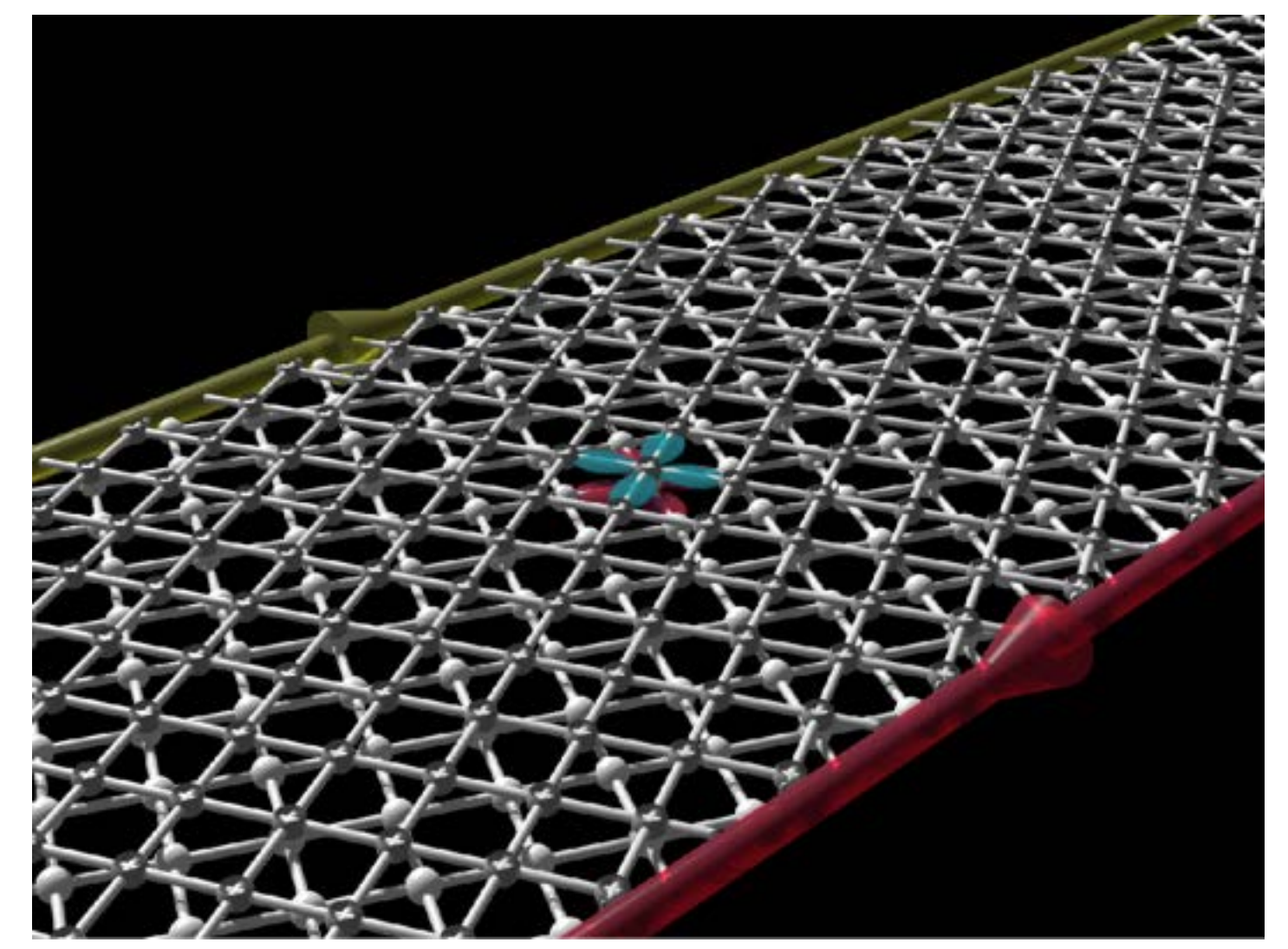
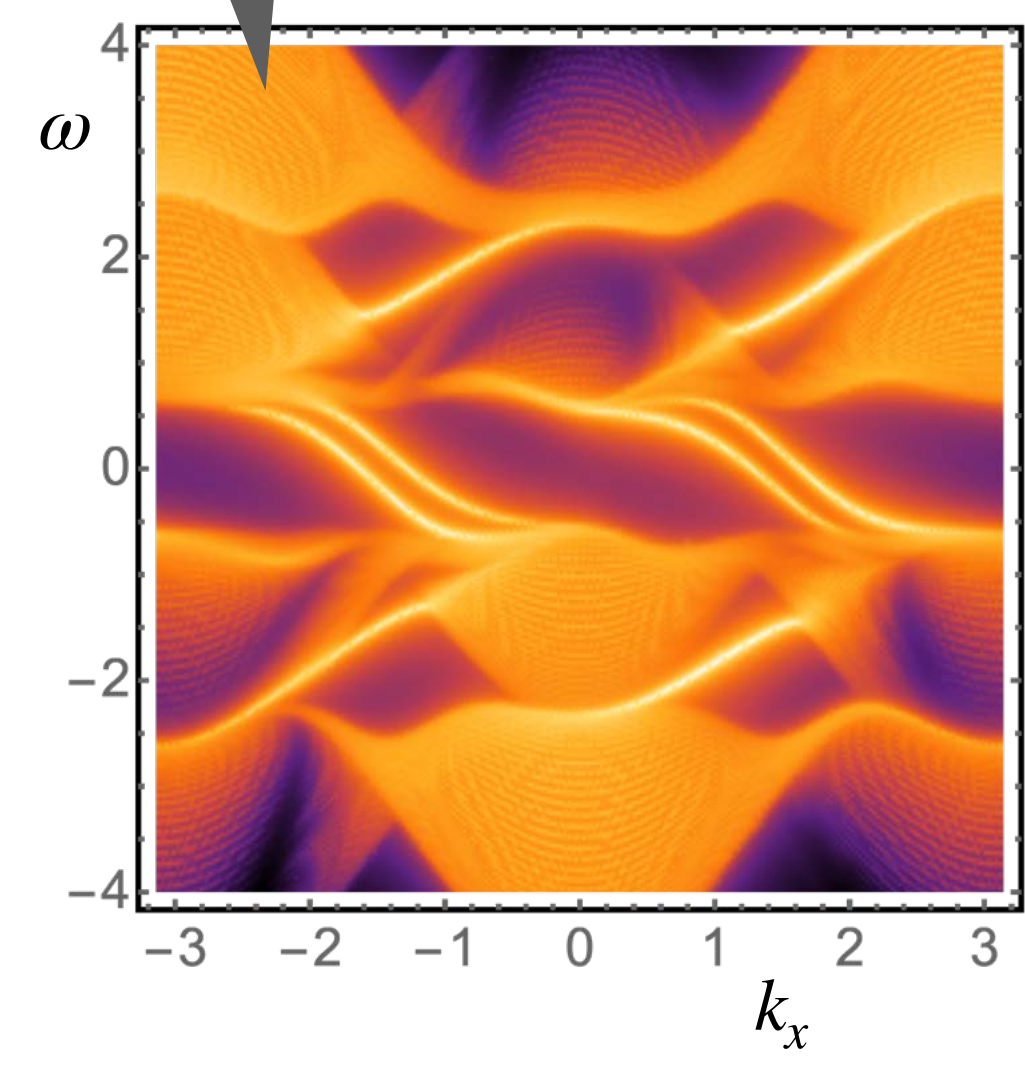
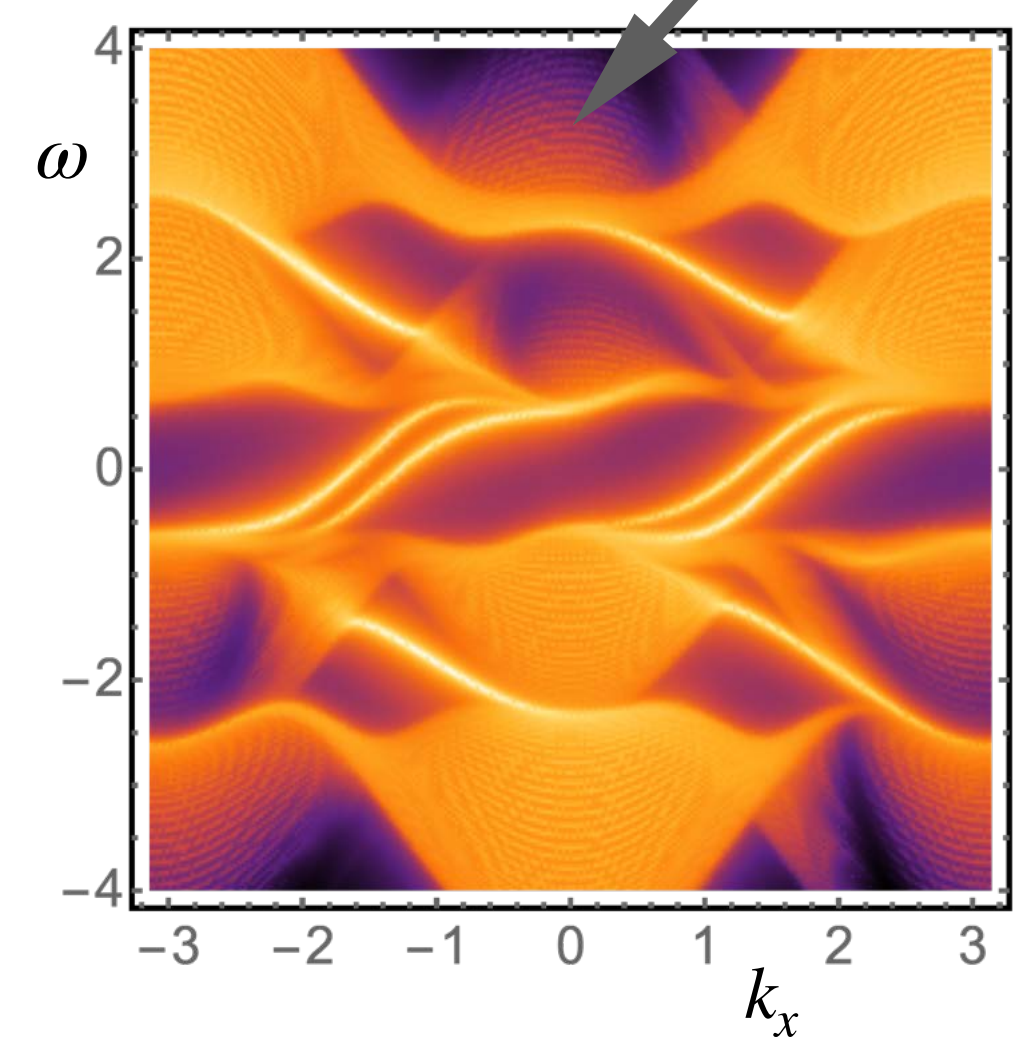
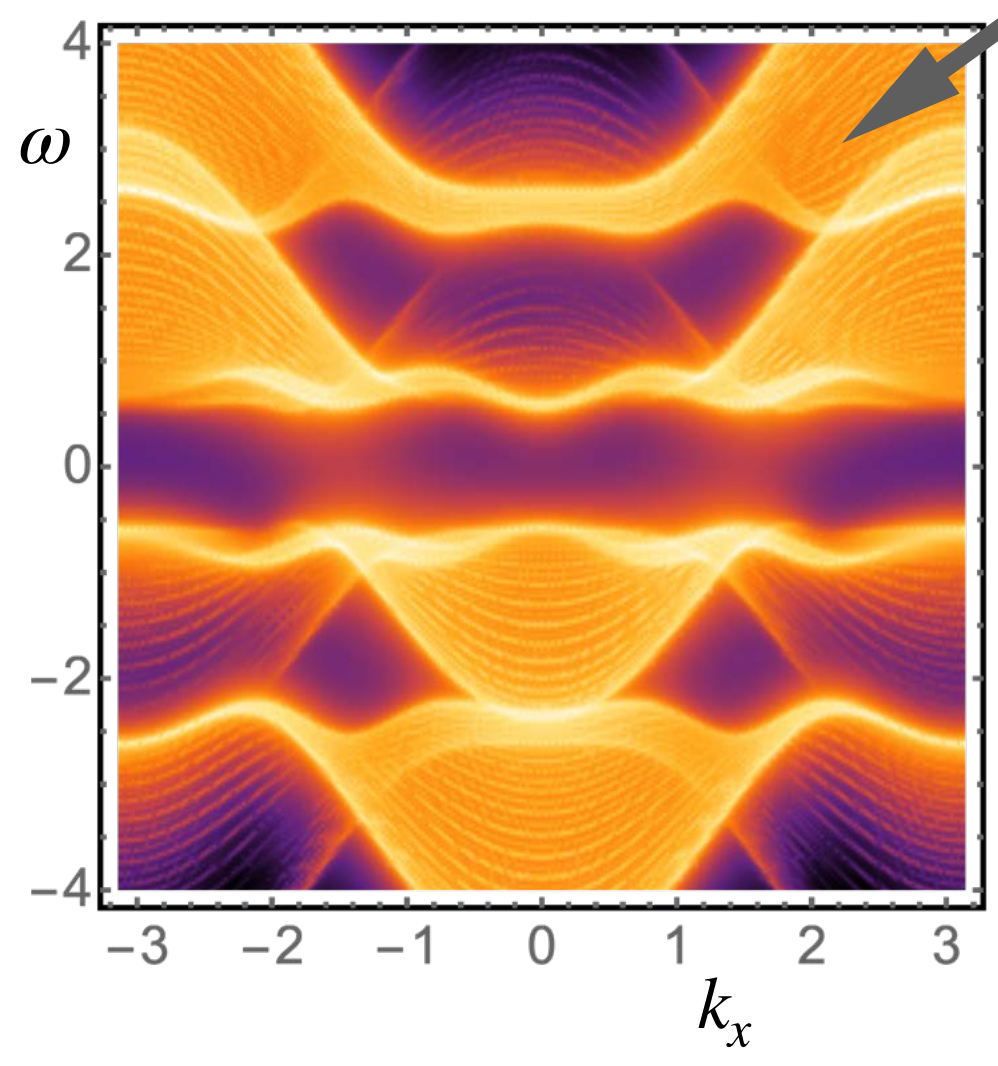
$$(d_{x^2-y^2} + e^{i\phi}d_{xy}) \xrightarrow{\mathcal{T}} (d_{x^2-y^2} + e^{-i\phi}d_{xy})$$

4. Topological superconductivity, protected edge modes

Consider a long strip geometry:



Spectral function
 $A(k_x, \omega)$



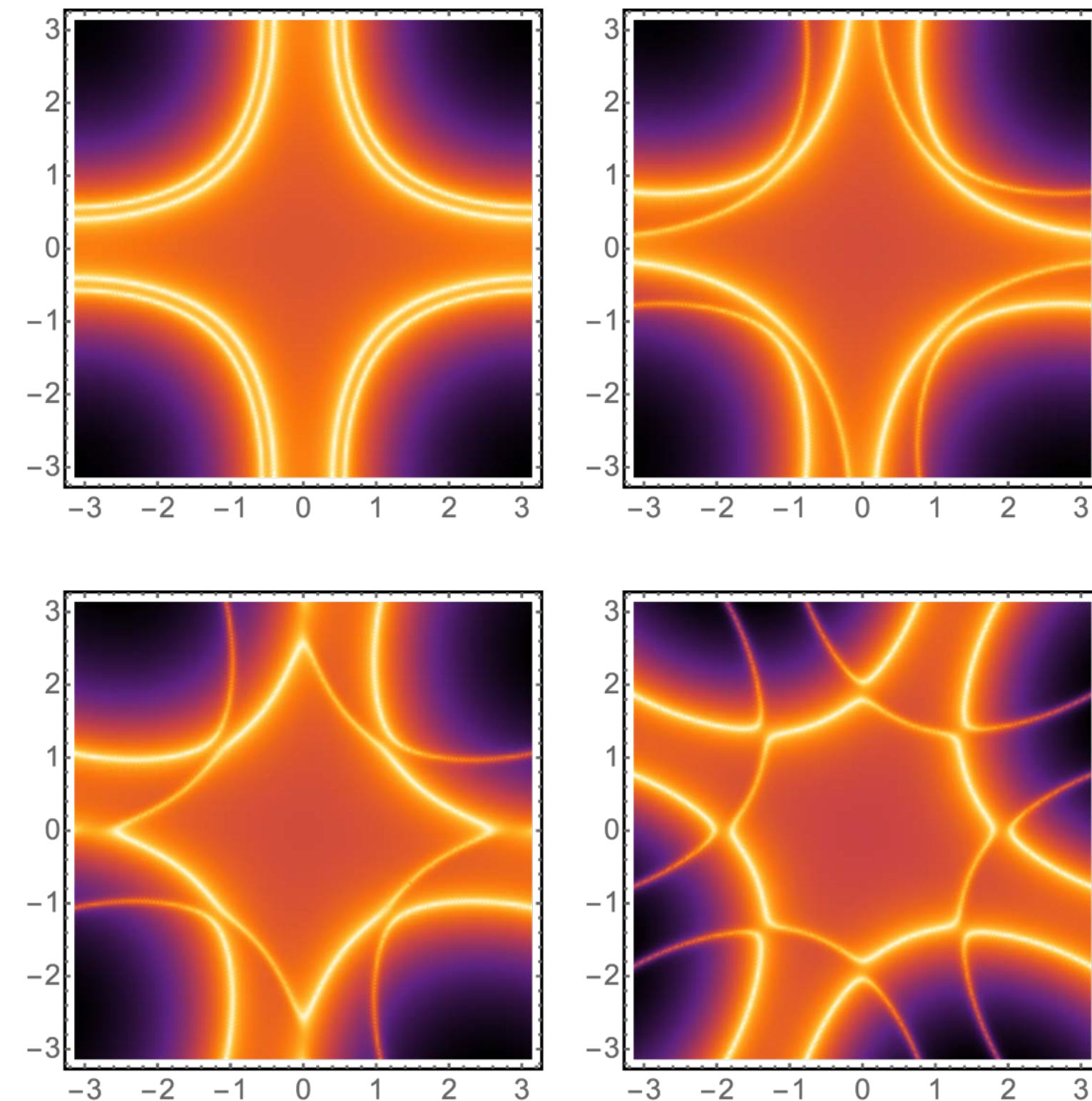
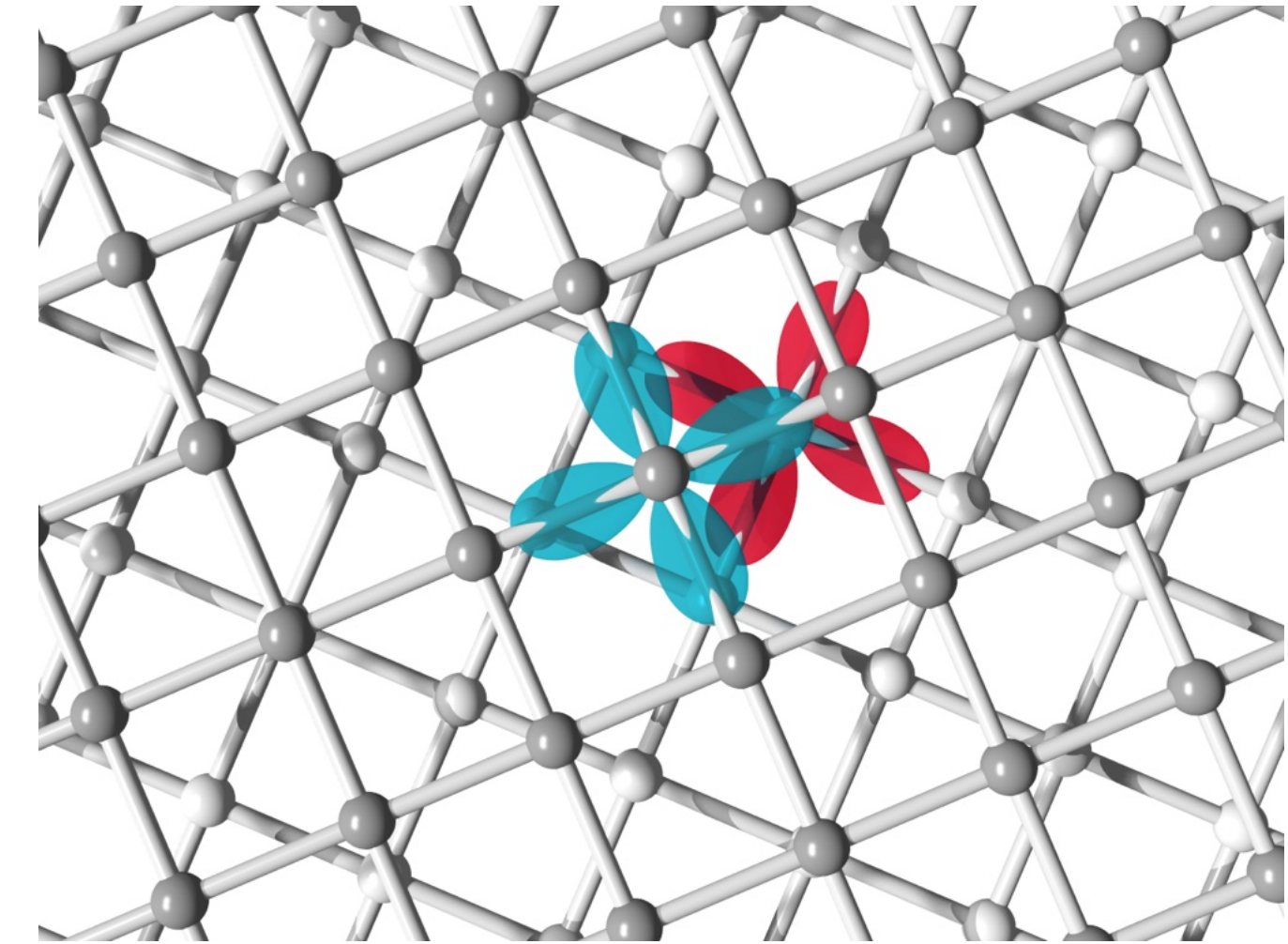
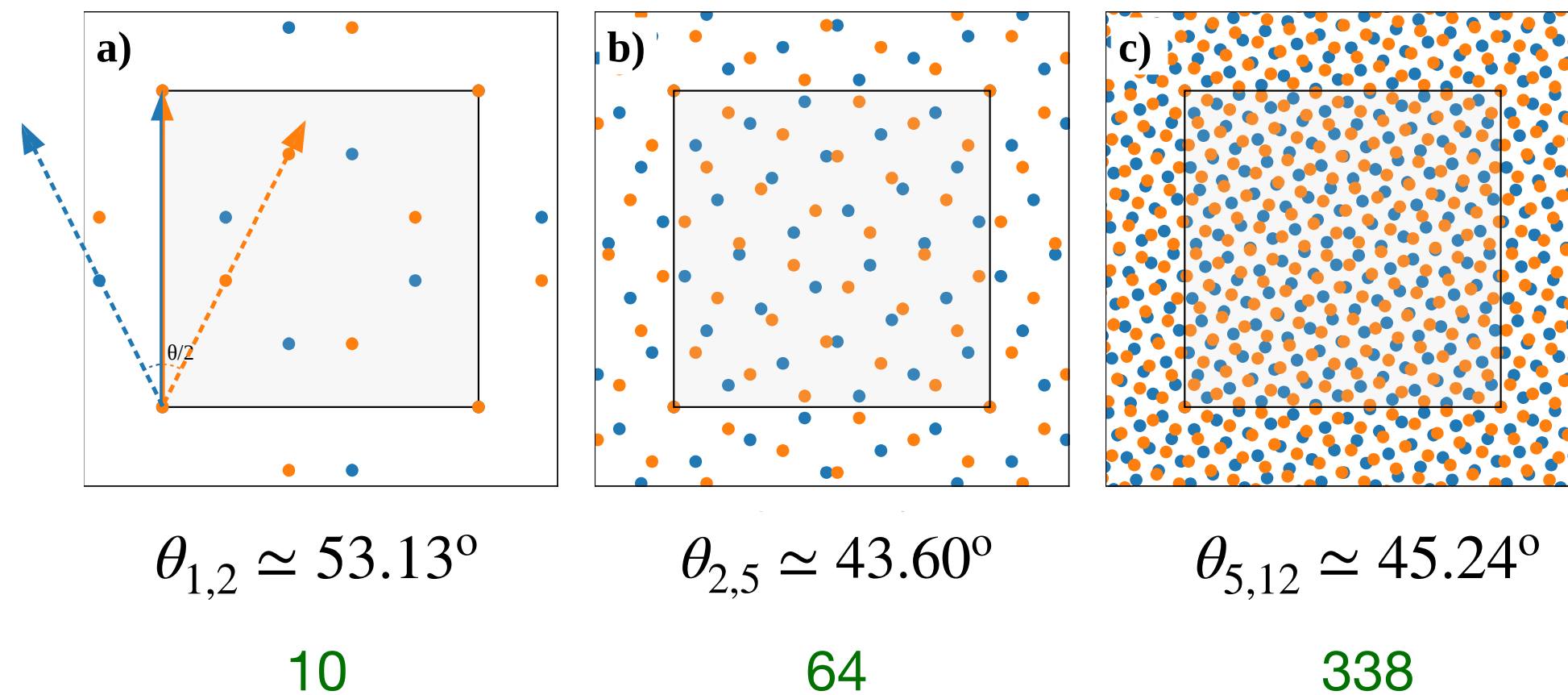
5. Self-consistent theory on the lattice

Hubbard model with nn attraction and on-site repulsion

$$H = - \sum_{ij, \sigma a} t_{ij} c_{i\sigma a}^\dagger c_{j\sigma a} - \mu \sum_{i\sigma a} n_{i\sigma a} + \sum_{ij, a} V_{ij} n_{ia} n_{ja} - \sum_{ij\sigma} g_{ij} c_{i\sigma 1}^\dagger c_{j\sigma 2},$$

Solve using standard mean-field decoupling in the pairing channel for commensurate twist angles

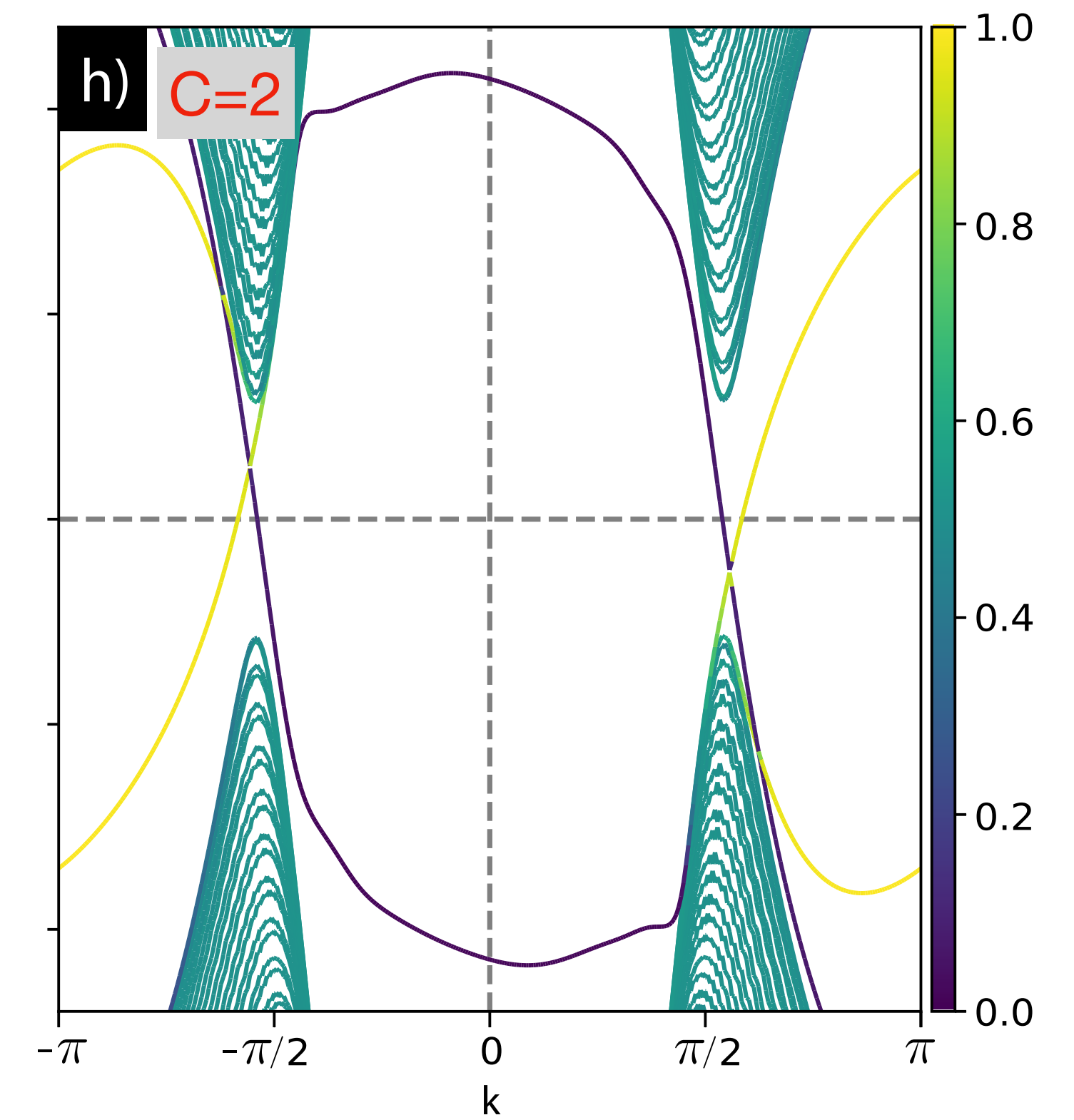
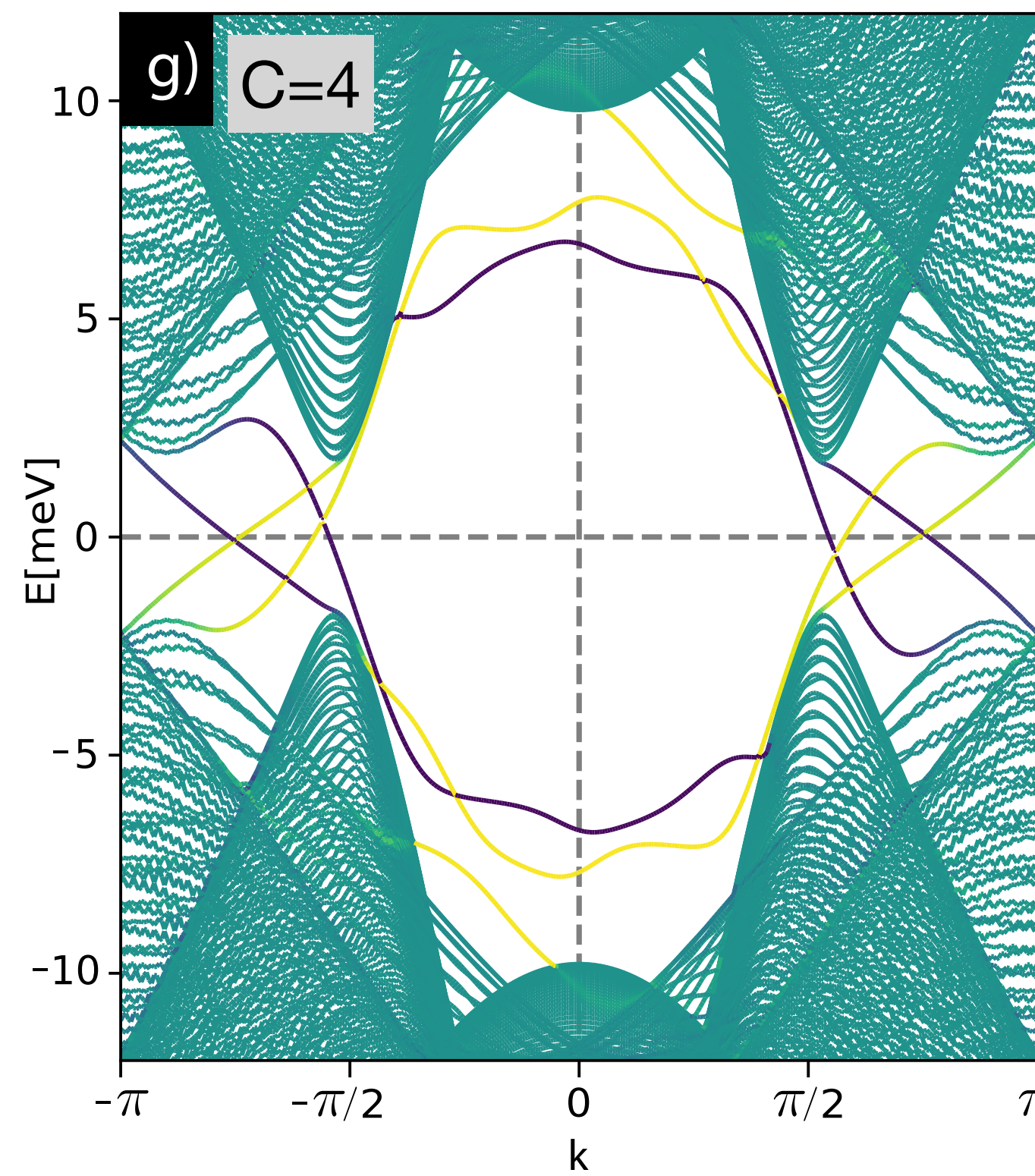
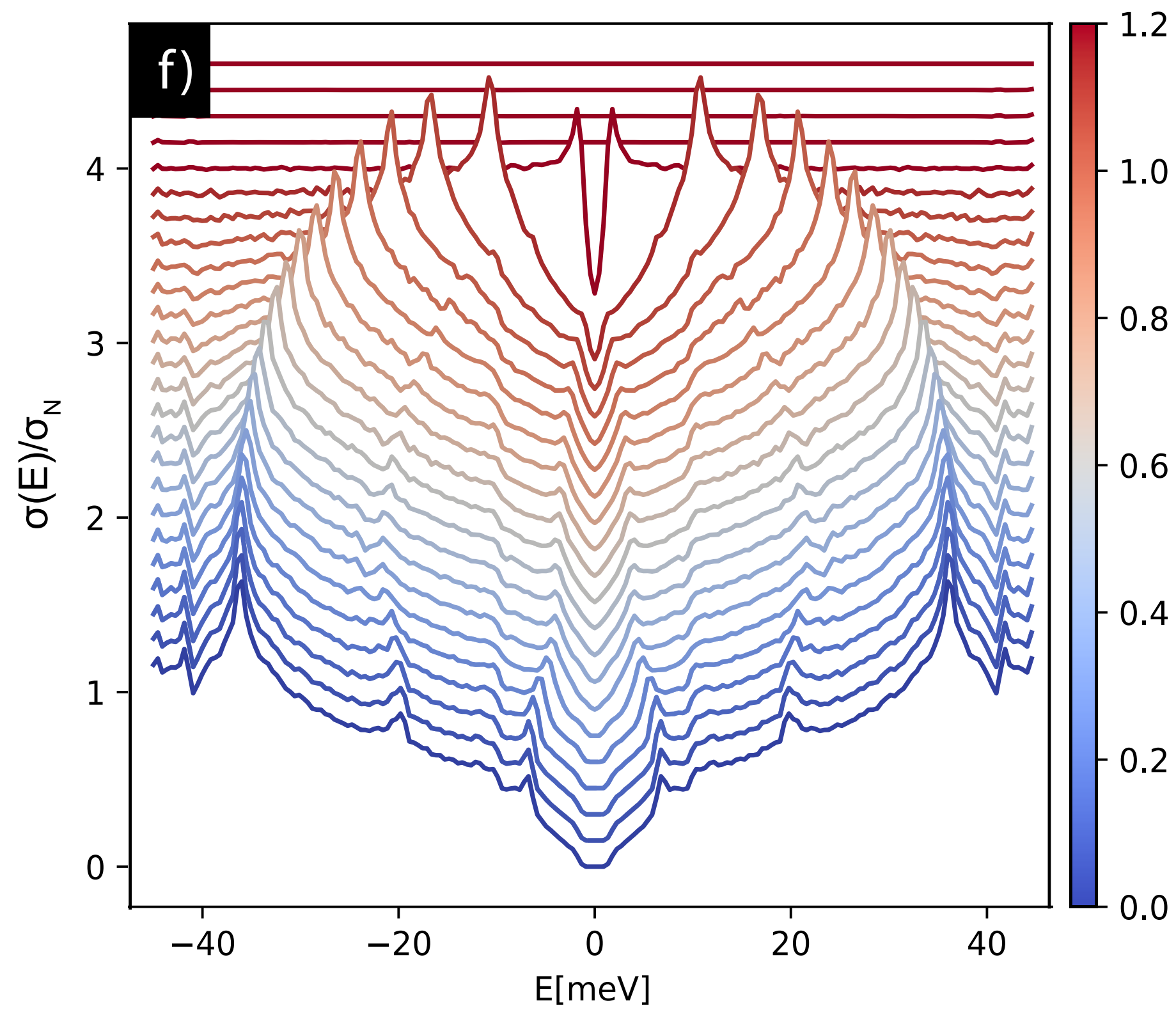
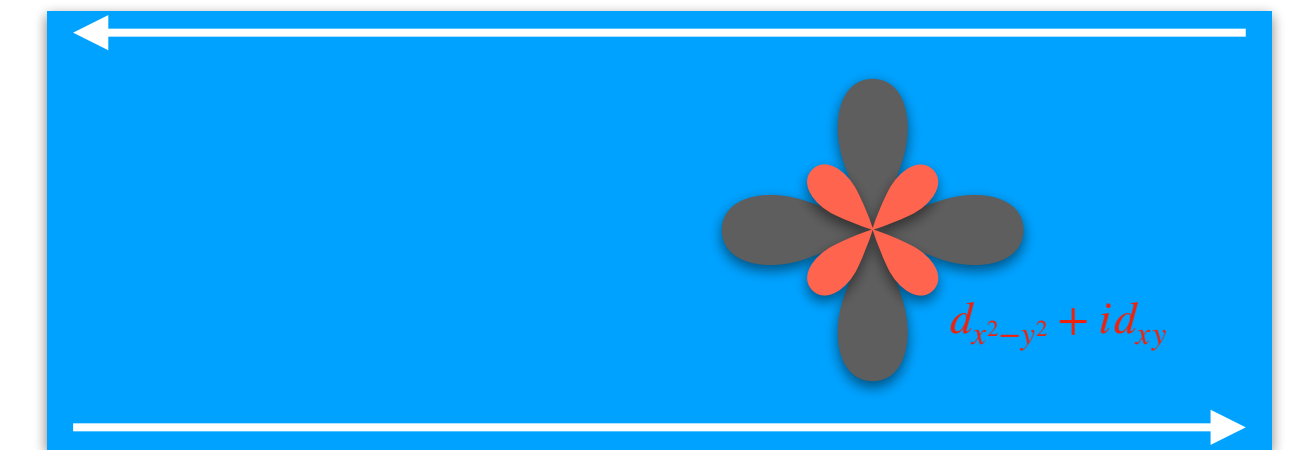
$$\theta_{m,n} = 2 \arctan(m/n)$$



Self-consistent theory on the lattice

Tunneling density of states and edge modes

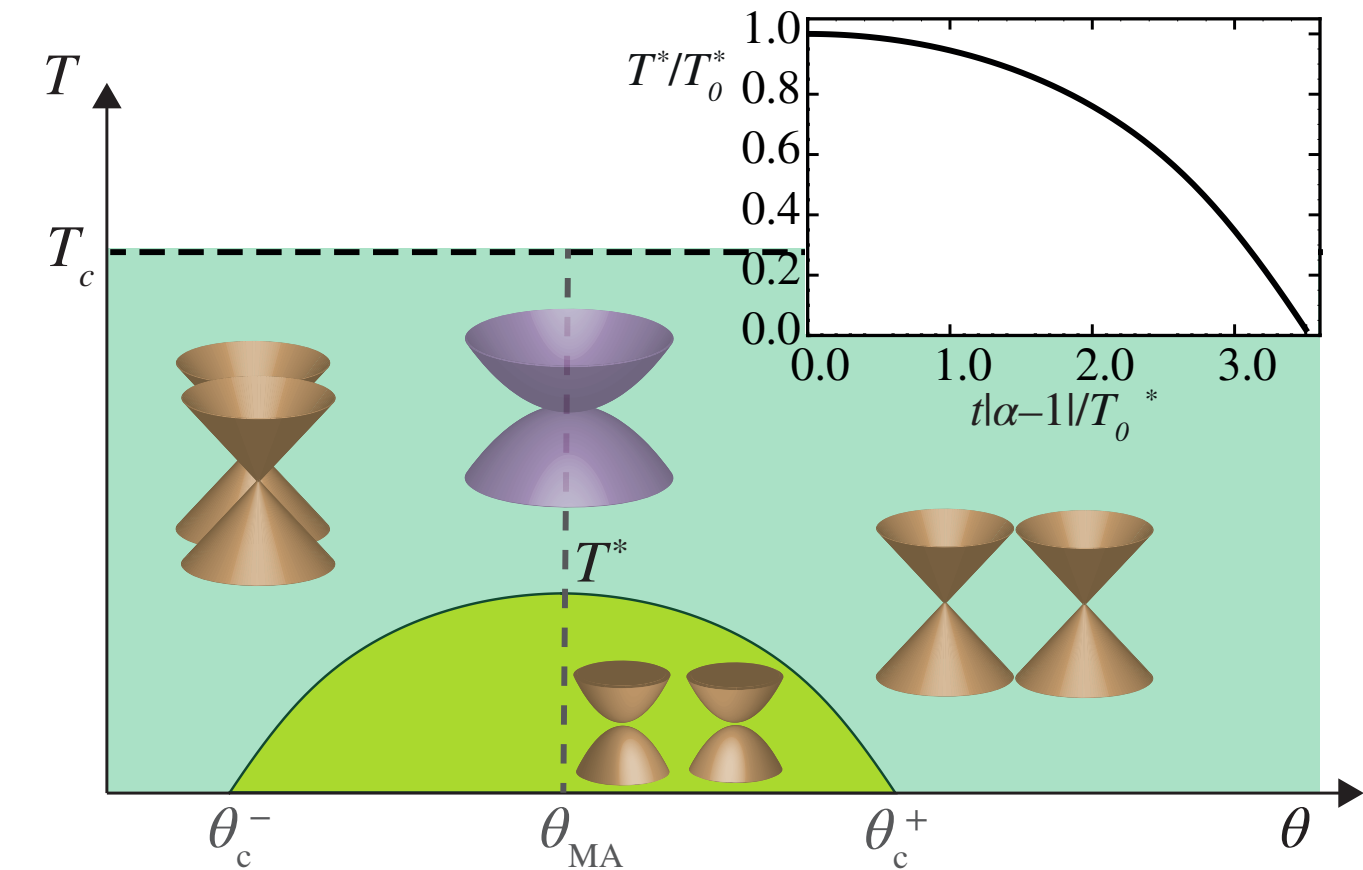
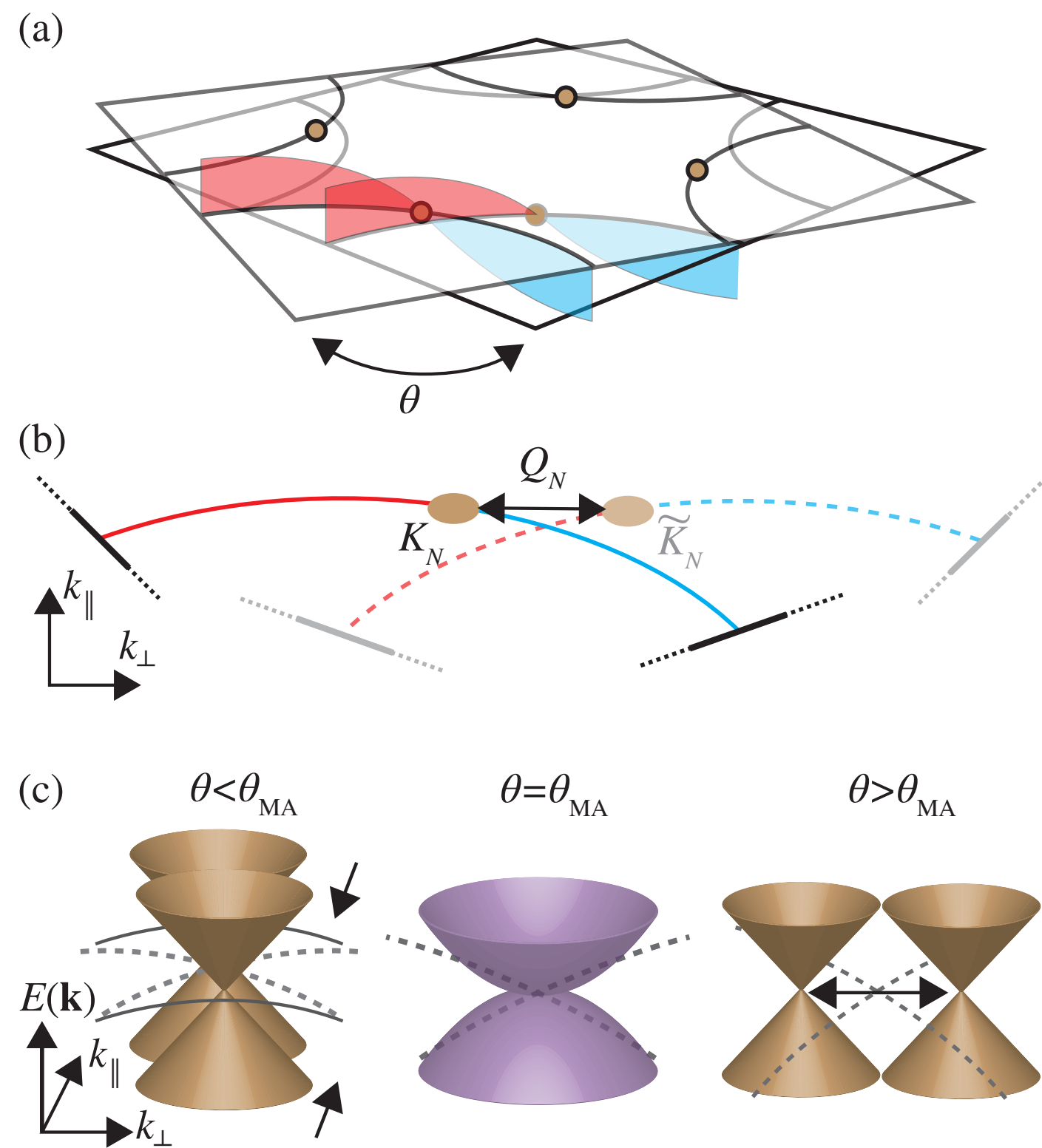
Long strip geometry



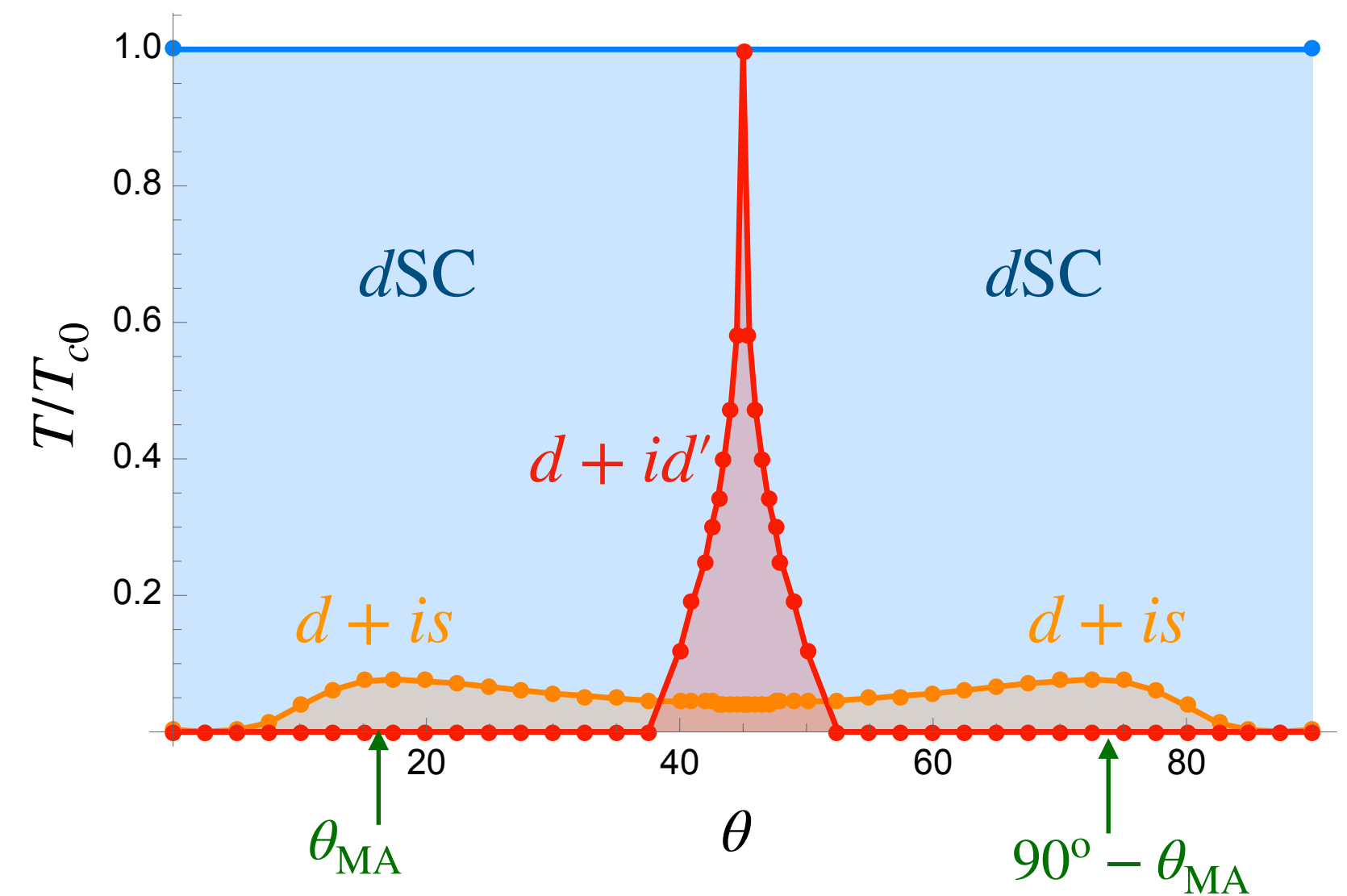
Interaction effects, flat bands, graphene similarities?

Magic angles and current-induced topology in twisted nodal superconductors

Pavel A. Volkov,* Justin H. Wilson, and J. H. Pixley
 Department of Physics and Astronomy, Center for Materials Theory,
 Rutgers University, Piscataway, NJ 08854, USA
 (Dated: December 16, 2020)

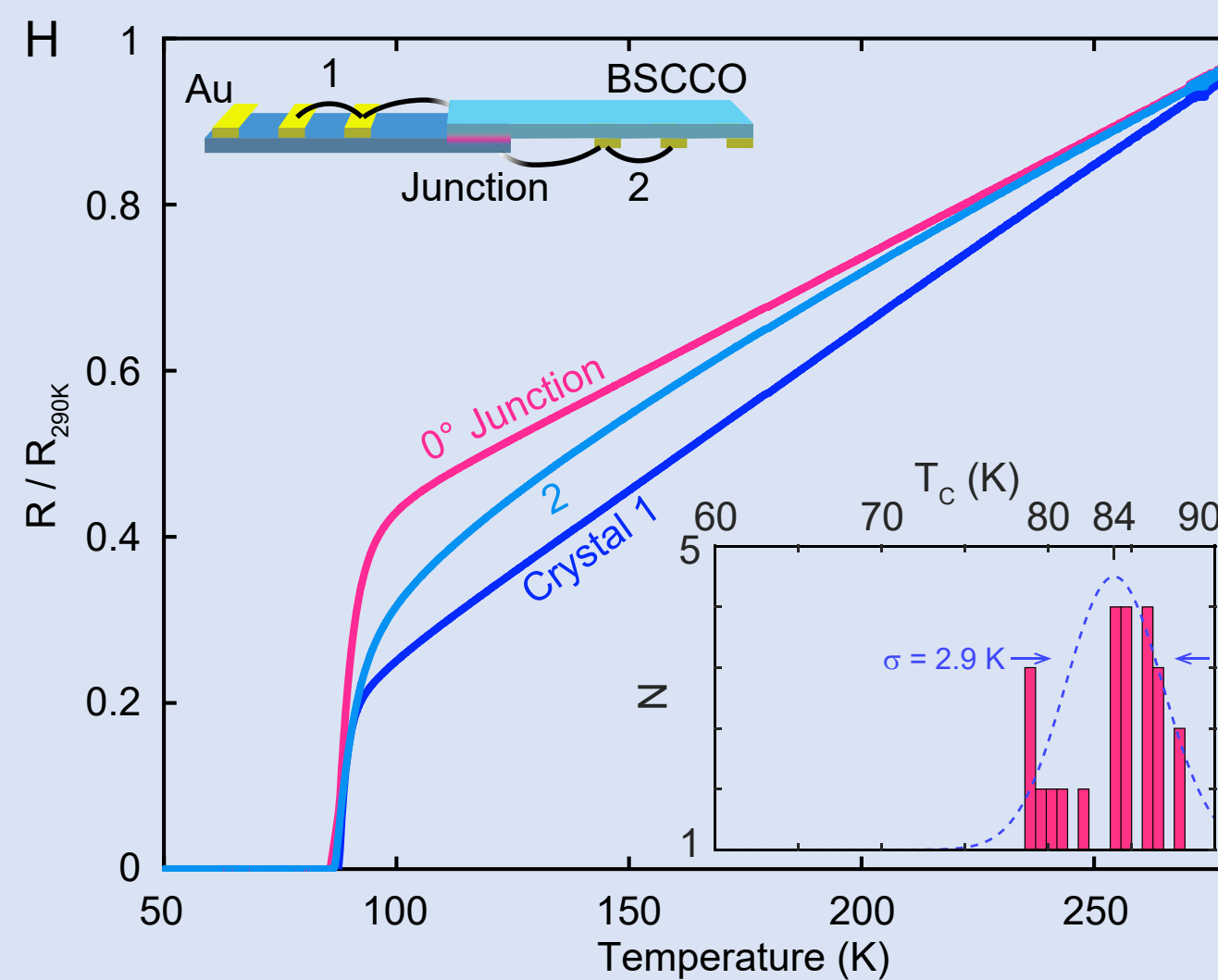
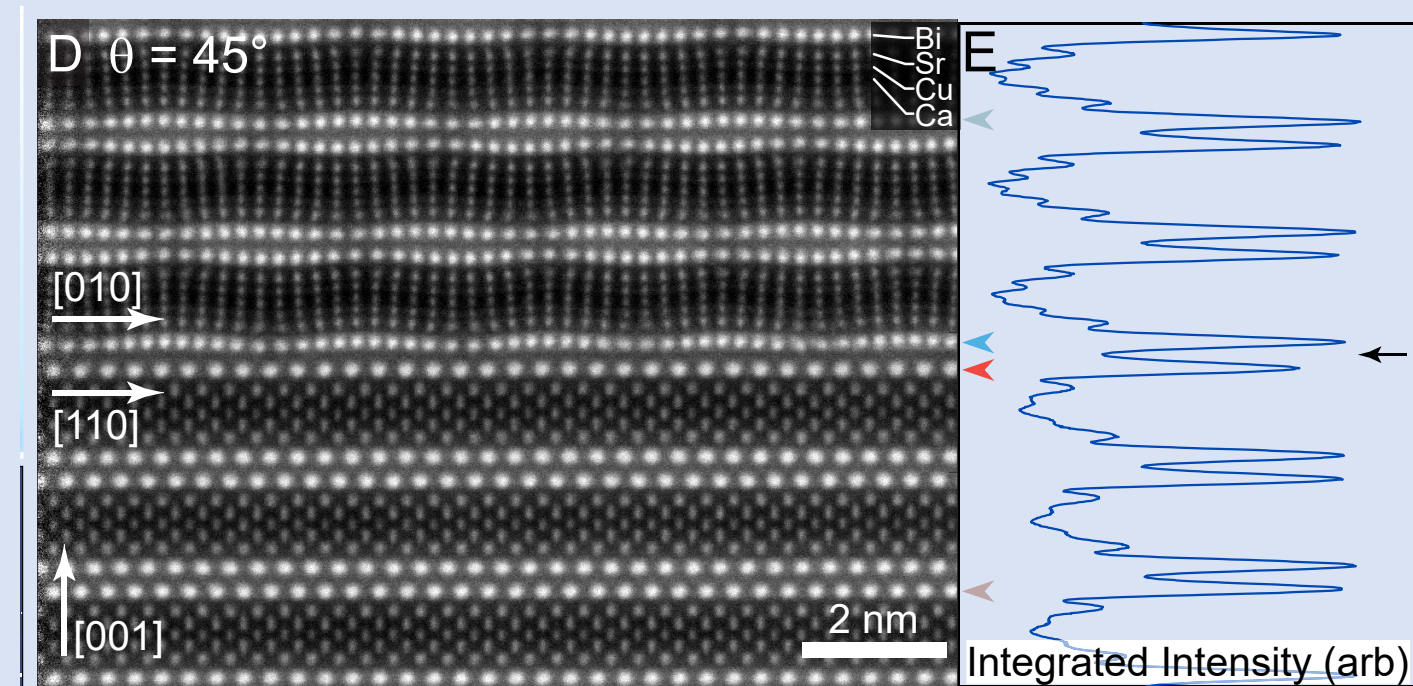


Self-consistently determined phase diagram - present work (assuming interaction-induced s-wave instability)

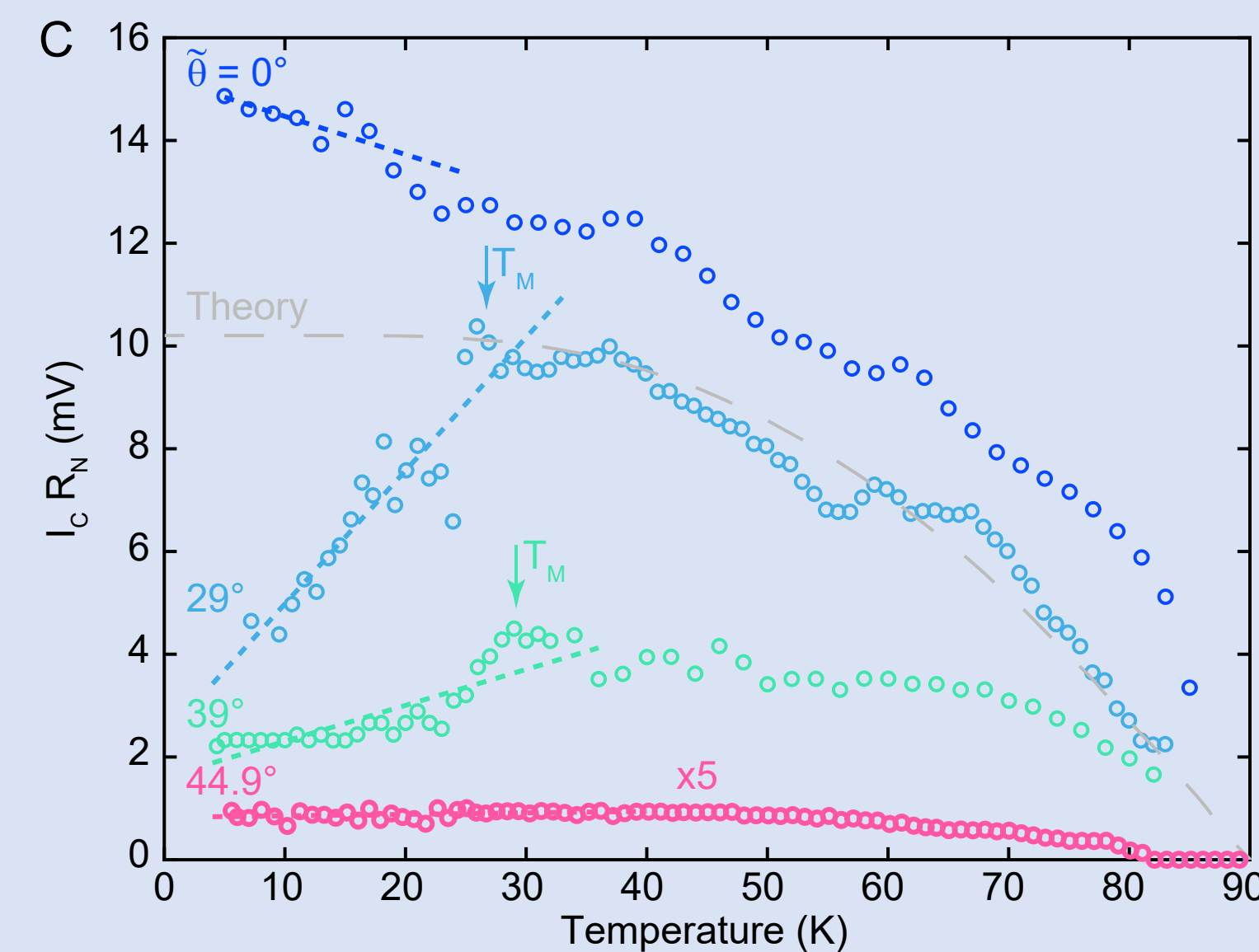
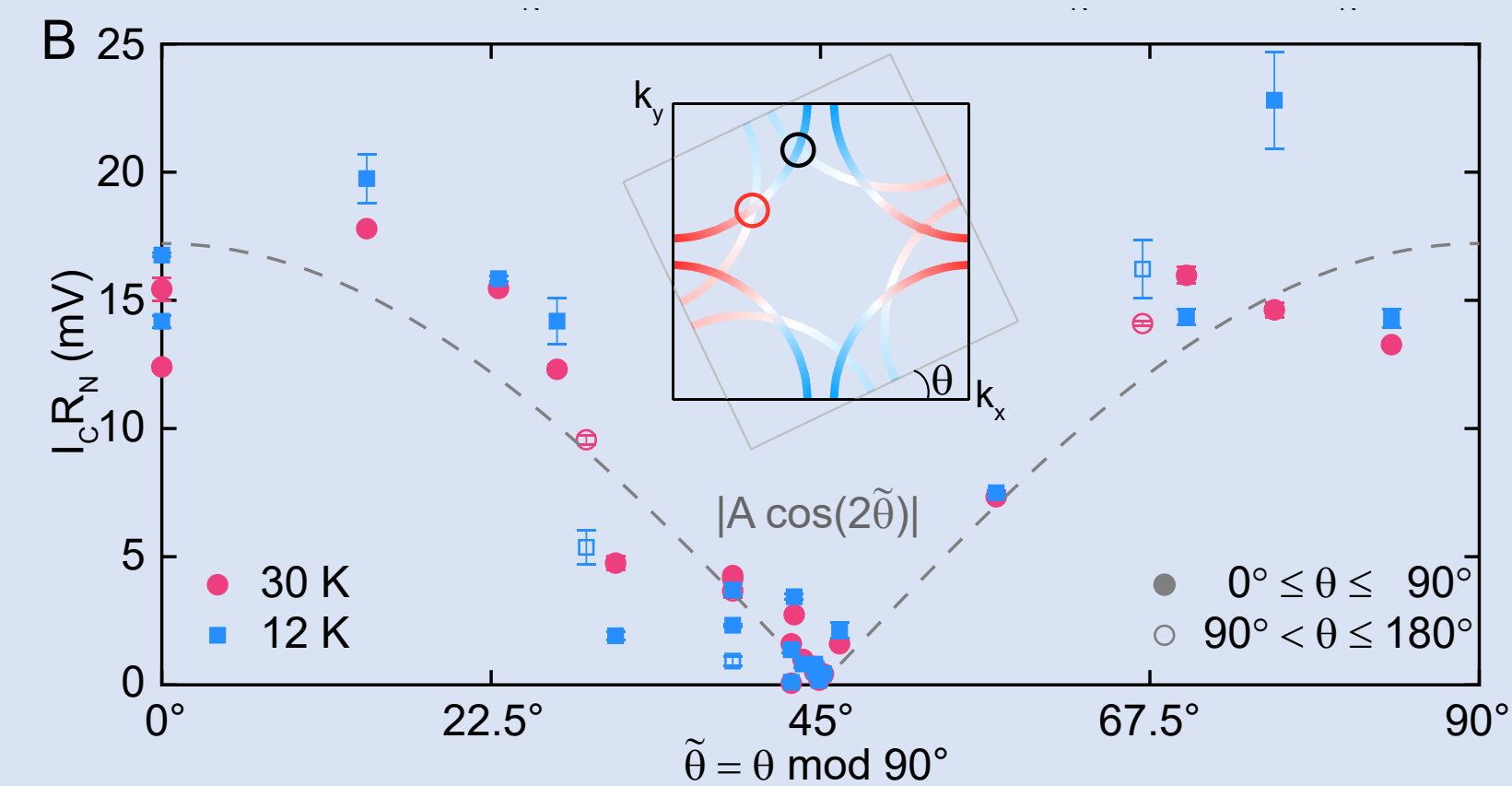


Experimental efforts

Exceptionally clean and ordered interfaces



Results



Microscopic model - Continuum Bogoliubov-de Gennes

We wish to understand how the anomalous increase in $I_c R_N$ follows from a theory of Josephson tunnelling between twisted d -wave superconductors.

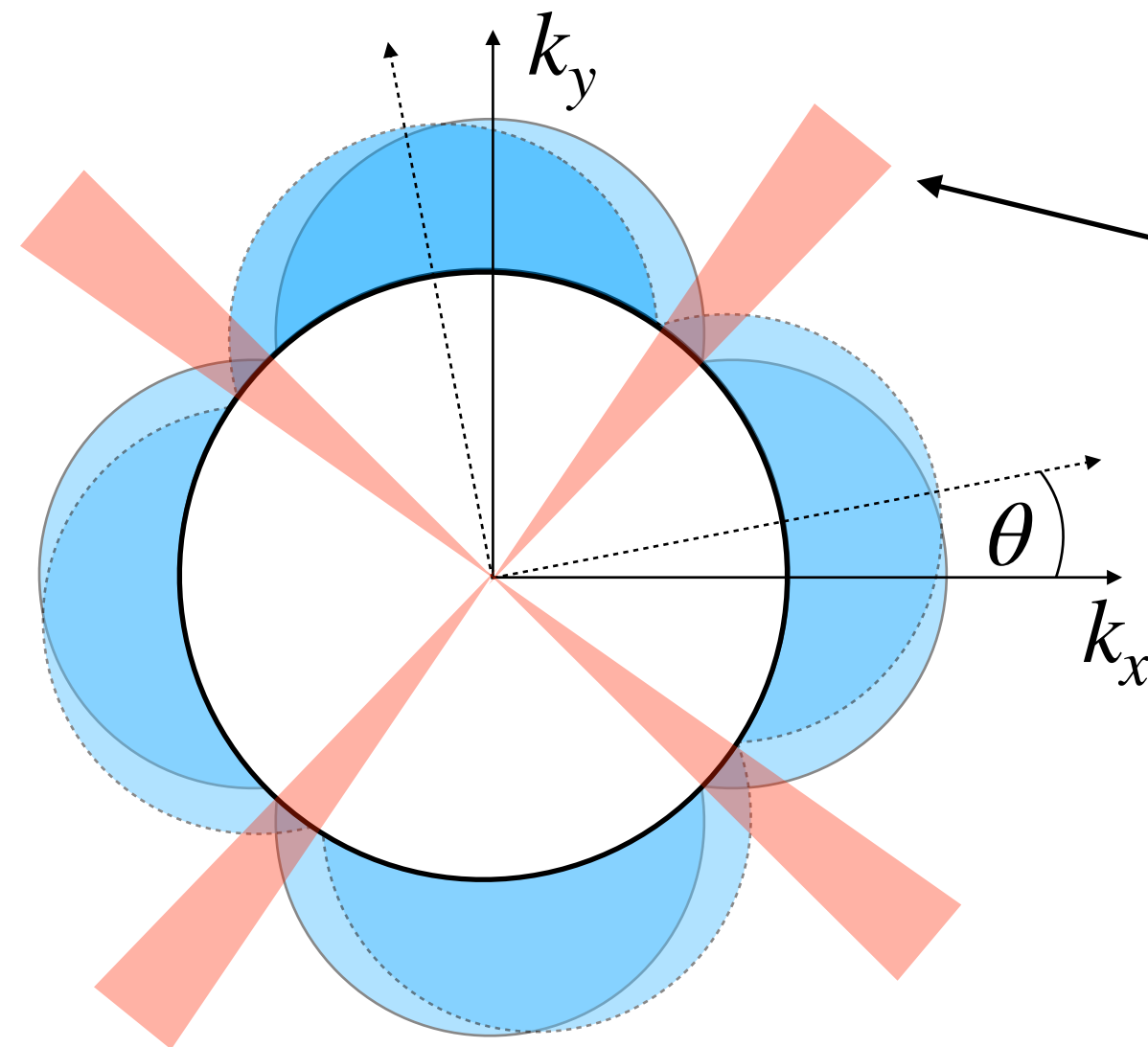
$$\mathcal{H} = \sum_{\mathbf{k}\sigma a} \xi_{\mathbf{k}a} c_{\mathbf{k}\sigma a}^\dagger c_{\mathbf{k}\sigma a} + g \sum_{\mathbf{k}\sigma} \left(c_{\mathbf{k}\sigma 1}^\dagger c_{\mathbf{k}\sigma 2} + \text{h.c.} \right) + \sum_{\mathbf{k}a} \left(\Delta_{\mathbf{k}a} c_{\mathbf{k}\uparrow a}^\dagger c_{-\mathbf{k}\downarrow a}^\dagger + \text{h.c.} \right) - \sum_{\mathbf{k}a} \Delta_{\mathbf{k}a} \langle c_{\mathbf{k}\uparrow a}^\dagger c_{-\mathbf{k}\downarrow a}^\dagger \rangle.$$

One can show that the interlayer critical current has the form

$$I_c(T) = \sum_{\mathbf{k}} \Delta_{\mathbf{k}1} \Delta_{\mathbf{k}2} \Omega(\xi_{\mathbf{k}}, T)$$

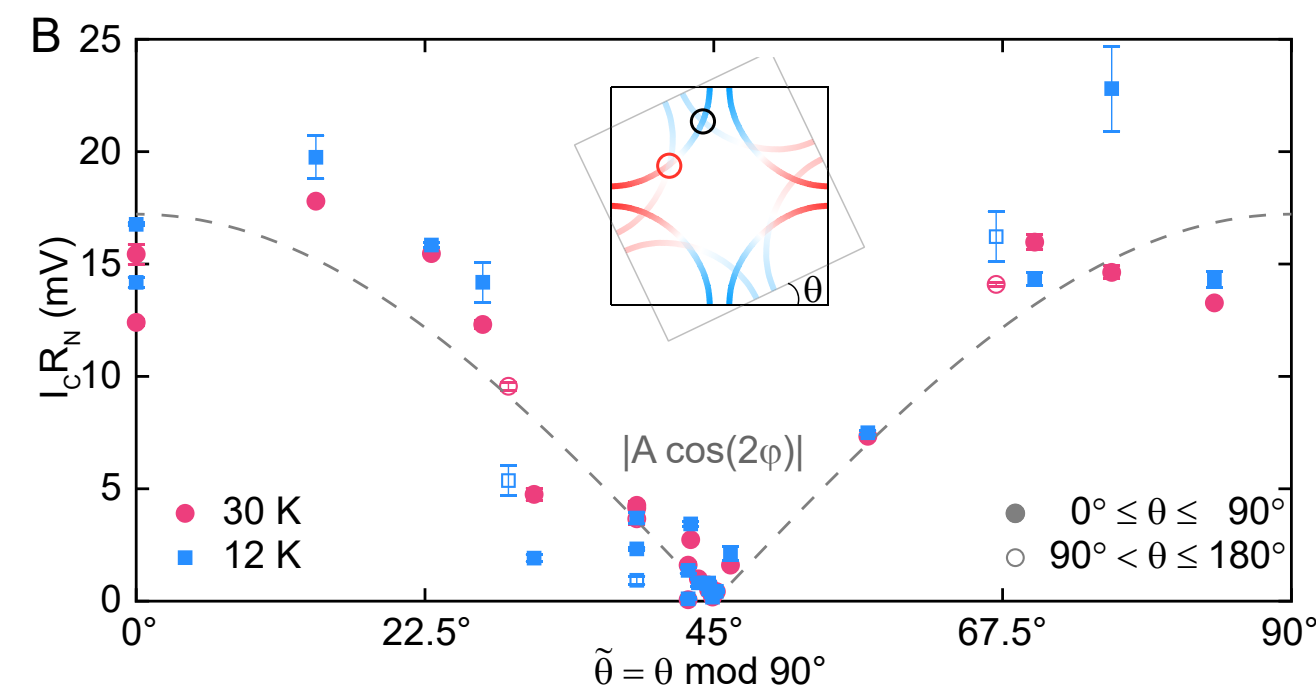
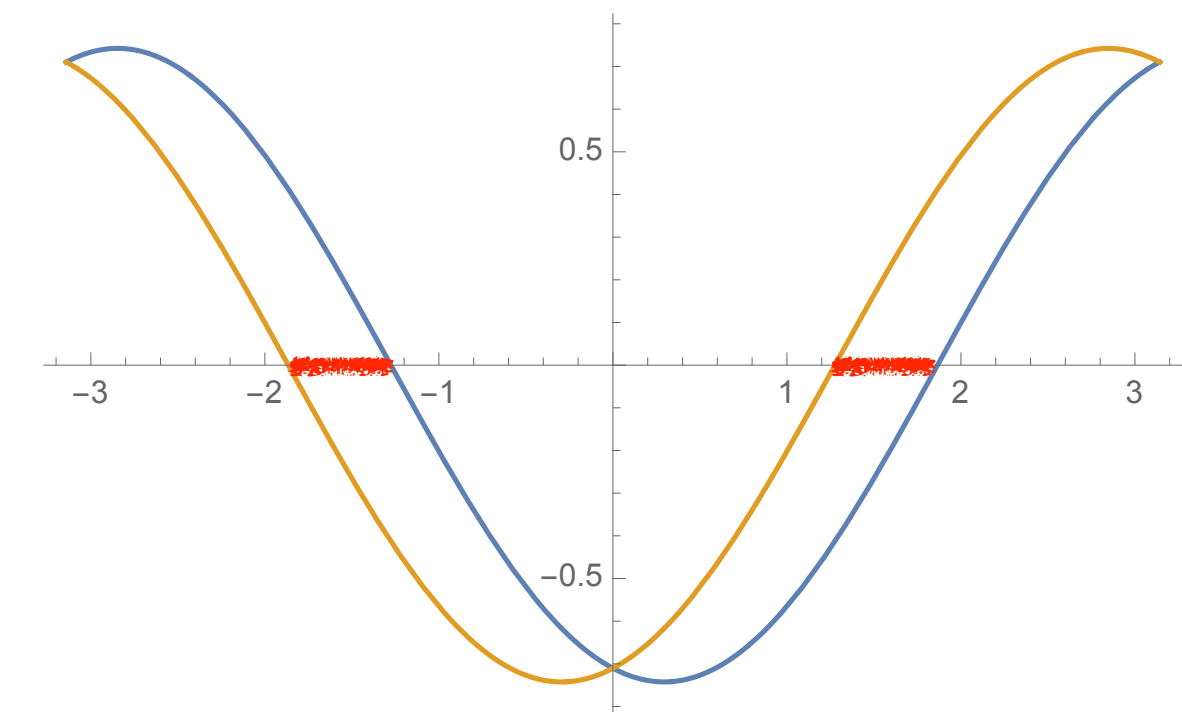
where $\Omega(\xi_{\mathbf{k}}, T) \geq 0$. In a dSC we have

$$\Delta_{\mathbf{k}1} \Delta_{\mathbf{k}2} = \Delta_0^2 \cos(2\alpha + \theta) \cos(2\alpha - \theta)$$



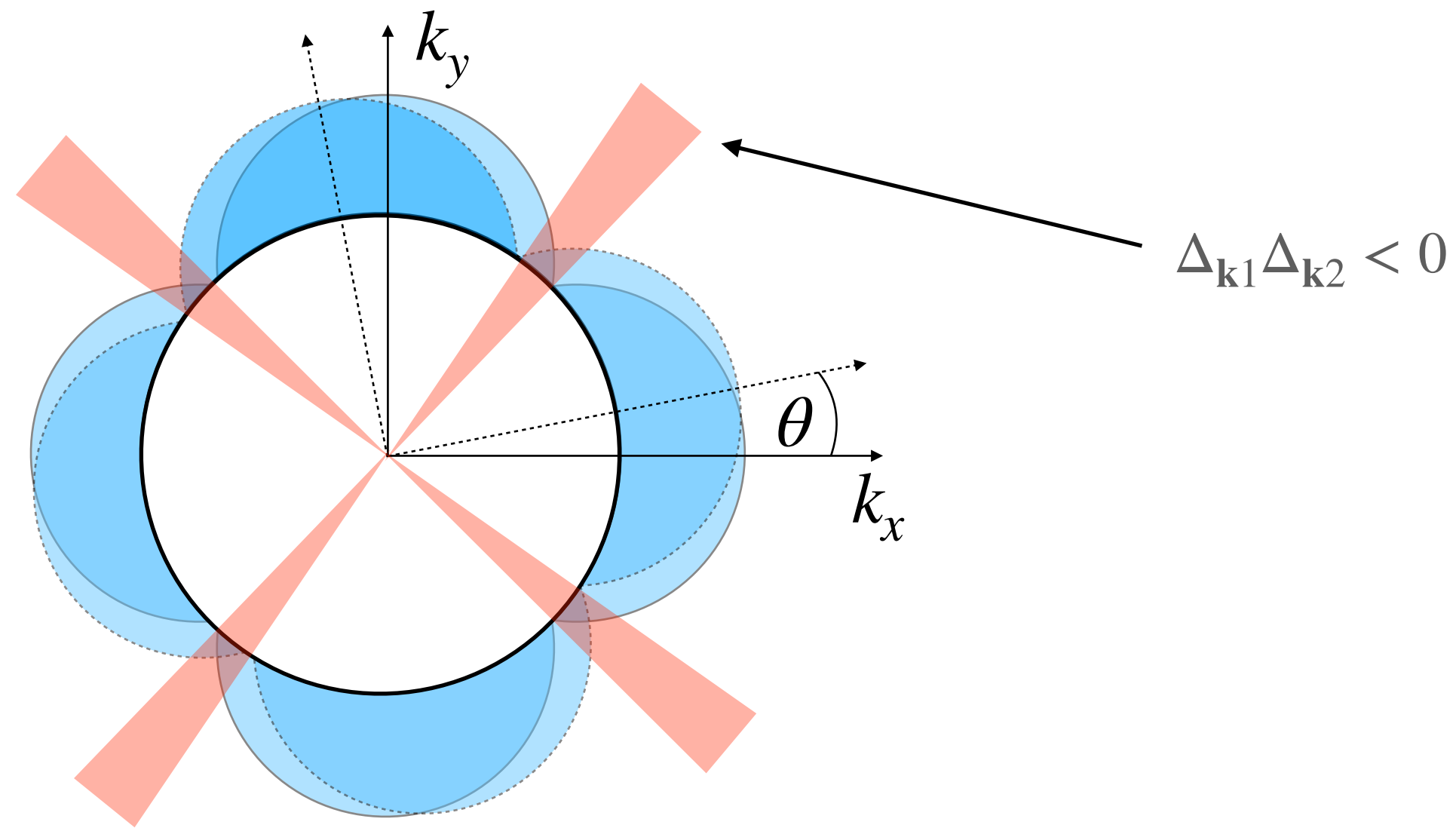
$$\Delta_{\mathbf{k}1} \Delta_{\mathbf{k}2} < 0$$

Nodal regions give **NEGATIVE** contribution to $I_c(T=0)$



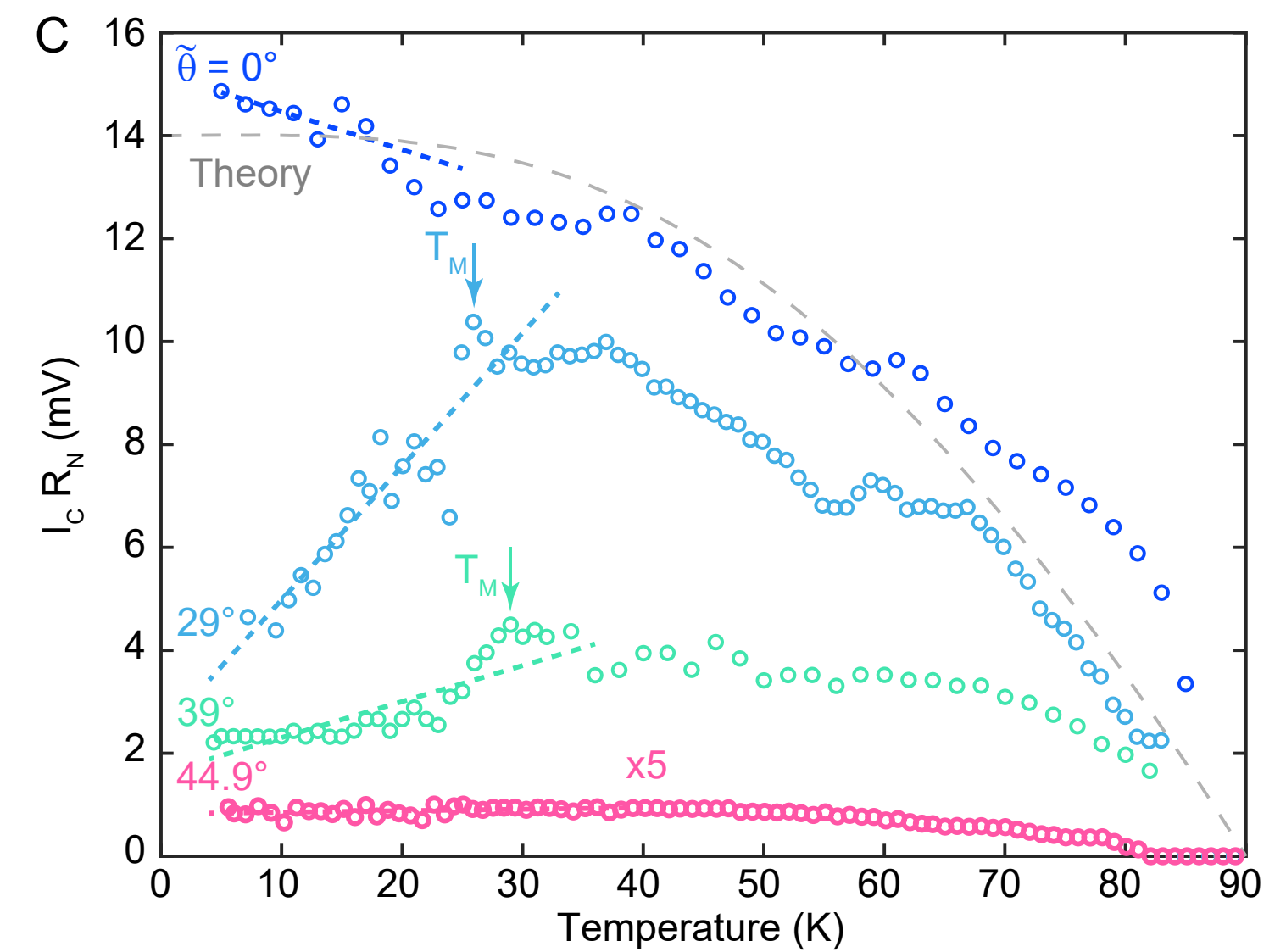
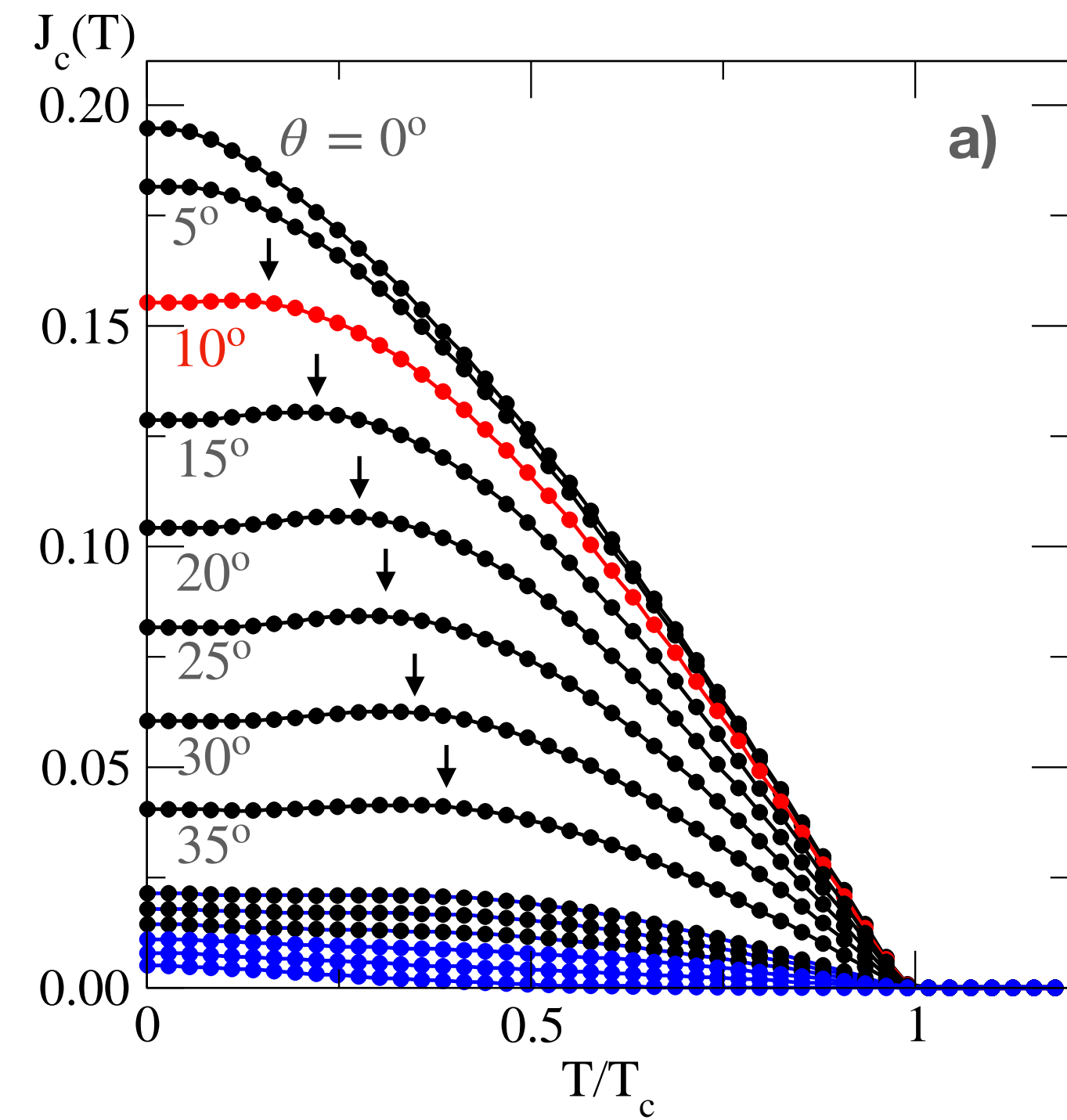
Effect of temperature

- Thermal excitations break Cooper pairs and remove their contribution to the supercurrent.
- At low T this happens primarily in the nodal regions
- Low- T thermal excitations therefore initially remove **NEGATIVE** contributions to I_c which is thus expected to **INCREASE** as a function of temperature



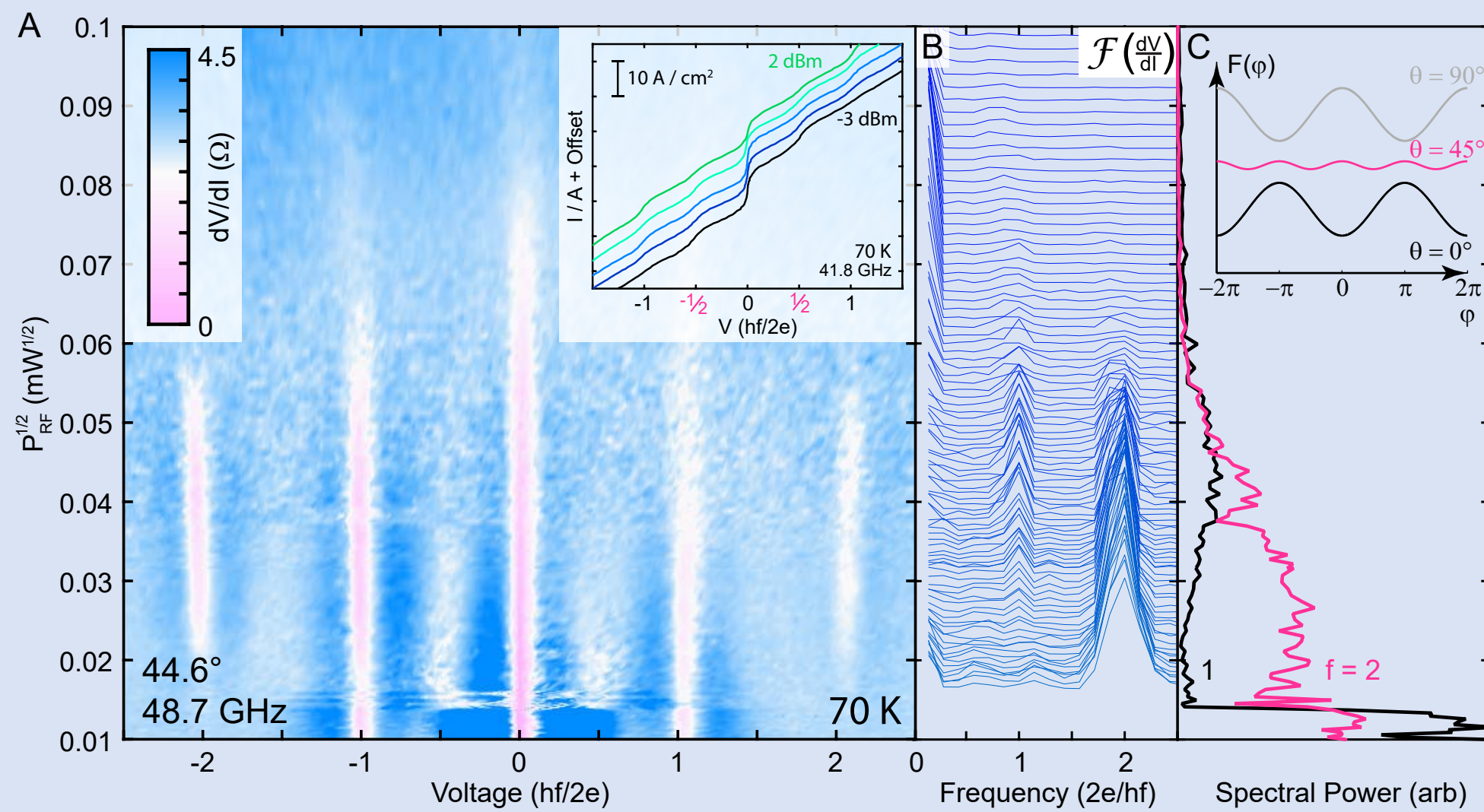
$$I_c(T) \simeq \frac{et^2}{2\hbar} \sum_{\mathbf{k}} \frac{\Delta_{\mathbf{k}1}\Delta_{\mathbf{k}2}}{D_{\mathbf{k}}(\pi/2)} \sum_{a=\pm} \left[\frac{-a}{E_{\mathbf{k}a}} \tanh \frac{1}{2} \beta E_{\mathbf{k}a} \right]_{\varphi \rightarrow \pi/2}$$

Calculation results:

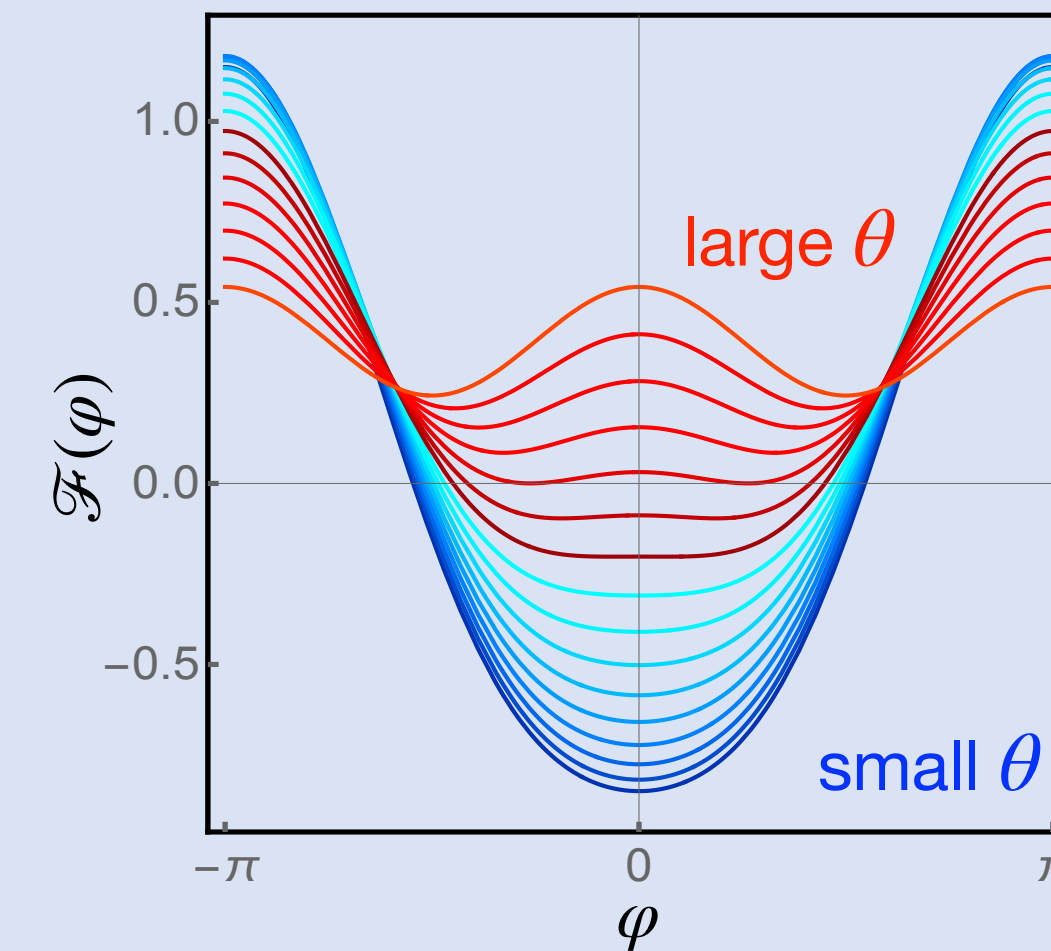


Experimental efforts: Evidence for T-broken phase

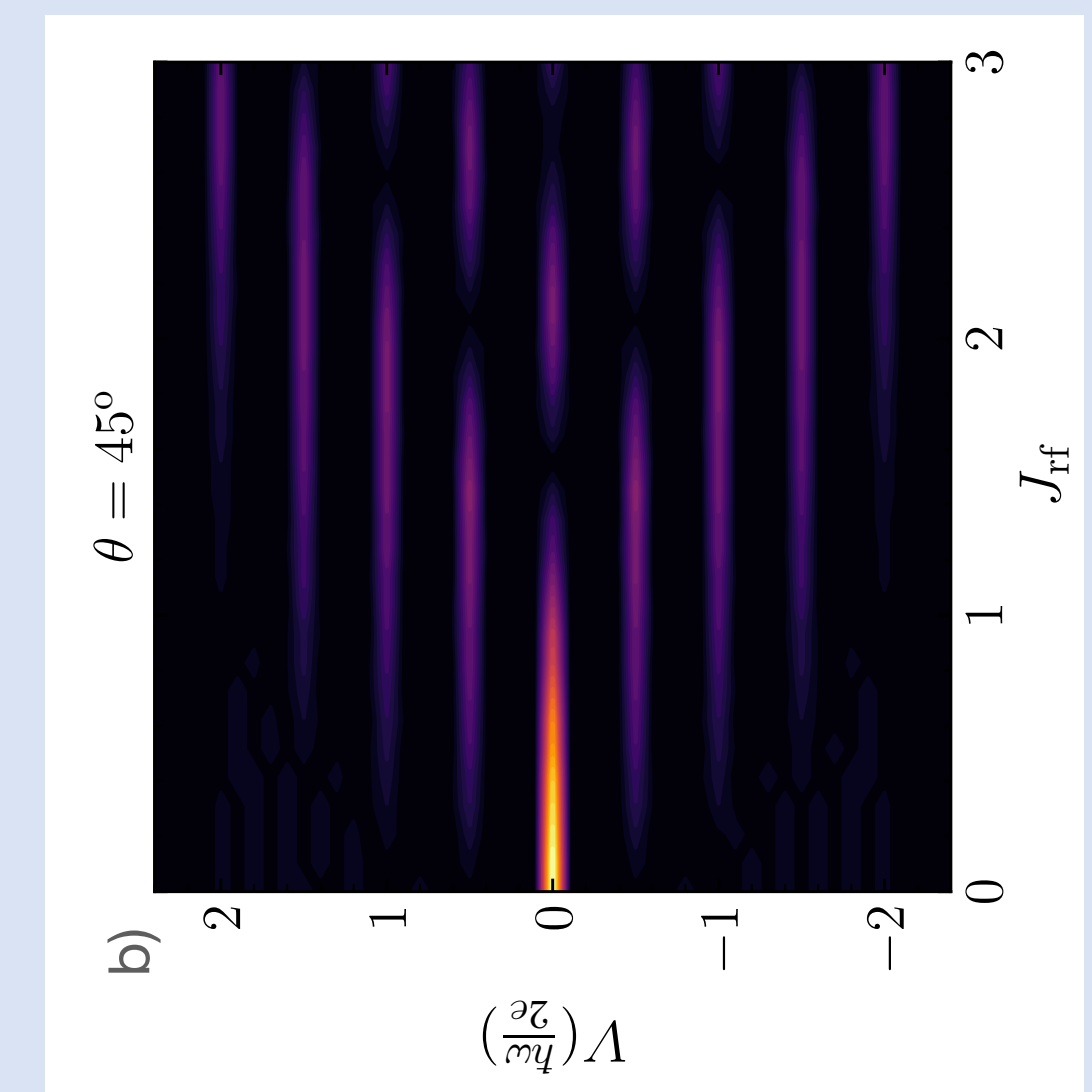
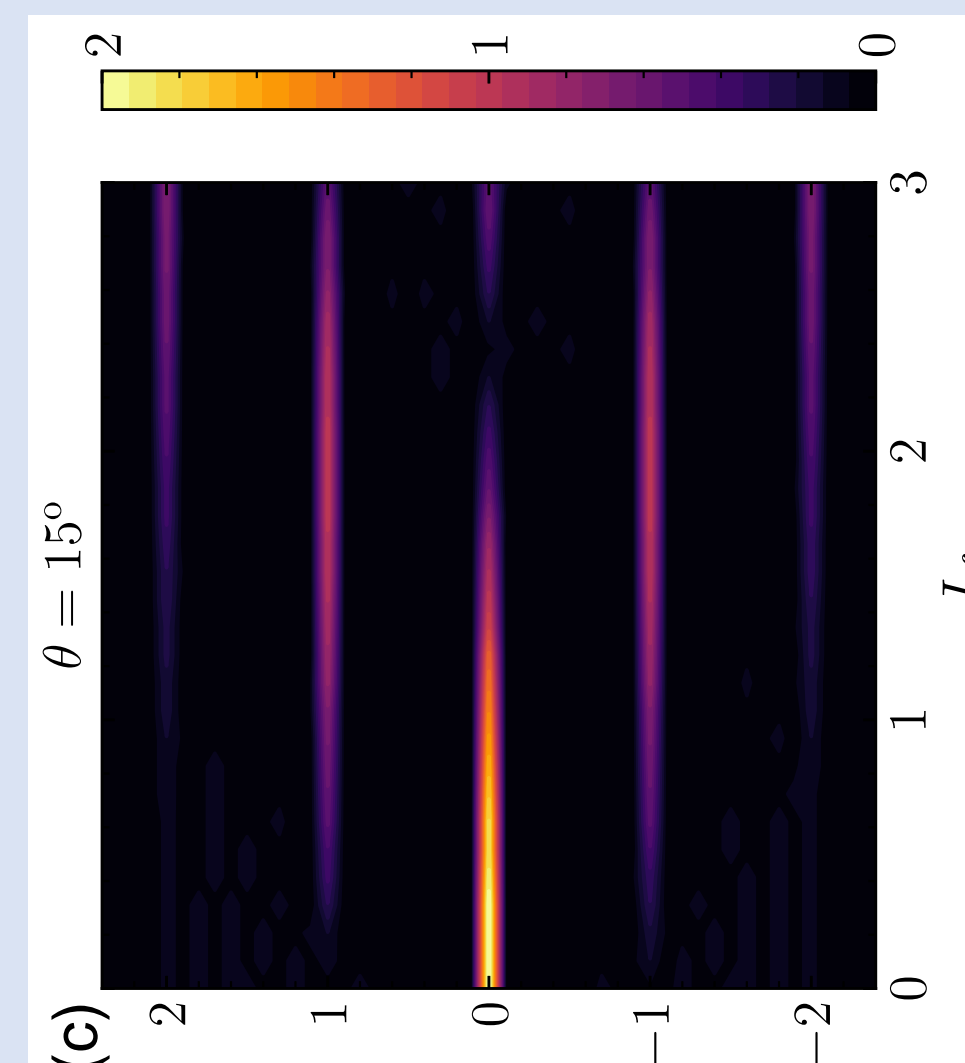
Experiment [Zhao et al., arXiv:2108.13455] observes “fractional Shapiro steps” near 45 degree twist



Fractional Shapiro steps can reflect the π -periodic I-V curves



$$\mathcal{F}[\psi_1, \psi_2] = f_0[\psi_1] + f_0[\psi_2] + A |\psi_1|^2 |\psi_2|^2 + B(\psi_1 \psi_2^* + c.c.) + C(\psi_1^2 \psi_2^{*2} + c.c.)$$

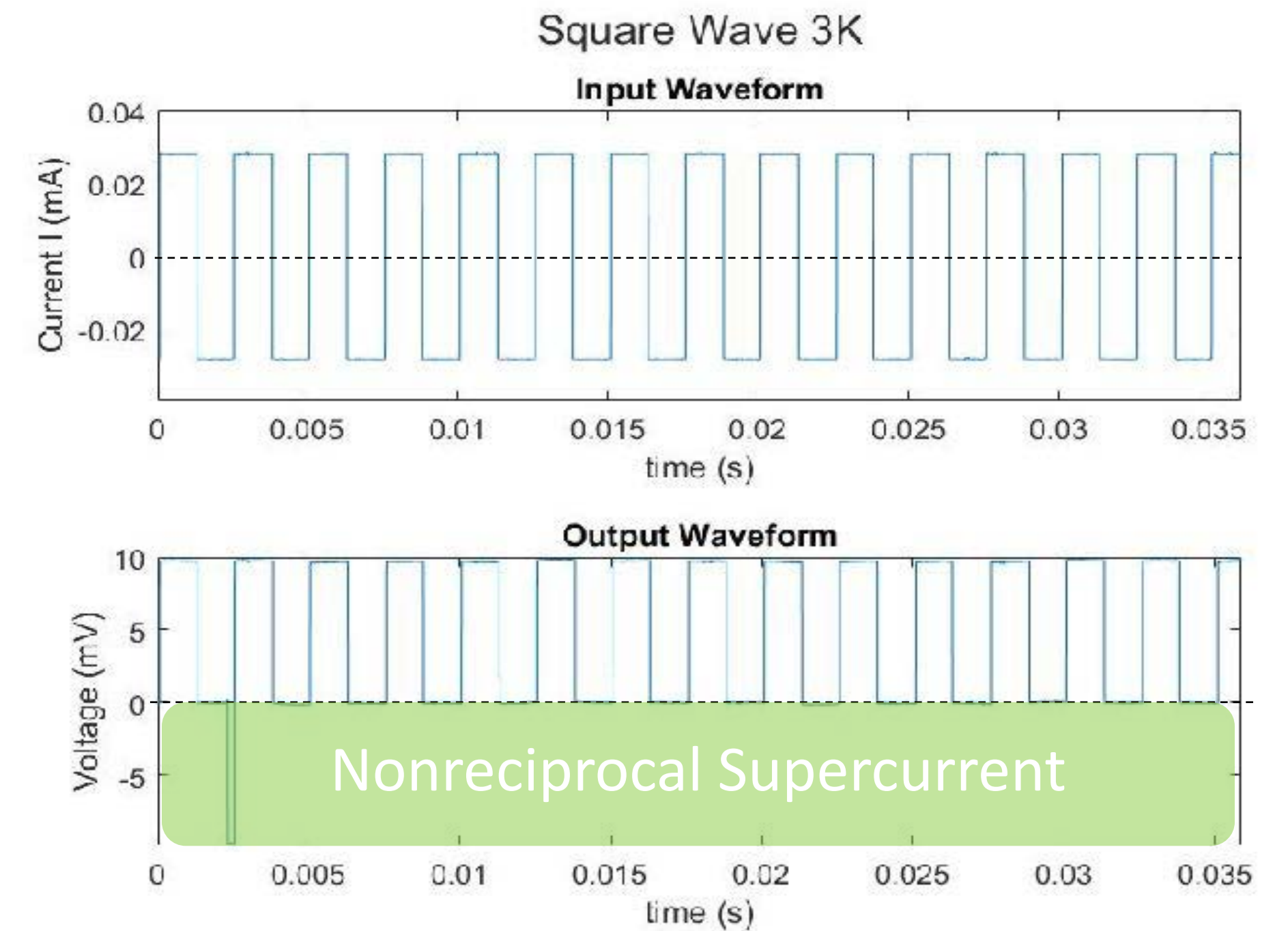
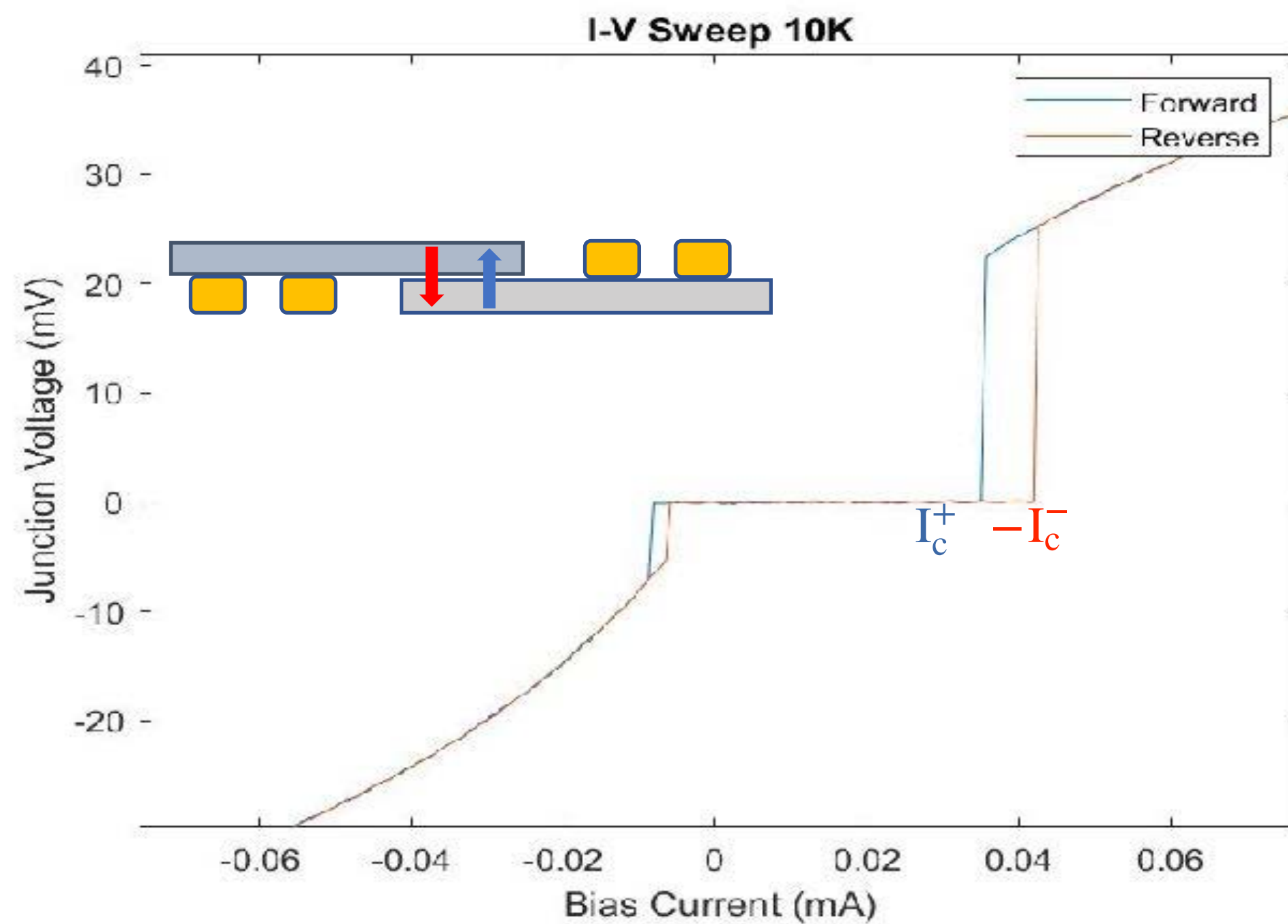


Smoking gun evidence: Field-Free Josephson Diode Effect in twisted BSCCO

[Zhao et al., *Science* 382,1422 (2023); Kim Group @ Harvard]

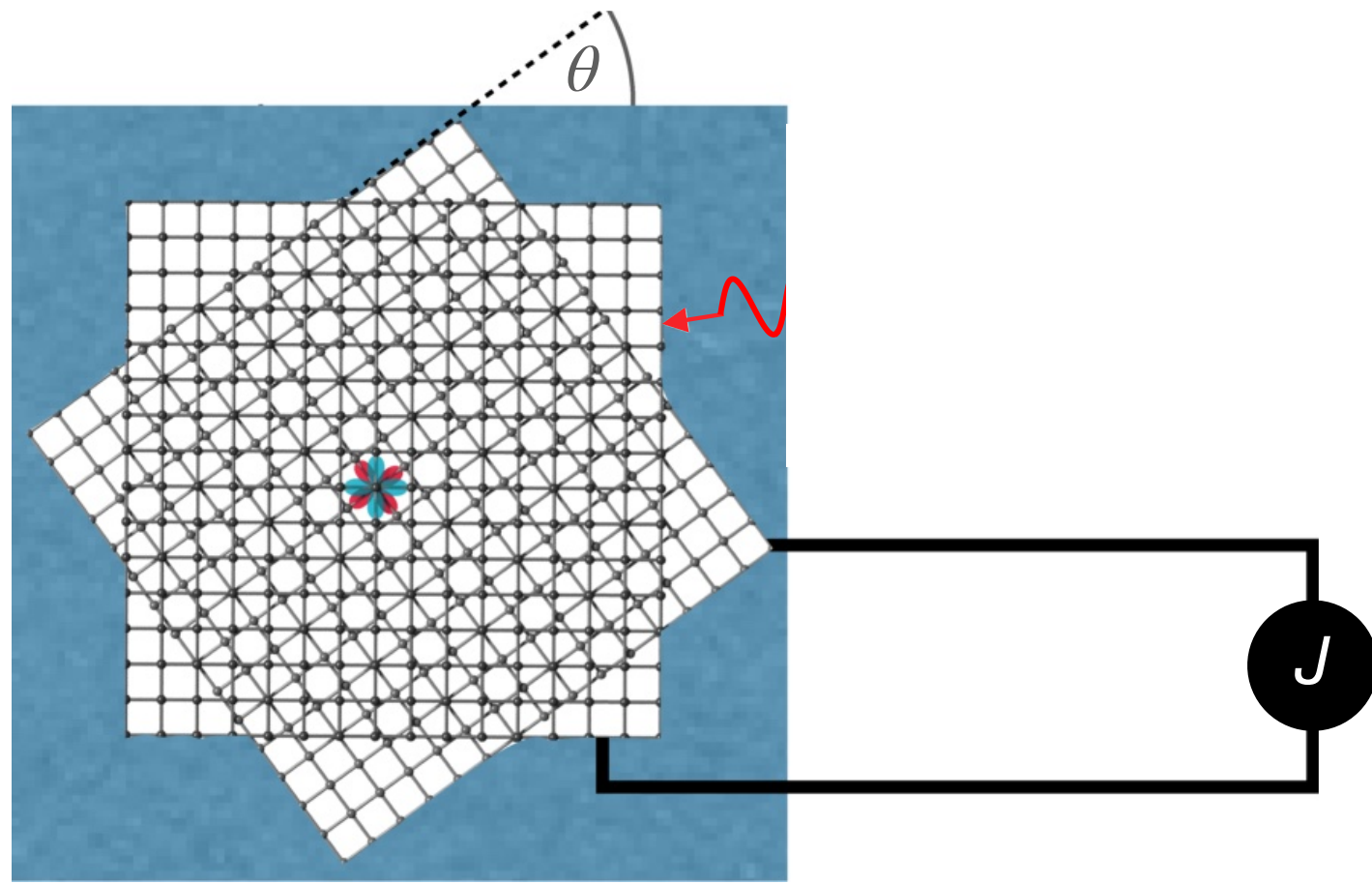
For samples with twist close to 45° they observe $|I_c^+| \neq |I_c^-|$

Josephson Diode: $I_c^+ < |I_{\text{bias}}| < I_c^-$



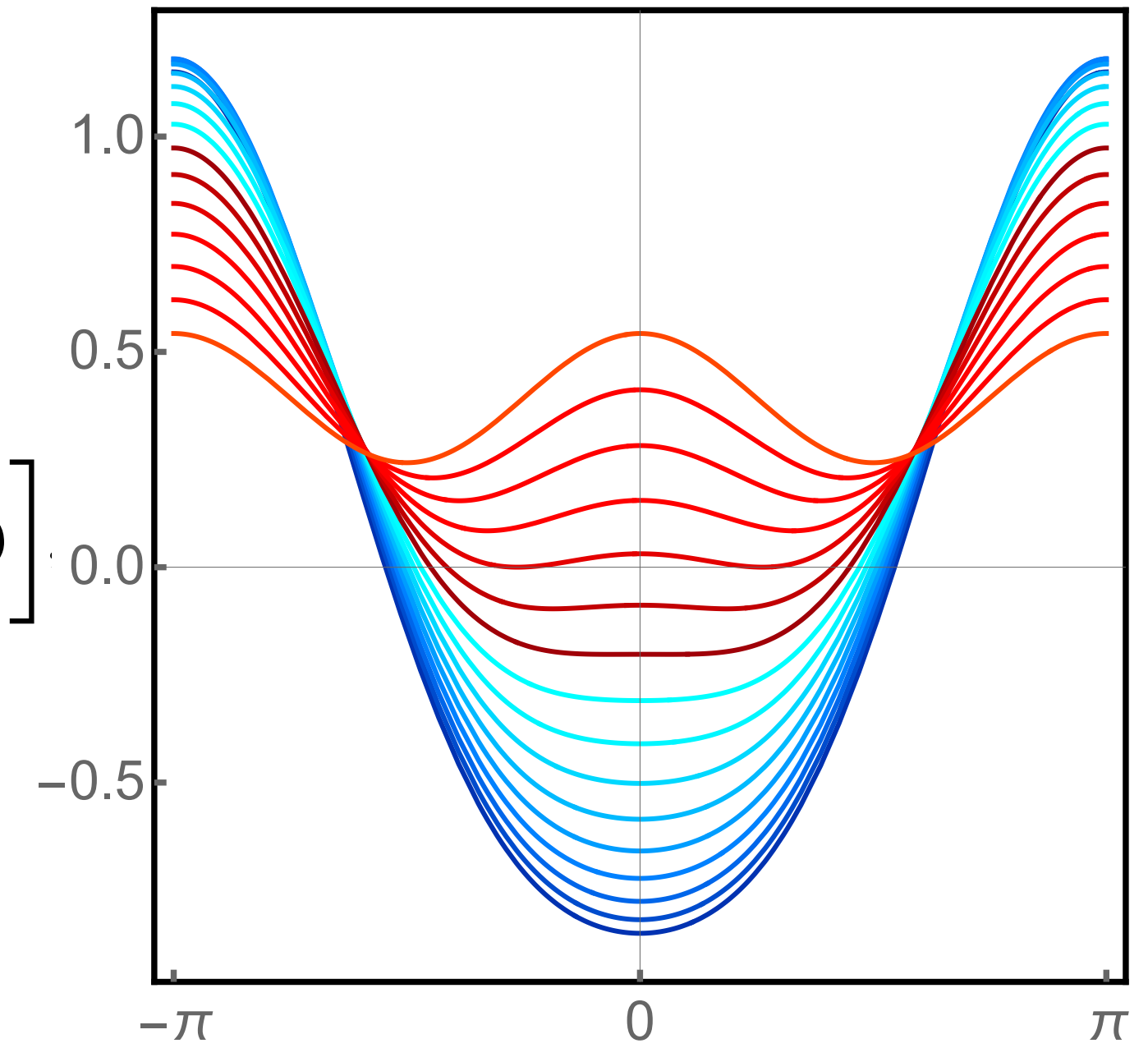
Because the current is odd under time reversal the non-reciprocal diode effect requires *broken time reversal symmetry*

Theory: Diode effect in twisted Bi2212 bilayers



$$\mathcal{F}(\varphi) = \mathcal{E}_0 - \frac{\hbar}{2e} \left[J_{c1} \cos \varphi - \frac{1}{2} J_{c2} \cos(2\varphi) \right]$$

$$J_{c1} \propto \cos(2\theta)$$

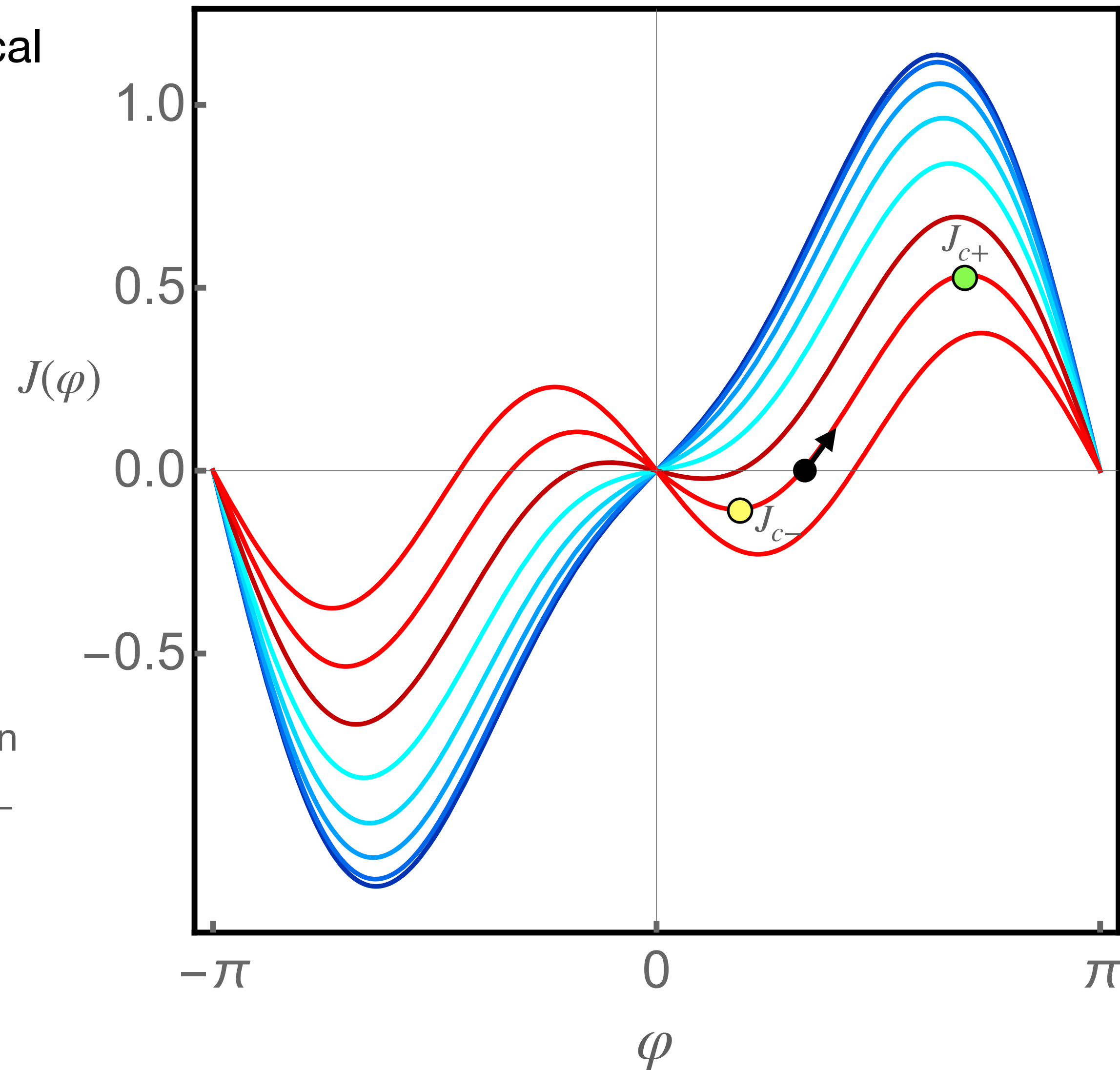


$$\begin{aligned} \mathcal{F}[\psi_1, \psi_2] = & \mathcal{F}_0[\psi_1] + \mathcal{F}_0[\psi_2] + A|\psi_1|^2|\psi_2|^2 \\ & + B(\psi_1\psi_2^* + \text{c.c.}) + C(\psi_1^2\psi_2^{*2} + \text{c.c.}) \end{aligned}$$

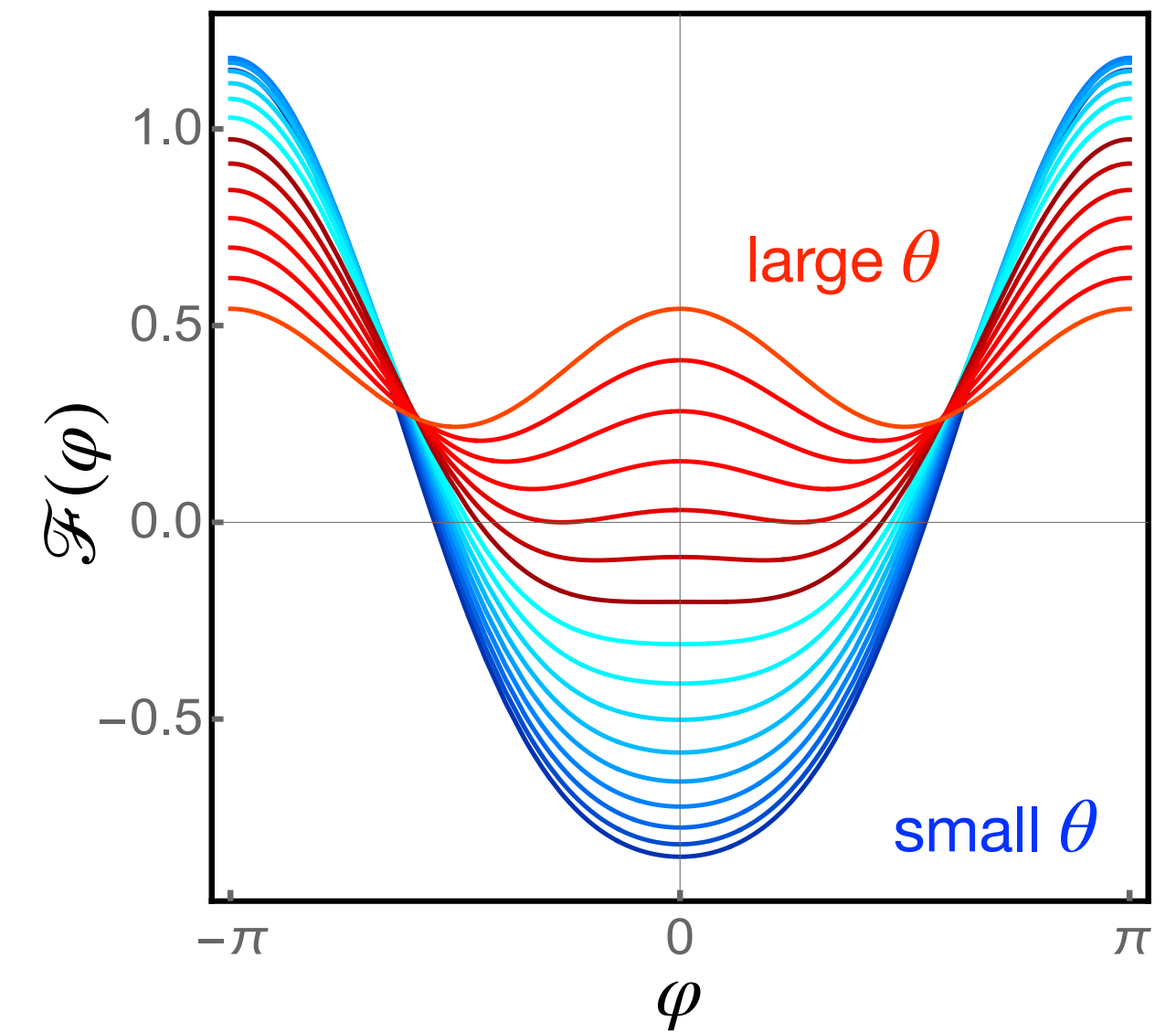
d -wave symmetry dictates $B = -B_0 \cos(2\theta)$

Theory: Diode effect in twisted Bi2212 bilayers

The origin of non-reciprocal critical current



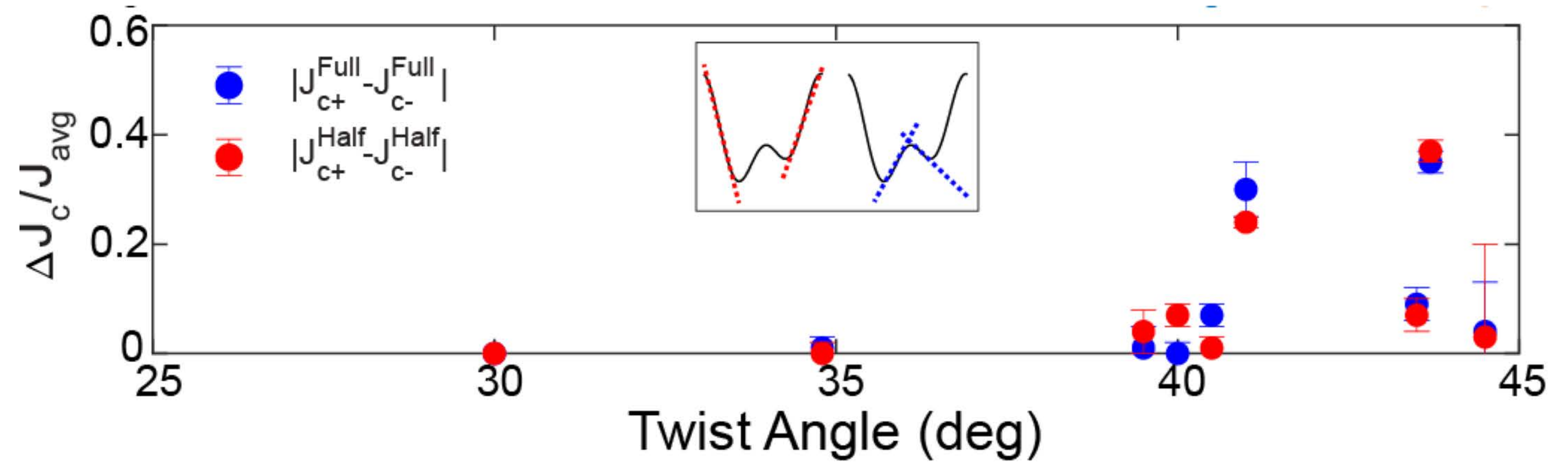
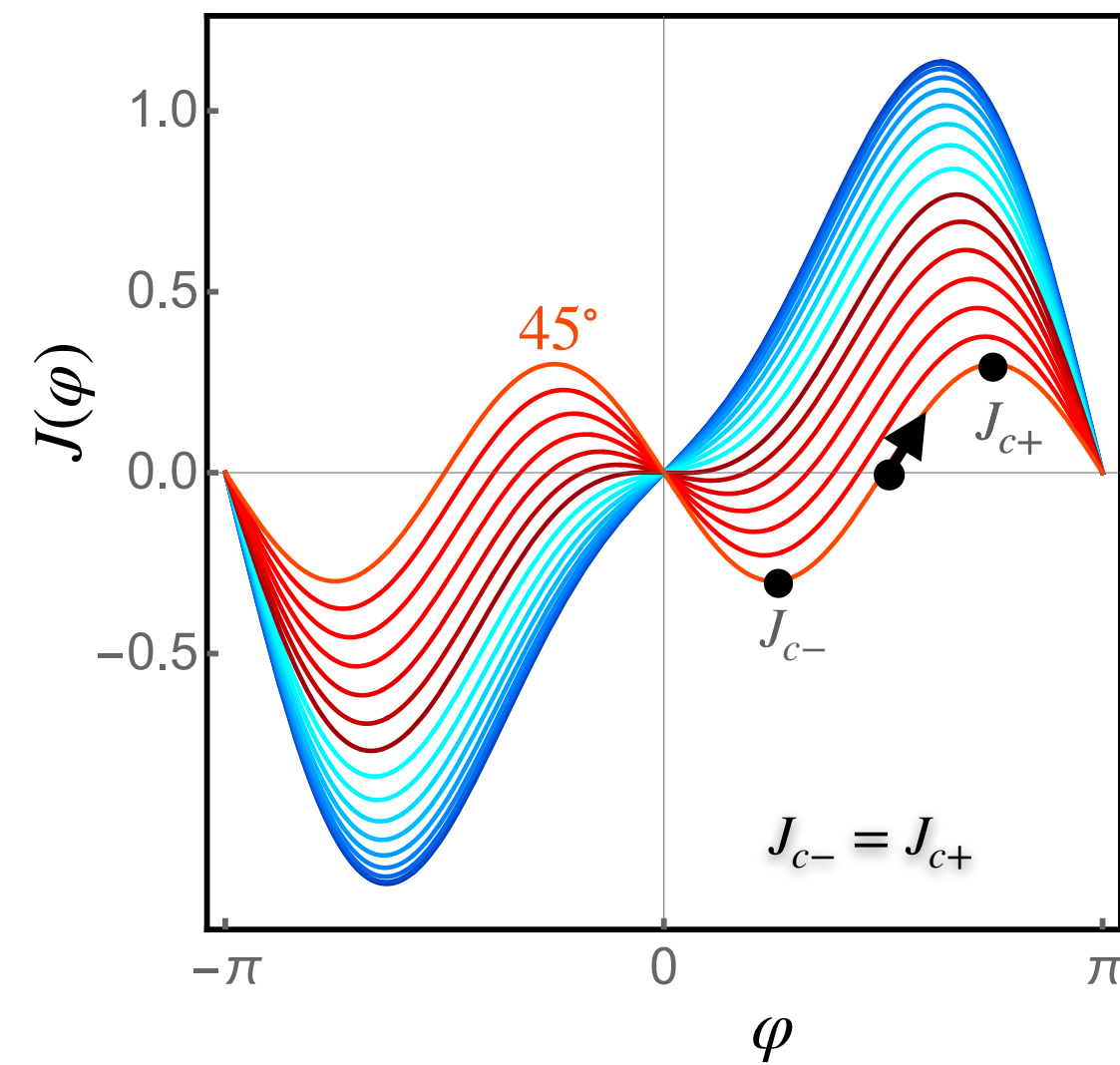
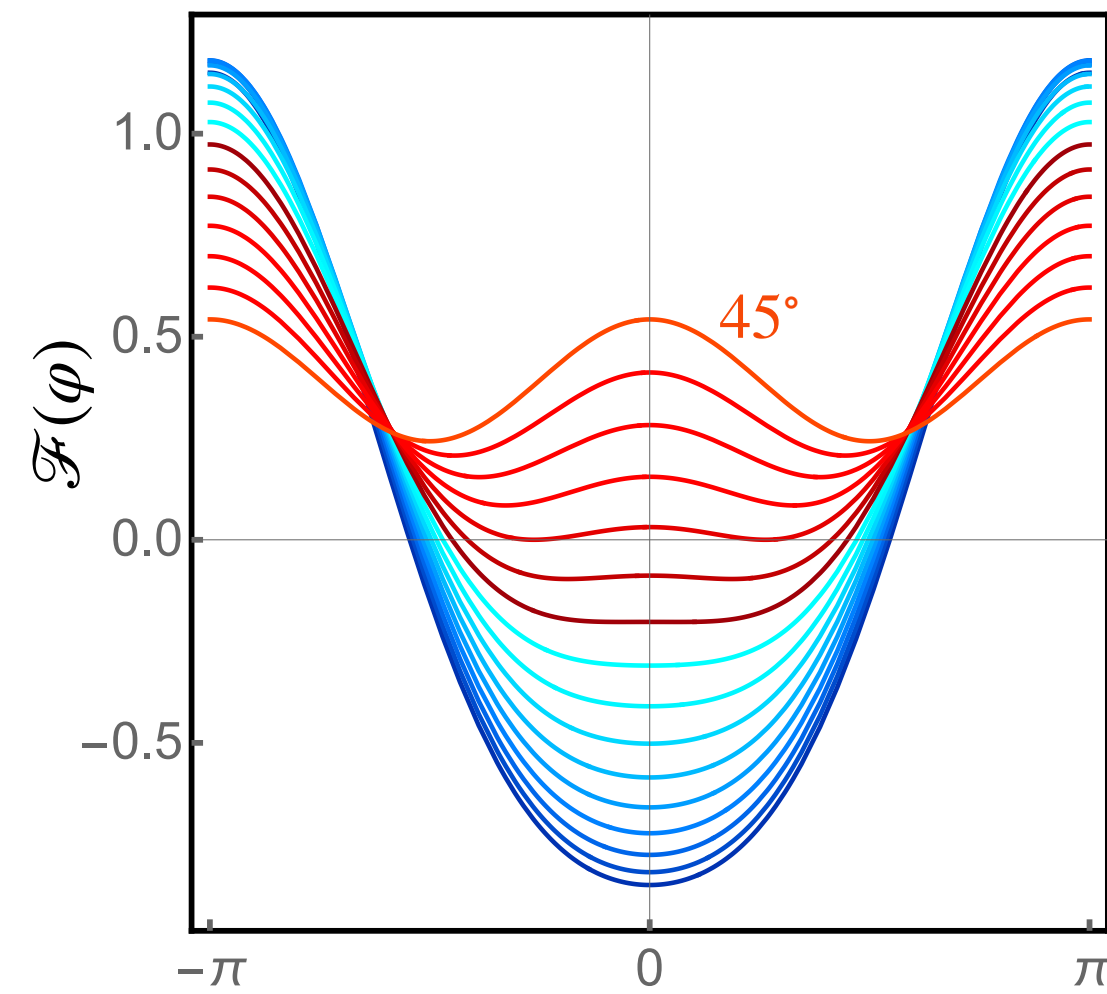
Whenever time reversal is broken
AND $\theta \neq 45^\circ$ we have $J_{c+} \neq J_{c-}$
which implies non-reciprocal
behaviour.



However, if the NORMAL state is \mathcal{T} -respecting one would expect to start randomly from either free-energy minimum.

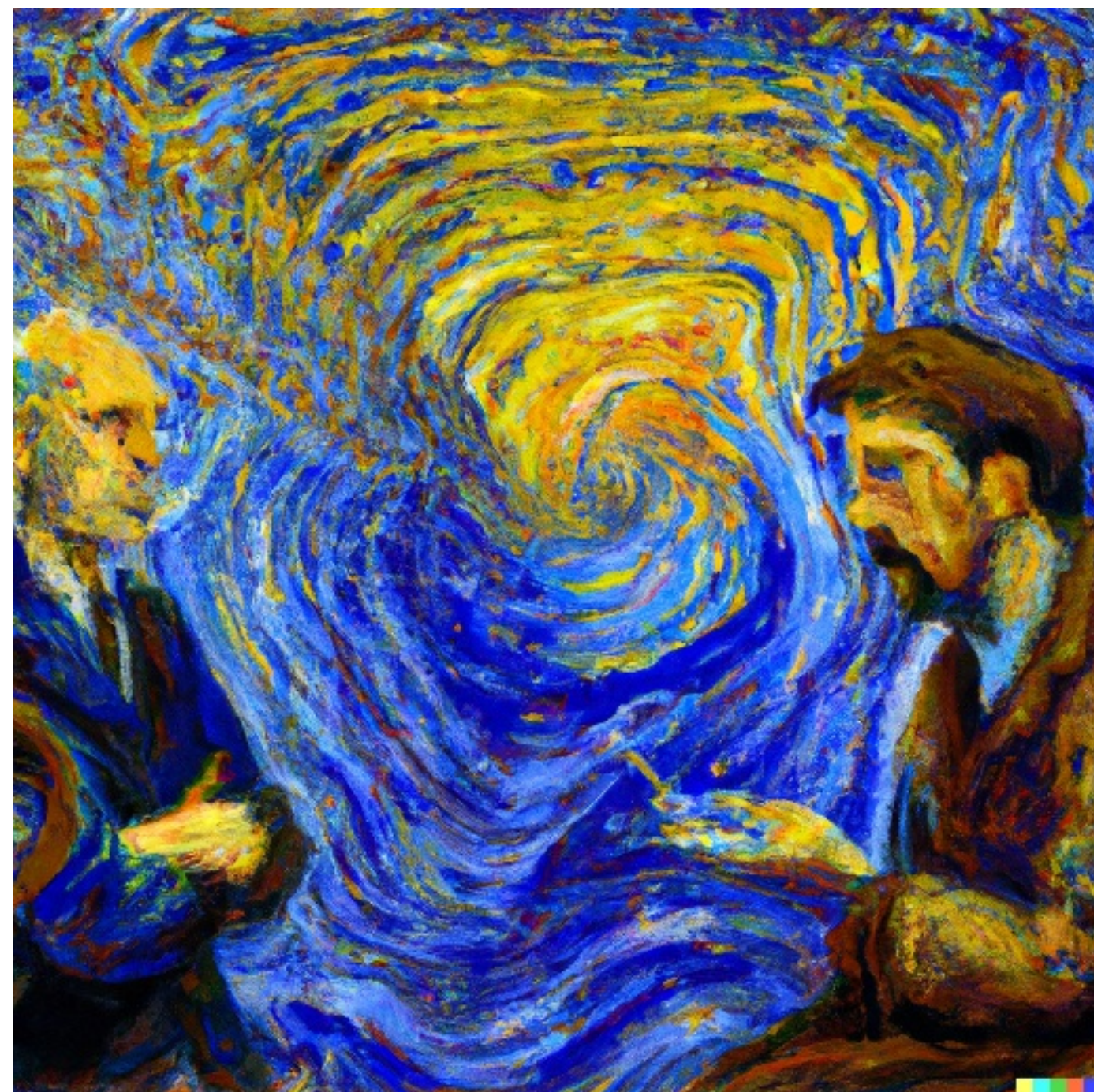
—> **One needs a measurement protocol that reproducibly initiates the system in the same minimum.**

A new theory prediction: The diode effect must vanish at exact 45° twist

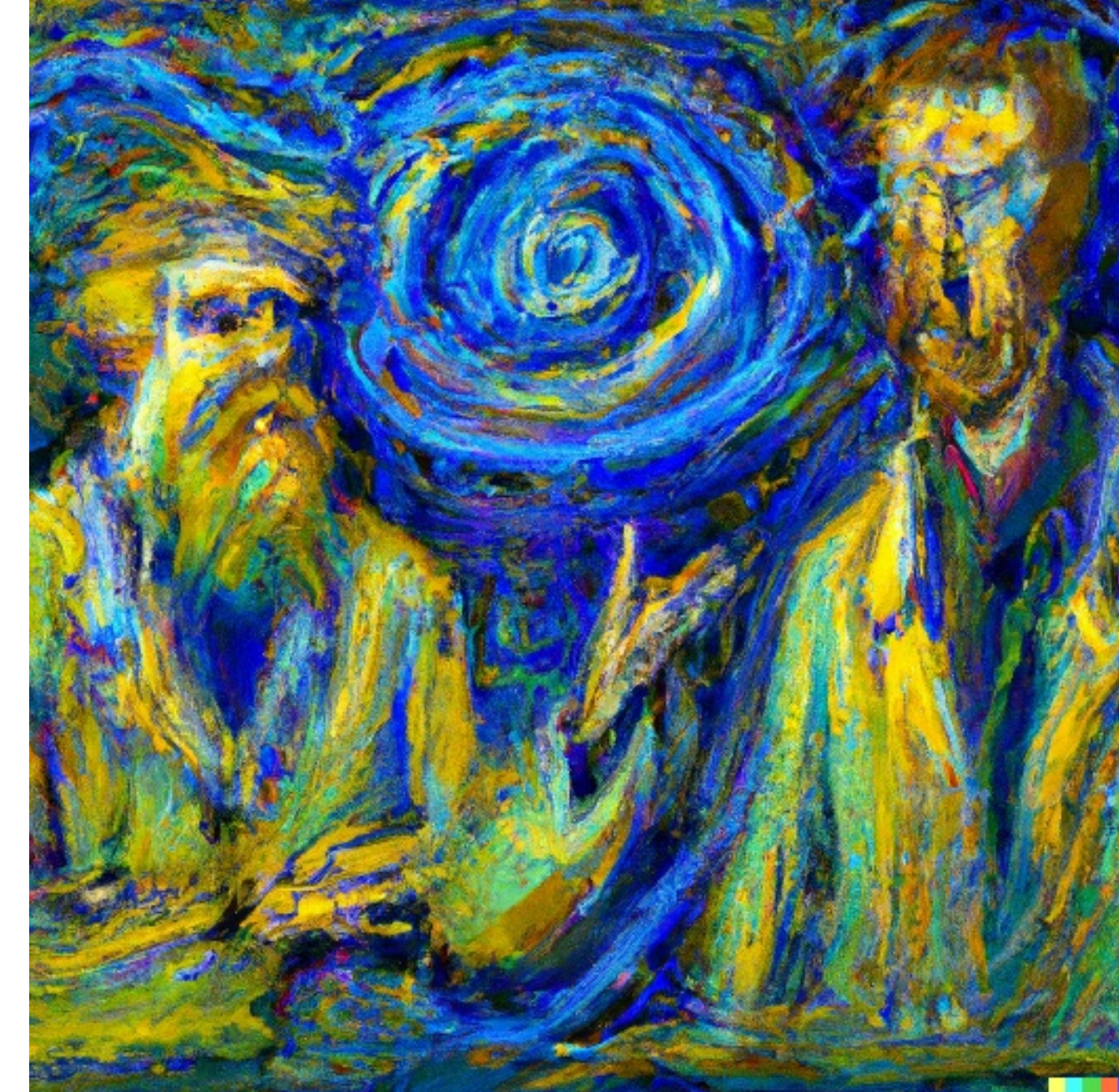


- Zero Field Diode Effect only appears near 45°
- Possible broken ground state degeneracy

Data courtesy of Alex Cui, Philip Kim group, Harvard



Thank you!



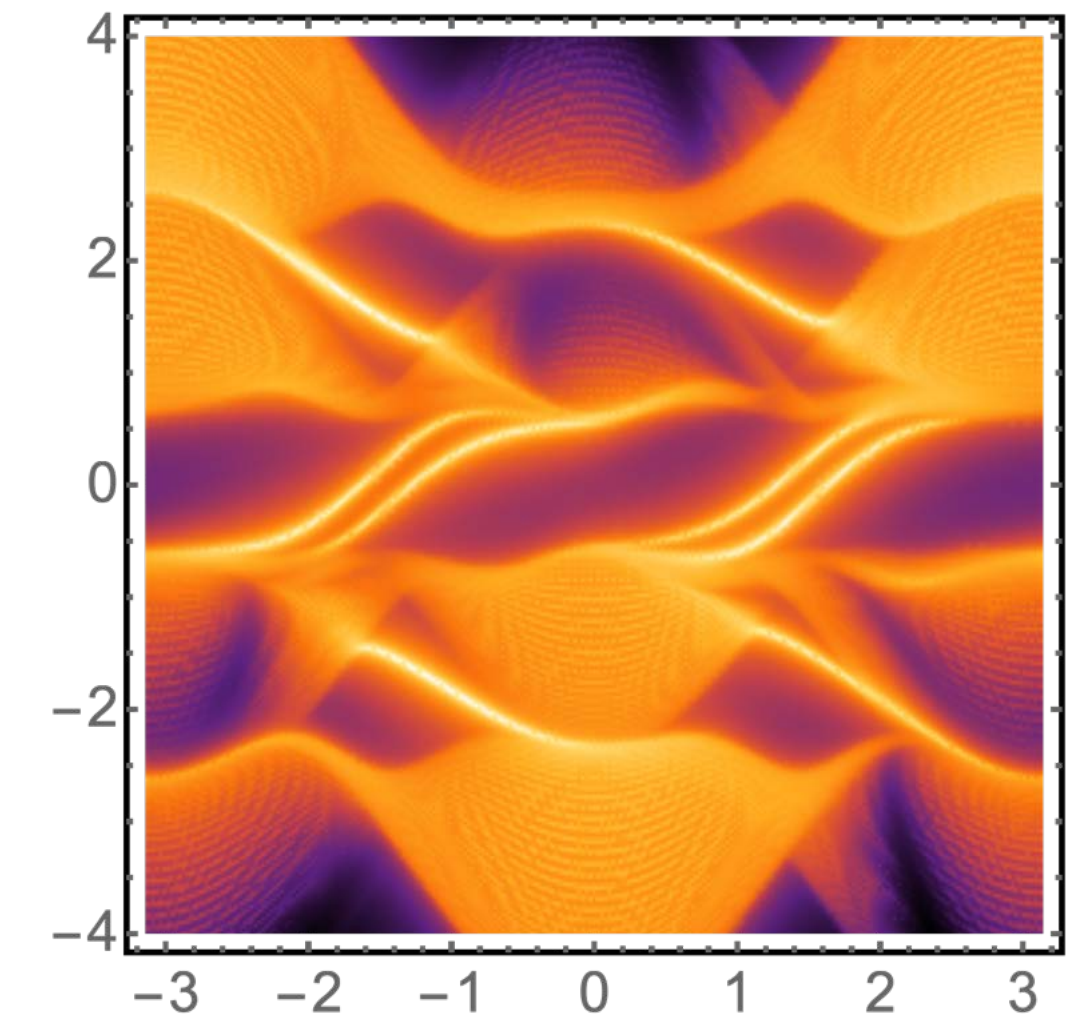
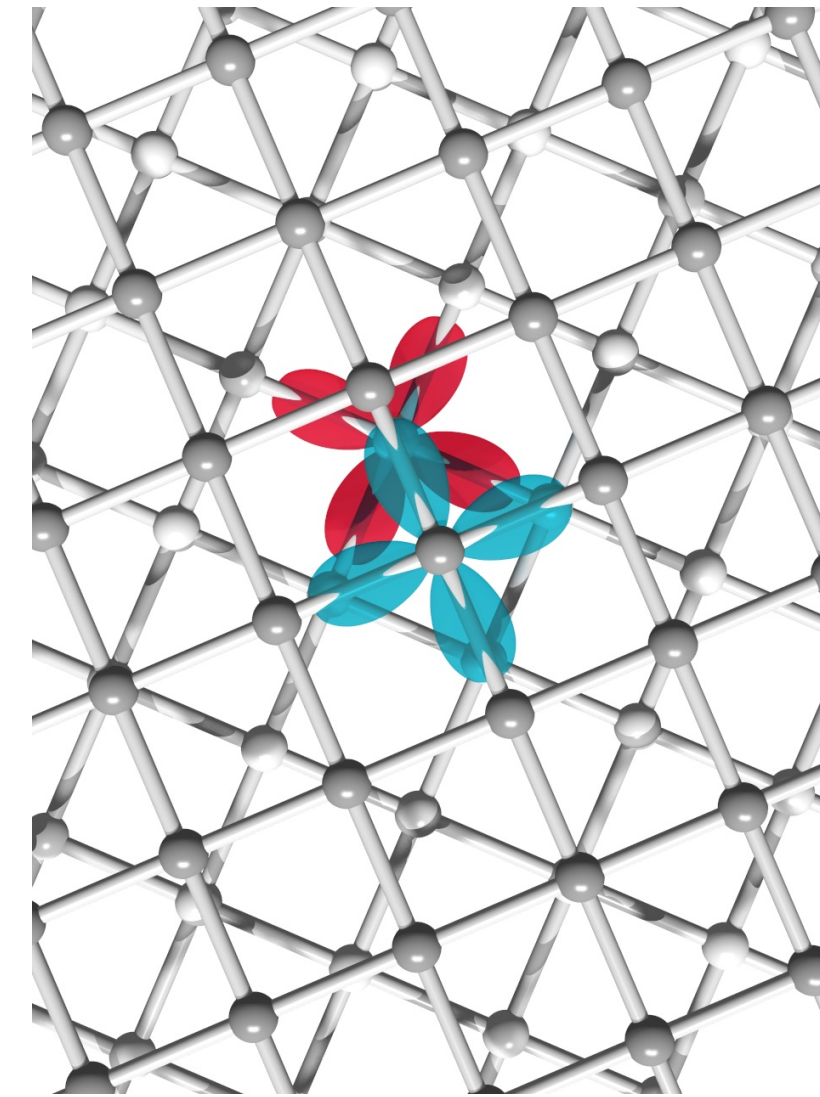
DALL-E: “Two scientists pondering twisted high- T_c cuprate superconductor”



Group members:
O. Can
T. Tummuru
E. Lantagne-Hurtubise

Summary and outlook

- Natural models of coupled layers of d -wave SC predict a T-broken phase when the twist angle is close to 45°
- The resulting phase is fully gapped and over much of the phase diagram also topologically non-trivial
- Topological phase will show an even number of protected chiral edge modes
- Gap opening can be detected through various spectroscopies (ARPES, STM)
- T-breaking can be probed directly (polar Kerr effect, SC diode effect, fractional Shapiro steps)



Some interesting open questions:

1. What is the best way to observe the topological phase experimentally?
2. Are there any interesting uses for this novel topological superconducting phase once identified?
3. Are there other 2D systems (beyond graphene, chalcogenides, cuprates) that will produce interesting new behaviors under twist or similar geometries?