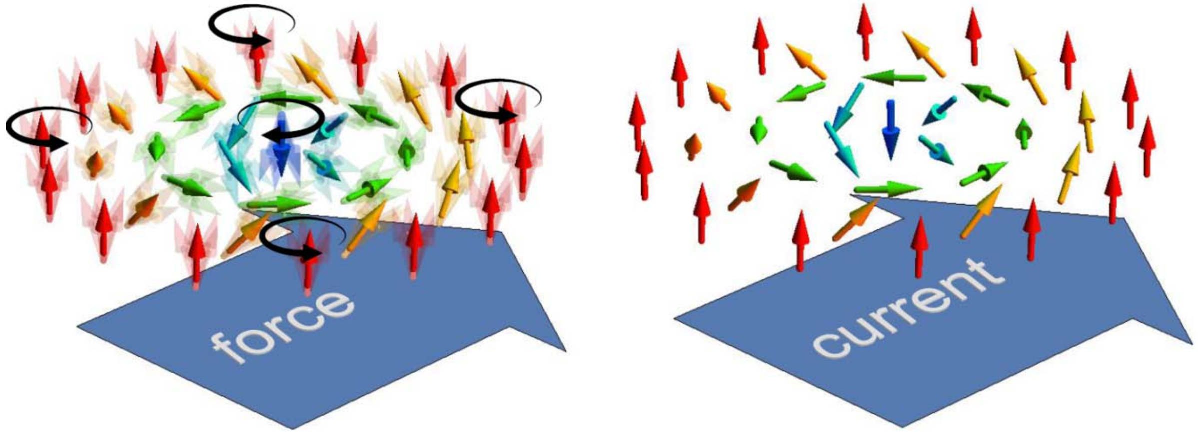
Lecture II:

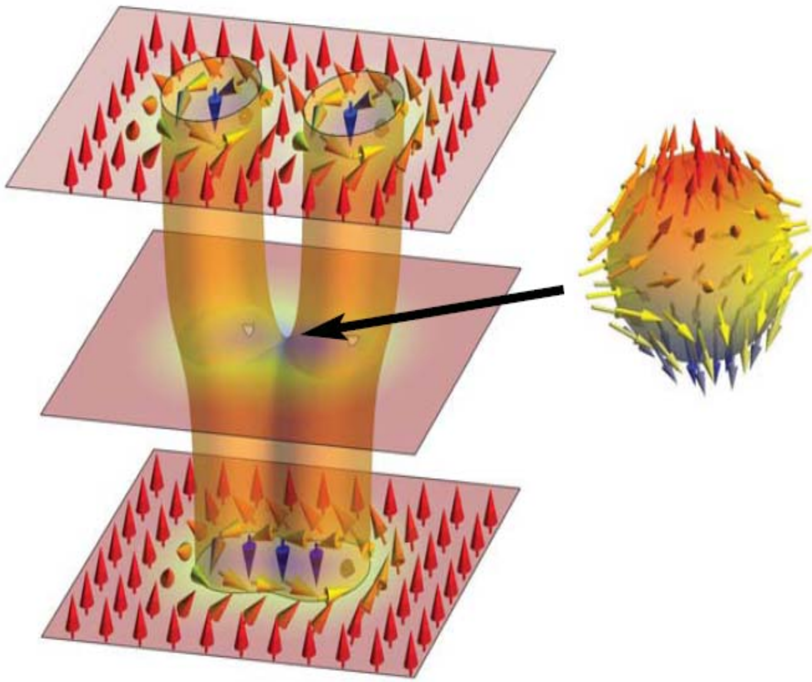
Inertia, Diffusion and Dynamics of a Driven Skyrmion

Schütte, Isagawa, A.R., Nagaosa (PRB 2014)

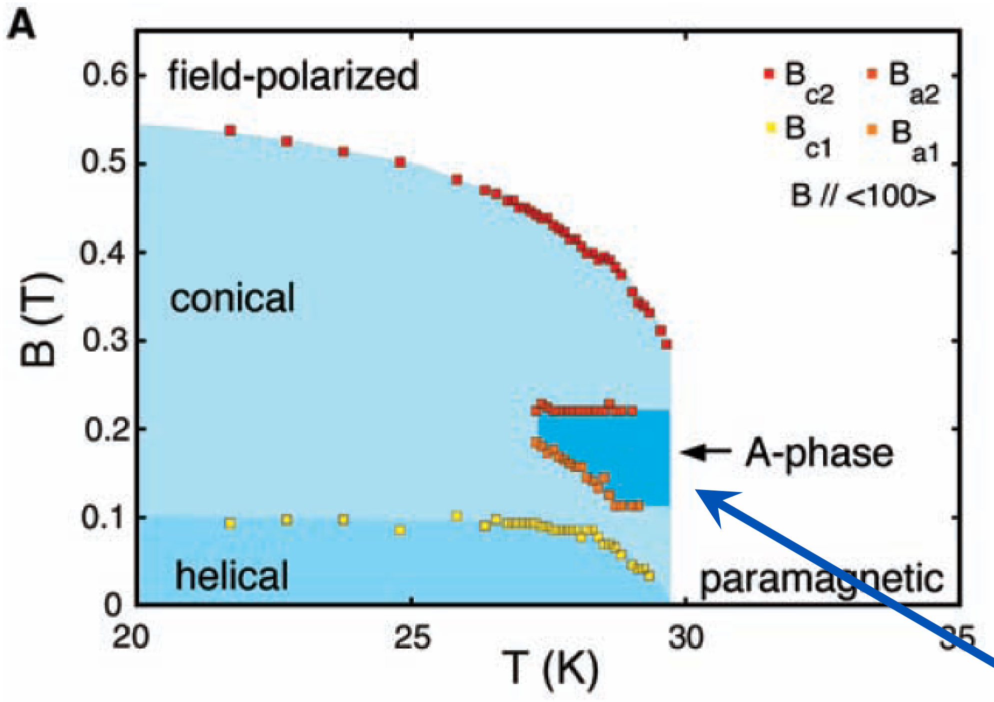


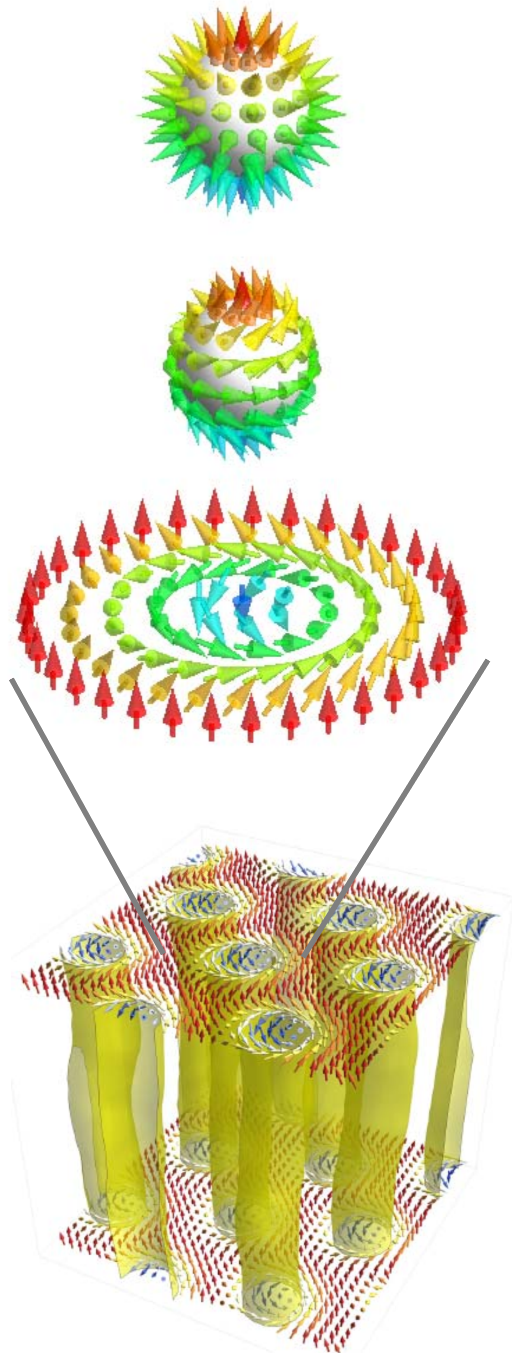
Changing topology: Emergent magnetic Monopoles

Milde, Köhler, Seidel,Eng, Bauer, Chacon, Pfleiderer, Buhrandt, A. R., Science (2013)



reminder I: skyrmion phase





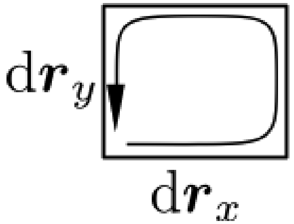
reminder II:
emergent electrodynamics & topological quantization

- effective electric charge:
  spin parallel/antiparallel to local magnetization

$$\mathbf{q}^{\mathbf{e}}_{\downarrow/\uparrow} = \mp\frac{1}{2}$$

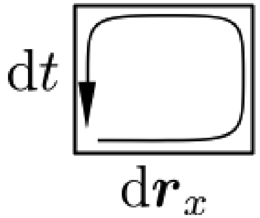
- **emergent magnetic & electric fields**:

Berry phase for loops in space



$$\mathbf{B}^{\mathbf{e}}_{\mathbf{i}} = \frac{\hbar}{2}\epsilon_{ijk}\hat{n}\cdot(\partial_j\hat{n}\times\partial_k\hat{n})$$

Berry phase for loops in space-time

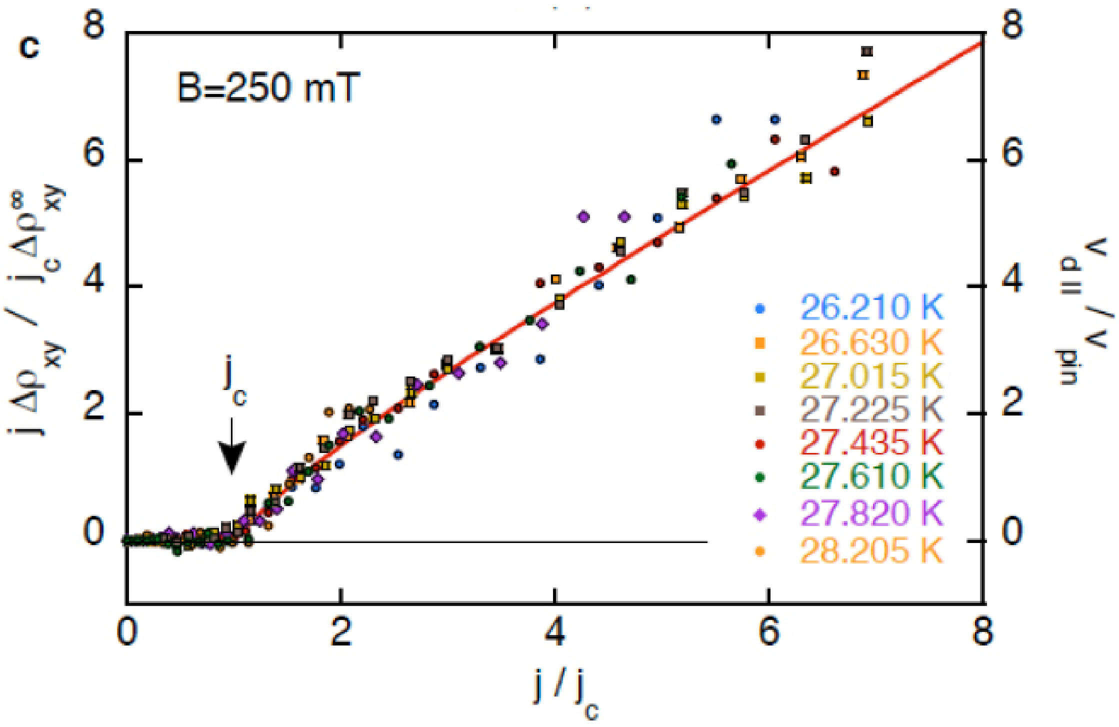


$$\mathbf{E}^{\mathbf{e}}_{\mathbf{i}} = \hbar\,\hat{n}\cdot(\partial_i\hat{n}\times\partial_t\hat{n})$$

- **topological quantization**:

winding number -1 ⟺ one flux quantum per skyrmion

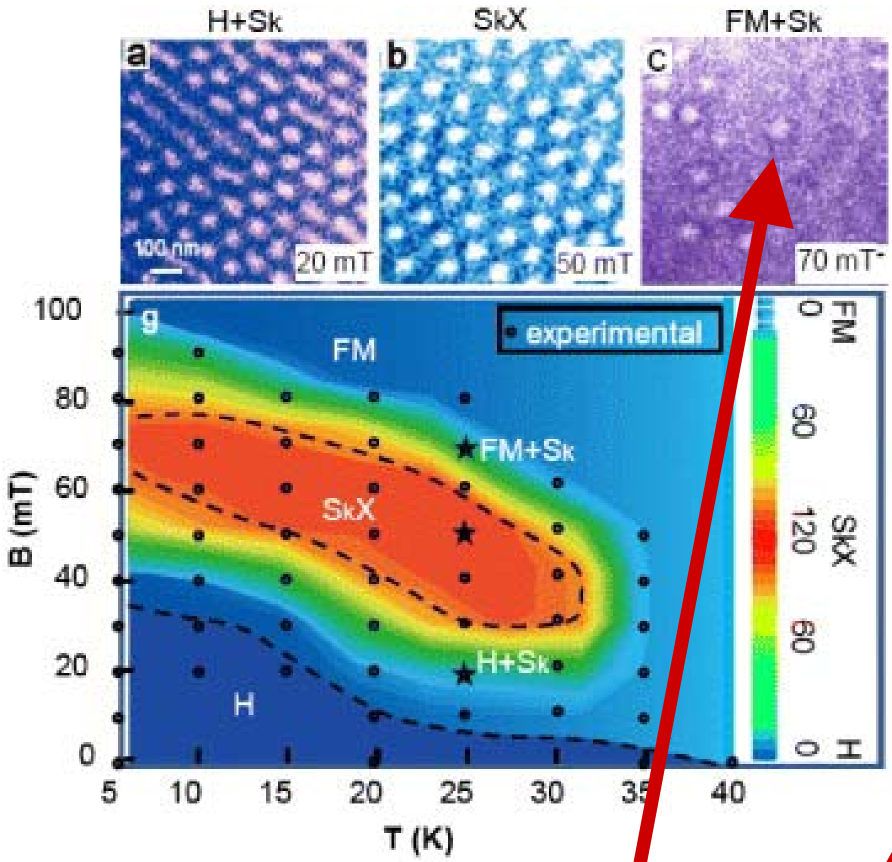
Emergent electric and magnetic fields directly **experimentally measurable**



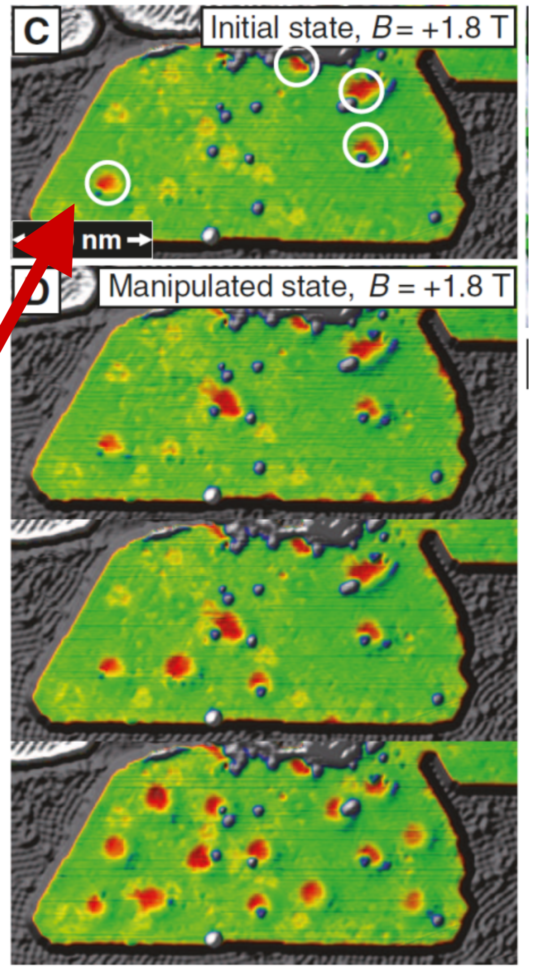
But: no conventional photons in skyrmion phase

why: charged matter (skyrmions) hang around
charged Wigner crystal in magnetic fields: quadratic dispersion!

Skyrmion lattices and single skyrmions

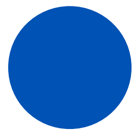


Tokura, Nagaosa et al. 2010



Wiesendanger group, Science 2013
writing of single skyrmions

now: single skyrmion in two dimensions
in ferromagnetic background



classical (or quantum) point particle in a medium
needed:

mass, friction, effective magnetic field, coupling to external fields,
internal excitations,….

first try:
guess Newton's equation by symmetry for slow motions
(low fequencies)

$$\mathcal{G} \times \dot{\mathrm{R}} + \alpha \mathcal{D}\, \dot{\mathrm{R}} + m\ddot{\mathrm{R}} + \alpha \Gamma \times \ddot{\mathrm{R}} = \mathbf{F}_c + \mathbf{F}_g + \mathbf{F}_{\mathrm{th}}$$

first time derivative

second time derivative

external forces from currents, field gradients, thermal fluctuations

first try:
guess Newton's equation by symmetry for slow motions
(low fequencies)

$$\mathcal{G} \times \dot{\mathrm{R}} + \alpha \mathcal{D} \dot{\mathrm{R}} + m \ddot{\mathrm{R}} + \alpha \Gamma \times \ddot{\mathrm{R}} = \mathbf{F}_c + \mathbf{F}_g + \mathbf{F}_{\mathrm{th}}$$

gyrocoupling = effective magnetic field = Magnus force	friction	inertia	new: "gyrodamping"

Landau-Lifshitz-Gilbert Gleichung

semiclassical dynamics of magnetization without current,

$$\partial_t \mathbf{M} = -\gamma \mathbf{M} \times [\mathbf{B}_{\text{eff}} + \mathbf{b}_{fl}(t)] + \frac{\alpha}{M} \mathbf{M} \times \partial_t \mathbf{M}$$

precession in effective fields (static + thermal noise)

damping due to spin-orbit

how modified by current? What terms allowed by symmetry?

Landau-Lifshitz-Gilbert equation

velocity of spin-currents

$$(\partial_t + v_s \nabla)\mathbf{M} = -\gamma \mathbf{M} \times [\mathbf{B}_{eff} + \mathbf{b}_{fl}(t)] + \frac{\alpha}{M}\mathbf{M} \times (\partial_t + \frac{\beta}{\alpha} v_s \nabla)\mathbf{M}$$

Berry phase spin torques

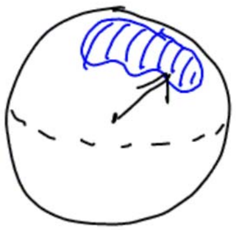
precession in effective fields

damping due to spin-orbit

Berry phase action

$$S_B = \int d^3r dt\, M \vec{A}(\hat{n})(\partial_t + \vec{\mathbf{v}}_s \vec{\nabla})\hat{n}$$

describes both topological Hall effect and main forces on magn. structure



Monopole vector field counts area on surface

Not covered: extra damping terms

spin velocity: $\mathbf{v}_s = \mathbf{j^e}/M$

(Zhang, Li, 2004)

$$(\partial_t + \mathbf{v}_s \boldsymbol{\nabla})\,\mathbf{M} = \mathbf{M} \times (-\frac{dF}{dM}) + \alpha\,\mathbf{M} \times \left(\partial_t + \frac{\beta}{\alpha}\mathbf{v}_s \boldsymbol{\nabla}\right)\mathbf{M}$$

emergent electromagnetism, Magnus forces

precession in effective fields

damping due to spin-orbit

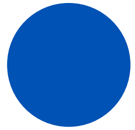
$$+\,\alpha' \left[\mathbf{M} \cdot (\partial_i \mathbf{M} \times (\partial_t + \frac{\beta'}{\alpha'}\mathbf{v_s} \boldsymbol{\nabla})\mathbf{M})\right] \partial_\mathbf{i} \mathbf{M}$$

ohmic damping due to currents induced by emerging electric field (Zhang,Zhang 09, Zang et al. 11)

higher gradients but: no spin orbit needed, enhanced by factor $k_F l$

$$|\mathbf{M}| = 1$$

first try:
guess Newton's equation by symmetry for slow motions
(low fequencies)



$$\mathcal{G} \times \dot{\mathrm{R}} + \alpha \mathcal{D}\,\dot{\mathrm{R}} + \xcancel{m\ddot{\mathrm{R}} + \alpha \Gamma \times \ddot{\mathrm{R}}} = \mathbf{F}_c + \mathbf{F}_g + \mathbf{F}_{\mathrm{th}}$$

first: consider only linear time derivative (Thiele 1973)

Effective equation for skyrmion coordinate (Thiele equation)

$$(\partial_t + v_s \nabla)\mathbf{M} = -\mathbf{M} \times \frac{\delta F}{\delta \mathbf{M}} + \alpha \mathbf{M} \times (\partial_t - \frac{\beta}{\alpha} v_s \nabla)\mathbf{M}$$

$$\Longleftrightarrow \qquad \mathbf{M} \times (\partial_t + v_s \nabla)\mathbf{M} = -\frac{\delta F}{\delta \mathbf{M}} + \alpha(\partial_t - \frac{\beta}{\alpha} v_s \nabla)\mathbf{M}$$

- ansatz: static skyrmion at position ***R(t)***: $\mathbf{M}(\mathbf{r}, t) \approx \mathbf{M}_0(\mathbf{r} - \mathbf{R}(t))$
- to project equation on translational motion:

multiply with $\frac{d\mathbf{M}}{d\mathbf{R}}$ and integrate over space:

$$\mathbf{G} \times \left(\dot{\mathbf{R}} - \vec{v}_s\right) + \mathcal{D}(\alpha \dot{\mathbf{R}} - \beta v_s) = -\frac{dF}{d\mathbf{R}}$$

$$(\mathbf{G}_\mathbf{R})_i = s\, \epsilon_{\mathrm{ijk}} \int \mathrm{d}^2 r\, \frac{1}{2} \mathbf{M}_0 \cdot \left(\frac{d\mathbf{M}_0}{dR_j} \times \frac{d\mathbf{M}_0}{dR_k}\right)$$ = spin density * winding number

$$(\mathcal{D}_\mathbf{R})_{ij} = s \int \mathrm{d}^2 r \frac{d\mathbf{M}_0}{dR_i} \cdot \frac{d\mathbf{M}_0}{dR_j}$$

strongest force:
Berry-phase coupling to electric currents,
= Magnus force = gyrocoupling (Thiele)

$$\mathbf{G} \times \left(\dot{\mathbf{R}} - \mathbf{v}_s \right)$$

“gyrocoupling“ — skyrmion velocity — electronic drift-velocity (spin-current /magnetization)

$$|\mathbf{G}| = 4\pi M \frac{\hbar}{a^2} \sim \frac{\text{flux quantum}}{\text{skyrmion size}} \times \frac{\text{spins}}{\text{skyrmion}} \sim 100.000\,\mathrm{T}/e$$

first try:
guess Newton’s equation by symmetry for slow motions
(low fequencies)

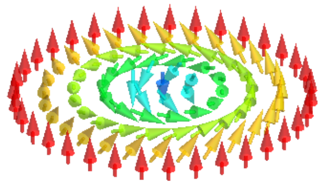
$$\mathcal{G} \times \dot{\mathbf{R}} + \alpha \mathcal{D}\, \dot{\mathbf{R}} + m\ddot{\mathbf{R}} + \alpha \boldsymbol{\Gamma} \times \ddot{\mathbf{R}} = \mathbf{F}_c + \mathbf{F}_g + \mathbf{F}_{\mathrm{th}}$$

now: effective mass + frequency dependent damping

problem:
frequency dependencies strong
$\alpha \boldsymbol{\Gamma} \times \ddot{\mathbf{R}}$ violates causality (wrong sign = antidamping)

therefore:
full frequency dependence needed!

frequency dependent dynamics of skyrmions in d=2
(linear response)



velocity of skyrmion | velocity of spin-currents | field gradients | thermal fluctuations

$$\mathbf{G}^{-1}(\omega)\mathbf{V}(\omega) = \mathbf{S}_c(\omega)\boldsymbol{v}_s(\omega) + \mathbf{S}_g(\omega)\boldsymbol{\nabla} B_z(\omega) + \mathbf{F}_{\text{th}}(\omega)$$

strongly frequency-dependend
screening of external forces

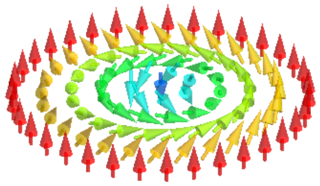
$$\mathbf{G}^{-1}(\omega) = \begin{pmatrix} \alpha\mathcal{D}(\omega) - i\omega\, m(\omega) & -\mathcal{G}(\omega) + i\alpha\omega\Gamma(\omega) \\ \mathcal{G}(\omega) - i\omega\alpha\Gamma(\omega) & \alpha\mathcal{D}(\omega) - i\omega\, m(\omega) \end{pmatrix}$$

damping

gyrocoupling

effective mass

frequency dependent dynamics of skyrmions in d=2
(linear response)



velocity of skyrmion	velocity of spin-currents	field gradients	thermal fluctuations

$$\mathbf{G}^{-1}(\omega)\mathbf{V}(\omega) \;=\; \mathbf{S}_c(\omega)\boldsymbol{v}_s(\omega) + \mathbf{S}_g(\omega)\boldsymbol{\nabla} B_z(\omega) + \mathbf{F}_{\text{th}}(\omega)$$

key for identification of dynamics: **fluctuation-dissipation theorem**

$$\langle \mathbf{F}^i_{\text{th}}(\omega)\mathbf{F}^j_{\text{th}}(\omega')\rangle = k_B T[\boldsymbol{G}^{-1}_{ij}(\omega) + \boldsymbol{G}^{-1}_{ji}(-\omega)]2\pi\delta(\omega+\omega')$$

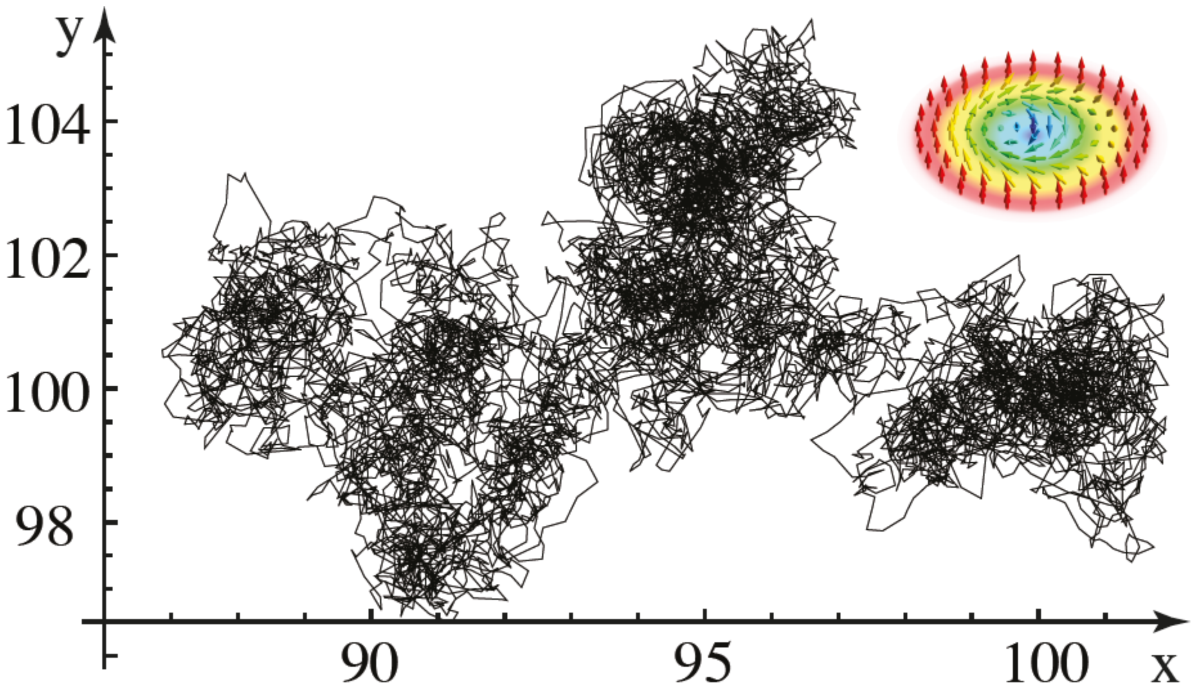
$$\boldsymbol{G}_{ij}(\omega) = \frac{1}{k_B T}\int_0^\infty \Theta(t-t')\langle \dot{R}_i(t)\dot{R}_j(t')\rangle e^{i\omega(t-t')} d(t-t')$$

Simulations (classically):
Landau Lifshitz Gilbert equation including thermal fluctuations

$$(\partial_t - v_s \nabla)\mathbf{M} = \gamma \mathbf{M} \times [\mathbf{B}_{eff} + \mathbf{b}_{fl}(t)] - \frac{\alpha}{M}\mathbf{M} \times (\partial_t - \frac{\beta}{\alpha} v_s \nabla)\mathbf{M}$$

with $< b_{fl,i}(t) >= 0$, $< b_{fl,i}(t) b_{fl,j}(t') >= 2D\delta_{i,j}\delta(t-t')$, $D = \frac{\alpha}{1+\alpha^2}\frac{k_B T}{\gamma M}$

Garcia-Palacios, Lazaro (1998)



$$G_{ij}(\omega) = \frac{1}{k_B T}\int_0^\infty \Theta(t-t')\langle \dot{R}_i(t)\dot{R}_j(t')\rangle e^{i\omega(t-t')} d(t-t')$$

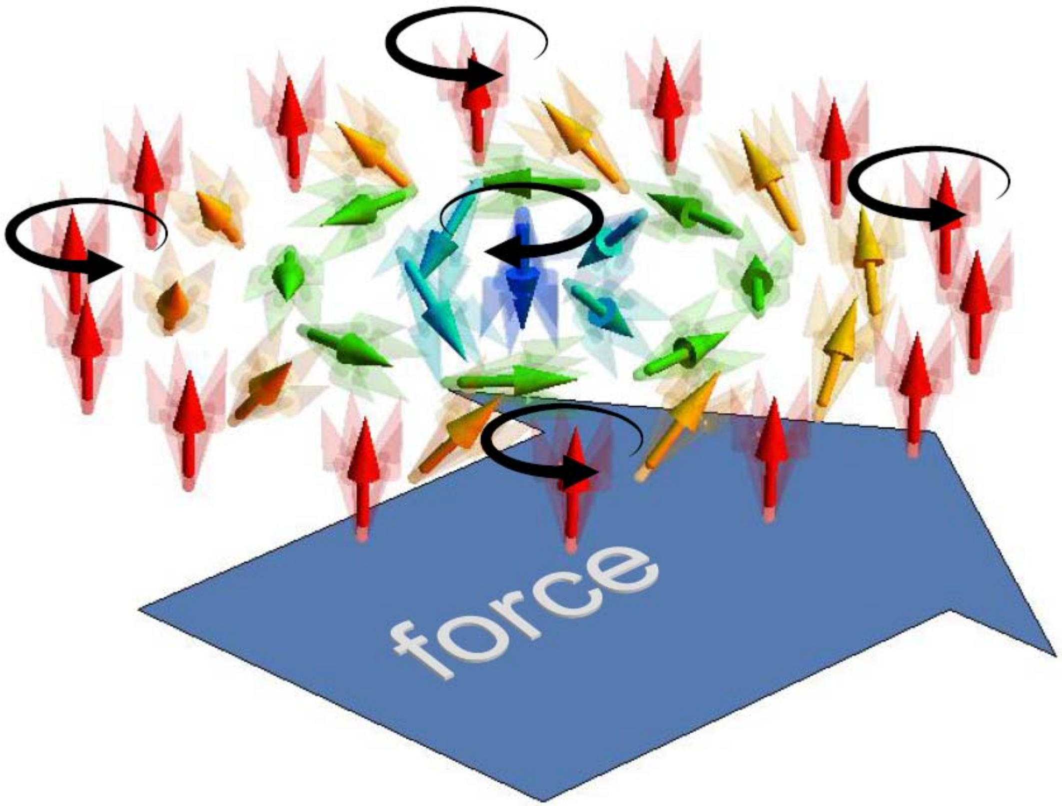
simulations in classical limit

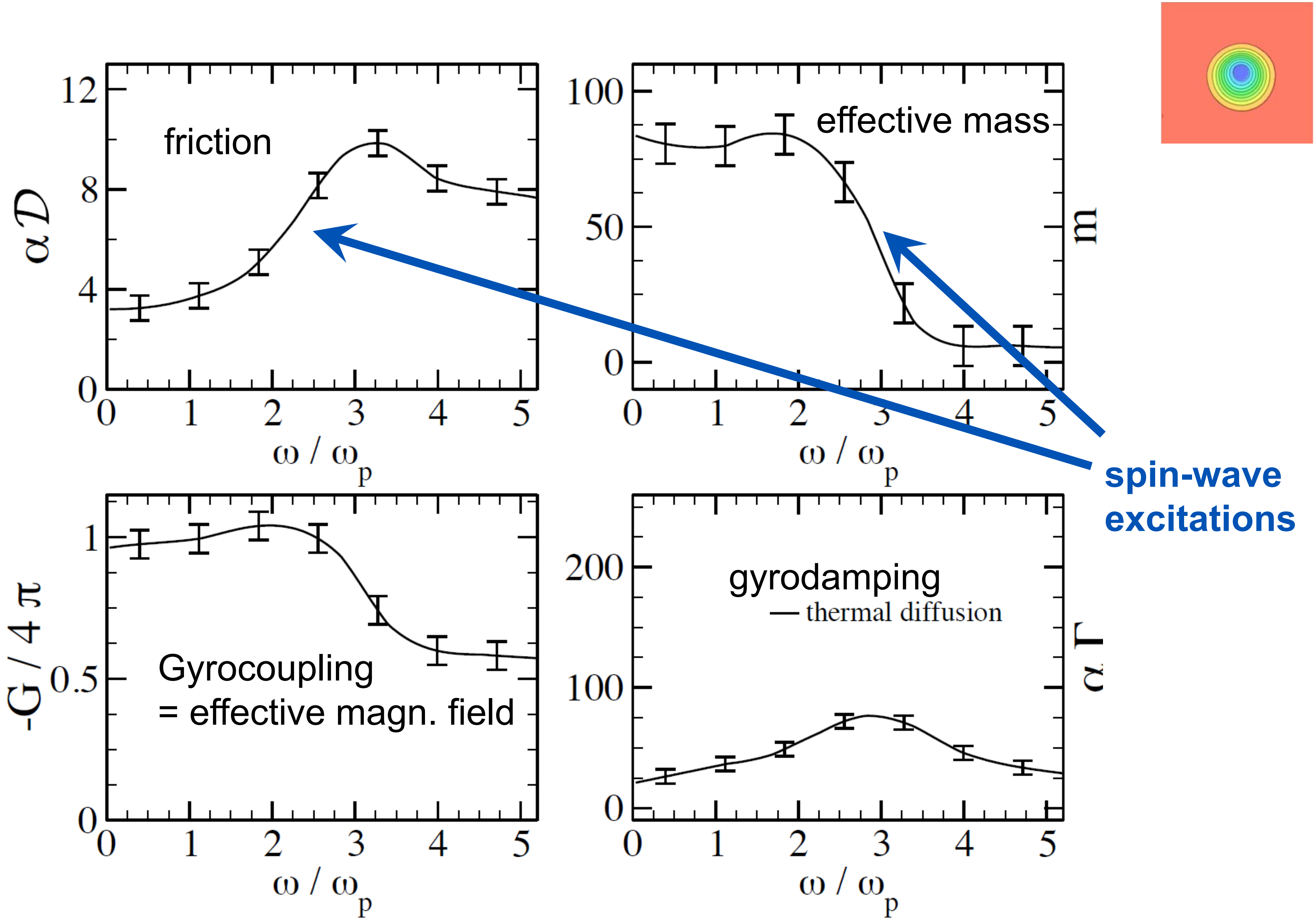
$$(\partial_t - v_s \nabla)\mathbf{M} = \gamma \mathbf{M} \times [\mathbf{B}_{eff} + \mathbf{b}_{fl}(t)] - \alpha \frac{\lambda}{M} \mathbf{M} \times (\partial_t - \frac{\beta}{\alpha} v_s \nabla)\mathbf{M}$$

linear equation: where is, e.g., mass coming from?

excitation of moving skyrmion

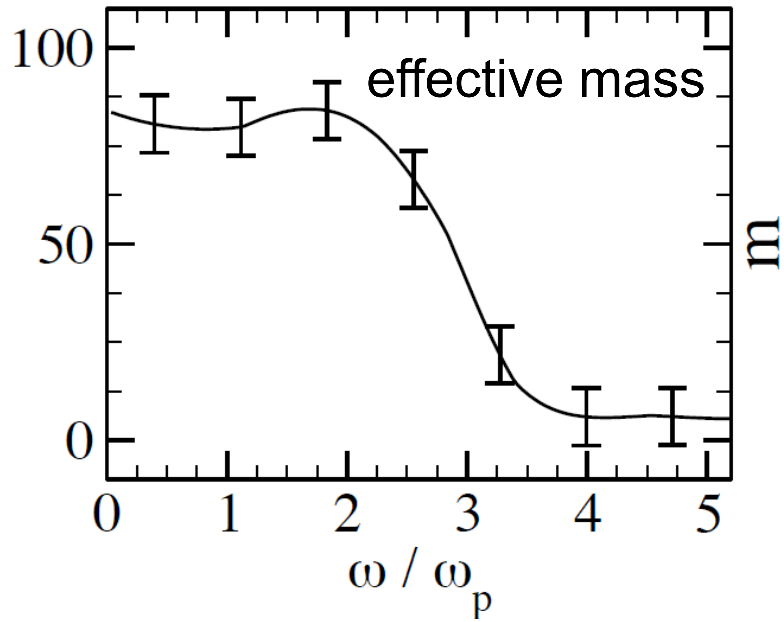
stores “kinetic” energy leads to retardation effects





units: gap of ferromagnet (~GHz)

- strong frequency dependencies set by excitations of spin wave
- huge effectiv mass (~ number of flipped spins of skyrmion)



Bad news?
No fast skyrmion dynamics?

No!
Depends on how skyrmion is manipulated!

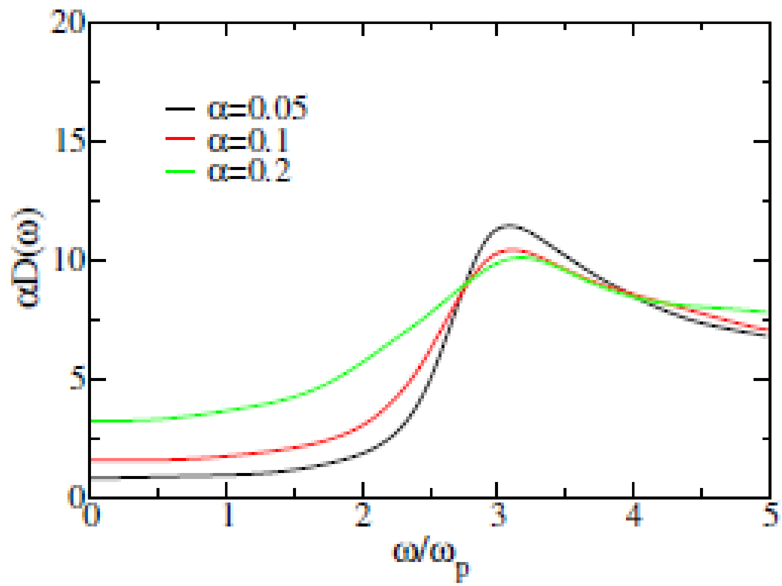
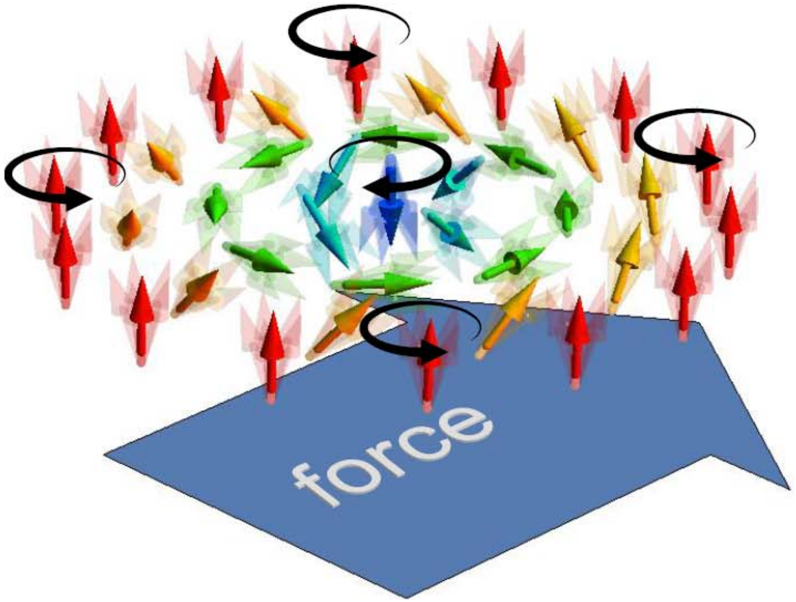
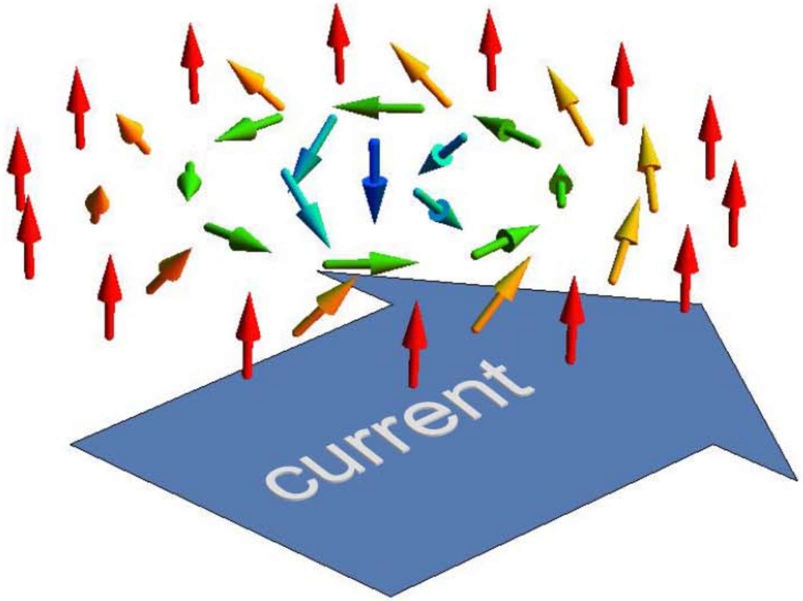


FIG. S6. Effective damping, $\alpha D(\omega)$ for $\alpha = 0.2$, 0.1 and 0.05.



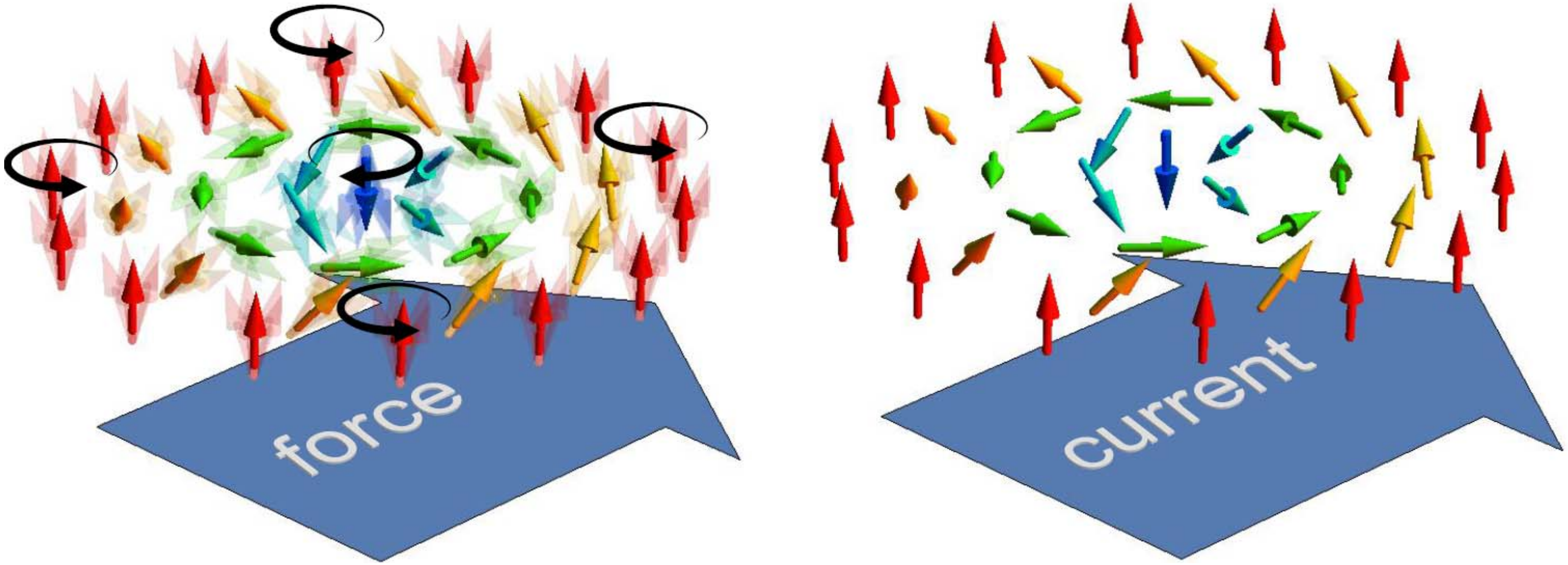


thermal fluctuations or
external forces by field gradients
excite internal modes

➡ large mass, delayed response

skyrmion flows approximately with
electric current, only weak excitation
of external modes

➡ small mass, fast response



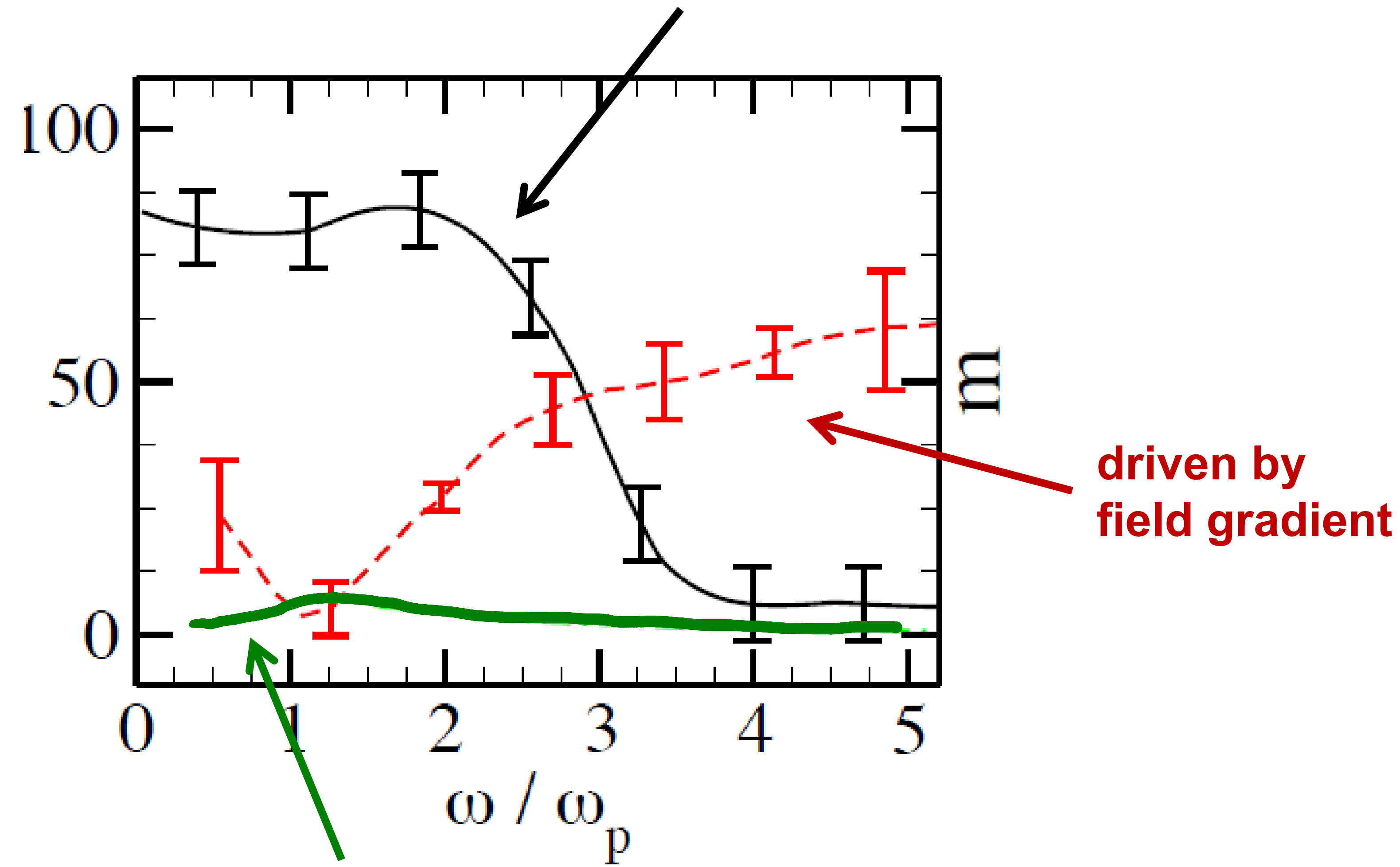
$$\mathbf{G}^{-1}(\omega)\mathbf{V}(\omega) \;=\; \mathbf{S}_c(\omega)\boldsymbol{v}_s(\omega) + \mathbf{S}_g(\omega)\boldsymbol{\nabla} B_z(\omega) + \mathbf{F}_{\rm th}(\omega)$$

"apparent mass" depends on driving mechanism
controlled by frequency dependence of screening

$$\boldsymbol{G}_a(\omega)\boldsymbol{S}_g(\omega = 0) = \boldsymbol{G}(\omega)\boldsymbol{S}_g(\omega)$$

apparent dynamics matrix

apparent mass:



current driven: tiny mass $\propto \alpha - \beta$

“nice to have” properties of skyrmions

- small “apparent” mass for ultra-fast manipulations by currents
- small friction
- tiny thermal diffusion constant (precession in huge effective B-field)

$$D = k_B T \frac{\alpha \mathcal{D}}{\mathcal{G}^2 + (\alpha \mathcal{D})^2}$$

Quantum dynamics of skyrmions

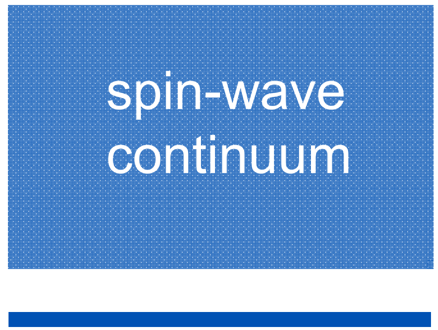
Garst, Schütte, 2014

- only relevant in insulators (e.g. Cu_2OSeO_3) at T<< gap
- Skyrmion: dynamics in effective B-field

  ⇒ skyrmion lives in single flat Landau level

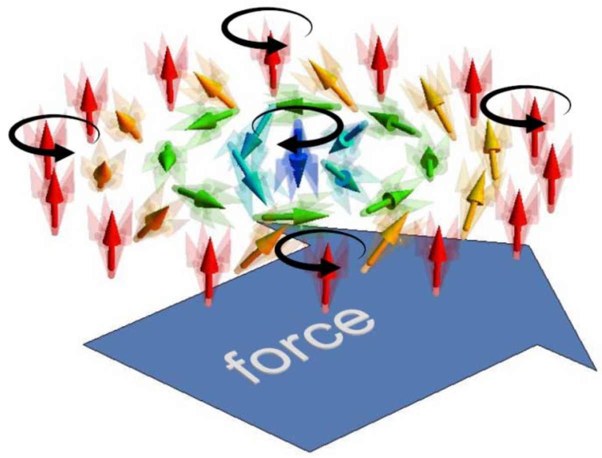
- close to boundary: chiral edge states
  coherent quantum dynamics close to edge state

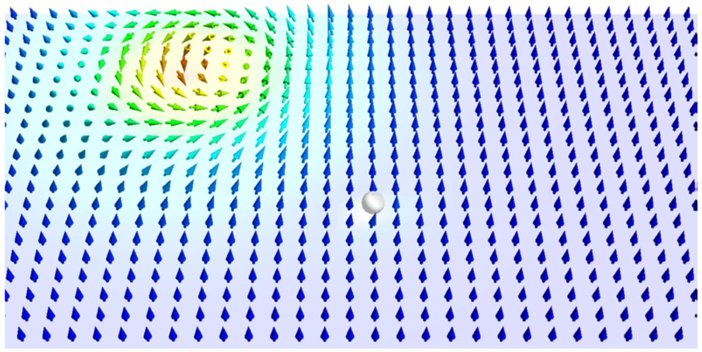
- also interesting: skyrmion & topological insulators,
  topological superconductors,…



conclusions: skyrmions as particles

- large intrinsic effective mass of skyrmions
- dynamics strongly frequency dependent due to emission of spin-waves
- nevertheless: rapid manipulation by currents possible
- quantum dynamics: particle in Landau level, edge states
- skyrmions & obstacles: pinning & depinning controllable by magnetic fields and currents speeding up by defects



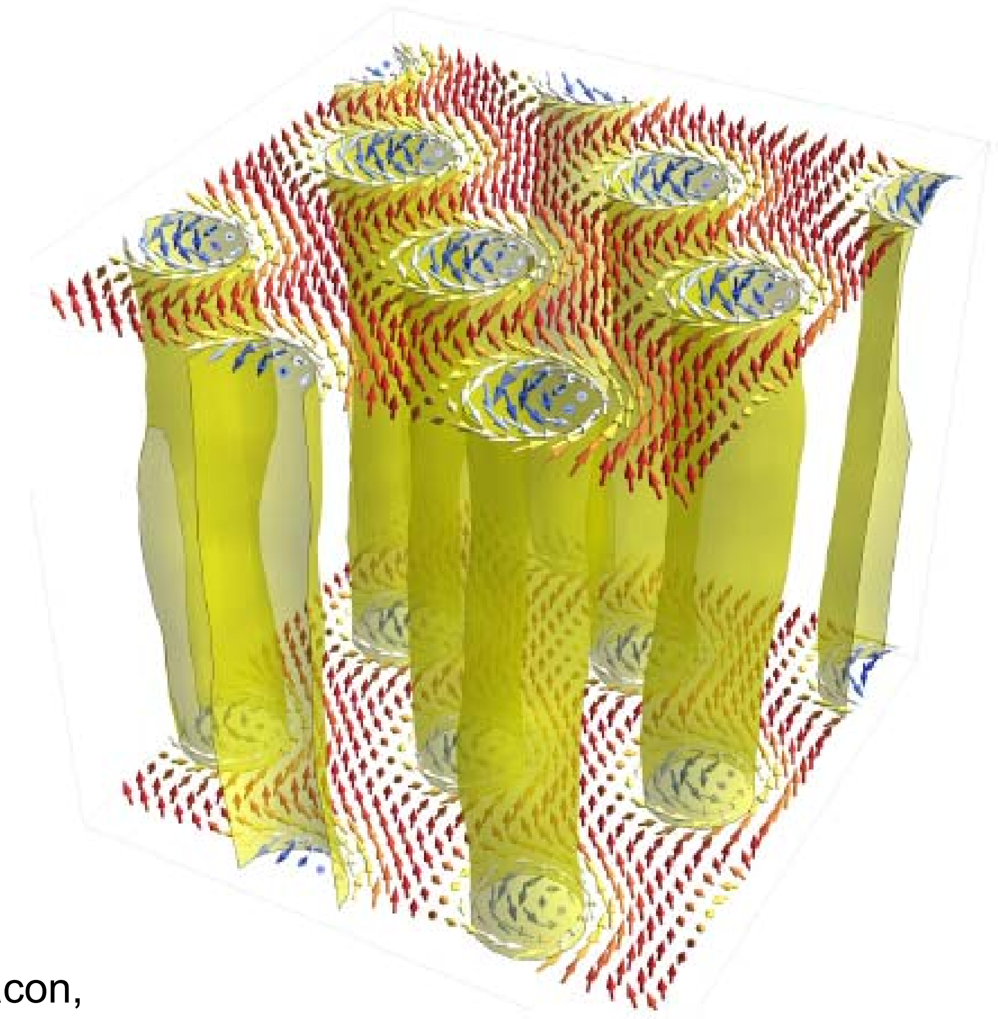


Schütte, Isagawa, Rosch, Nagaosa (2014)
Jan Müller, Achim Rosch (2014)

Back to 3d:

destroying skyrmions &
changing topologogy

➡ emergent
magnetic
monopoles

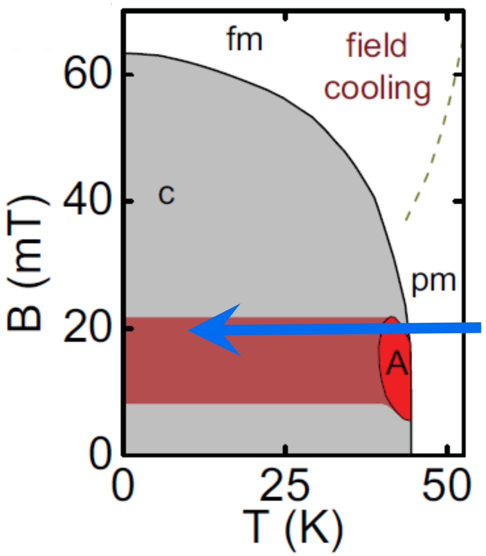


Milde, Köhler, Seidel,Eng, Bauer, Chacon,
Pfleiderer, Buhrandt, A. R., Science (2013)

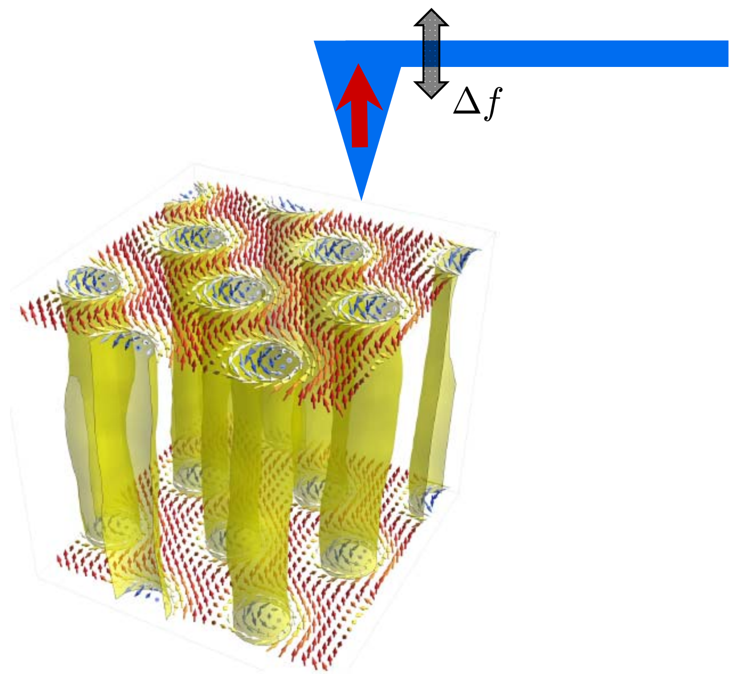
destruction of the skyrmion phase

experiment: track by magnetic force microscopy skyrmions on surface of $Fe_{0.5}Co_{0.5}Si$

Milde, Köhler, Seidel, Eng, TU Dresden



step 1: cool system down to 10 K at B=20 mT
measure z-component of magnetization
by **magnetic force microscopy**



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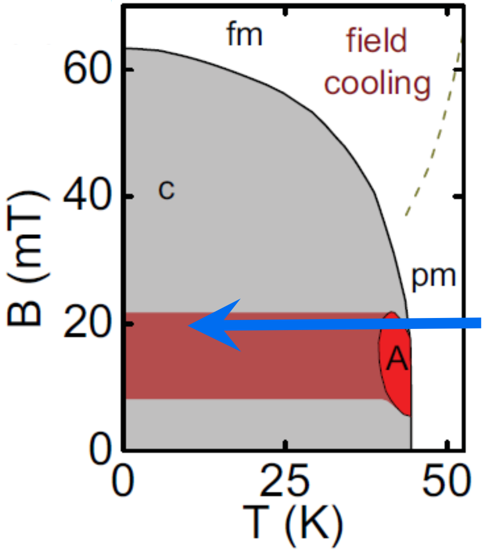
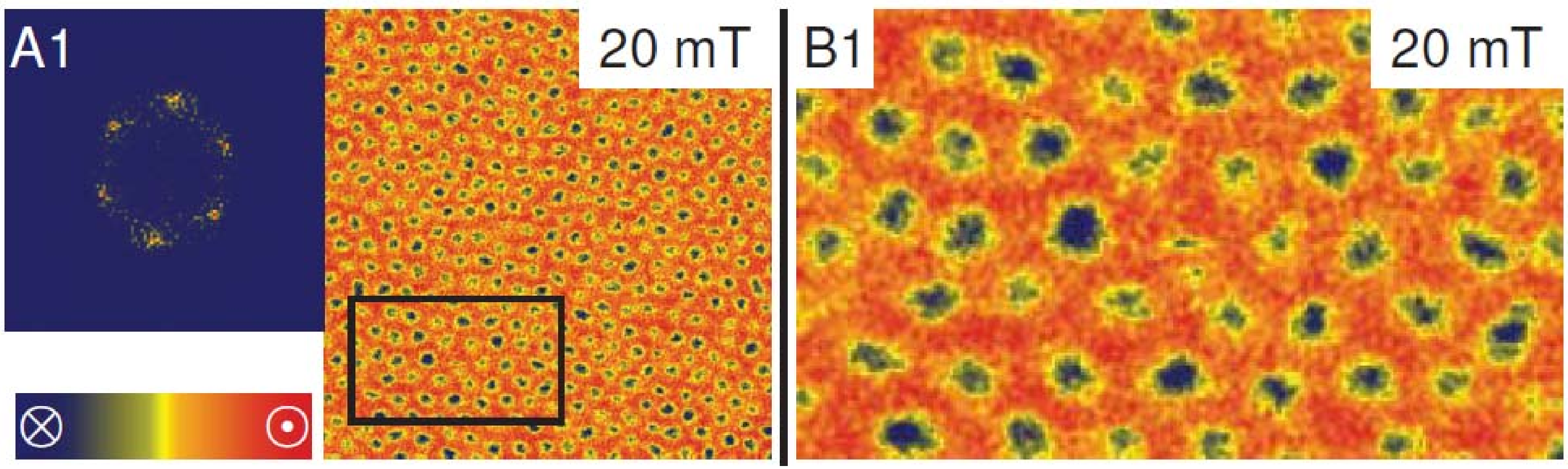


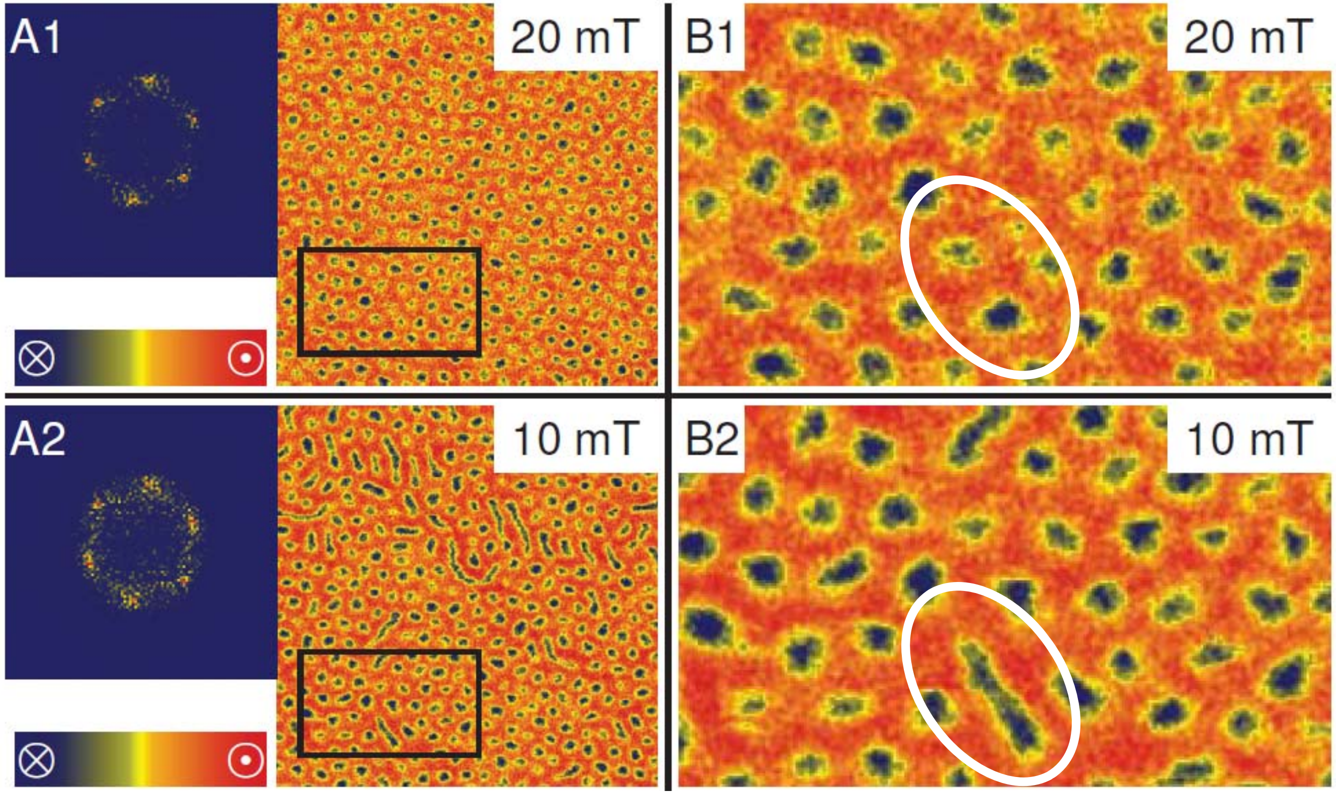
step 1: cool system down to 10 K at B=20 mT
measure z-component of magnetization
by **magnetic force microscopy**

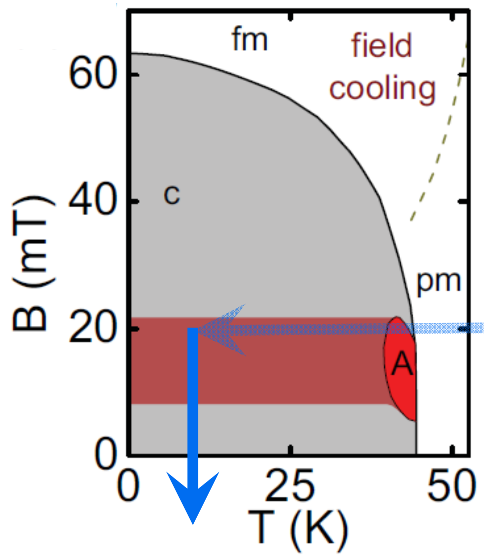
result: metastable skyrmion lattice, slightly disordered
good contrast due to low temperature
few fluctuations (high topolog. stability)



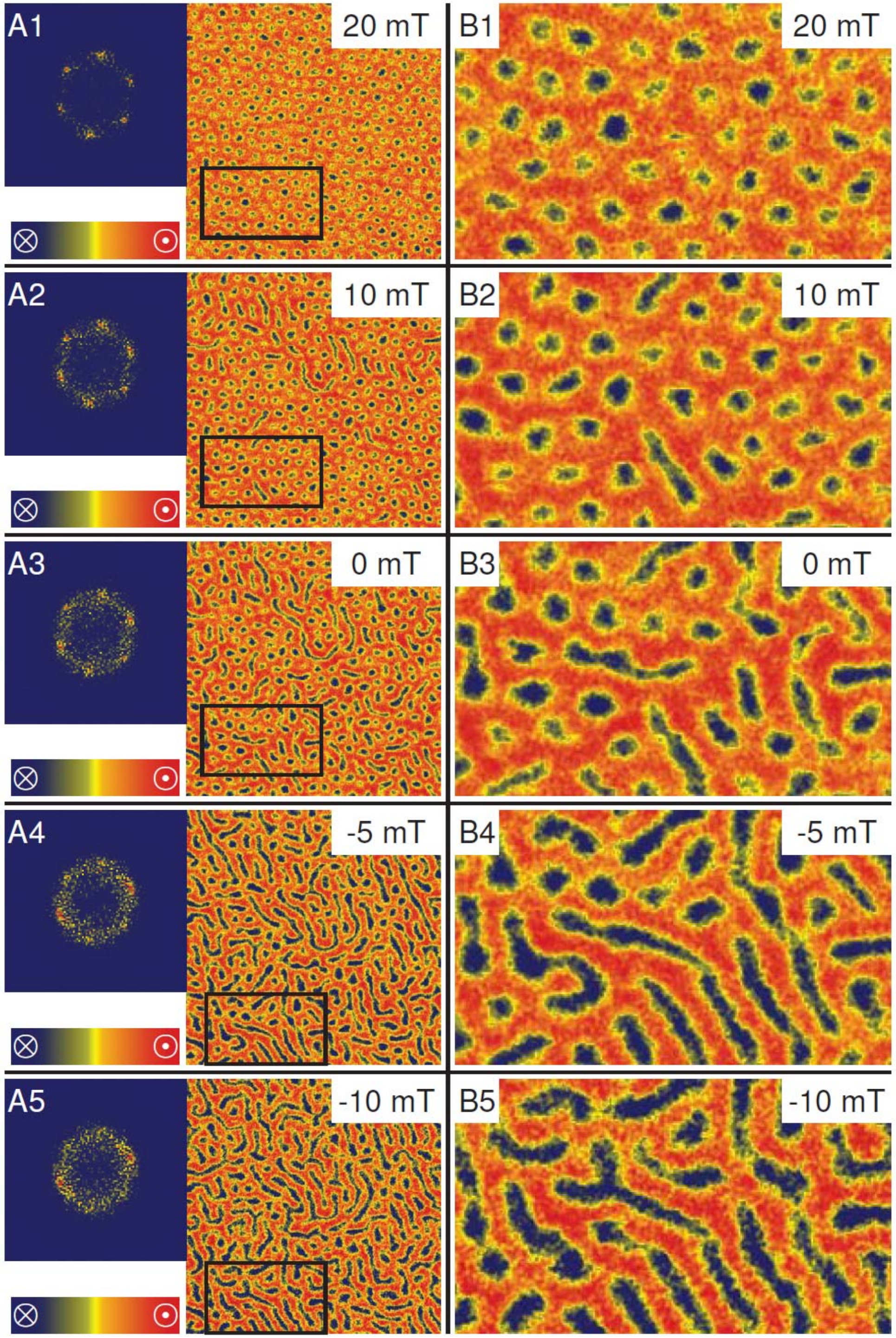
destruction of the skyrmion phase

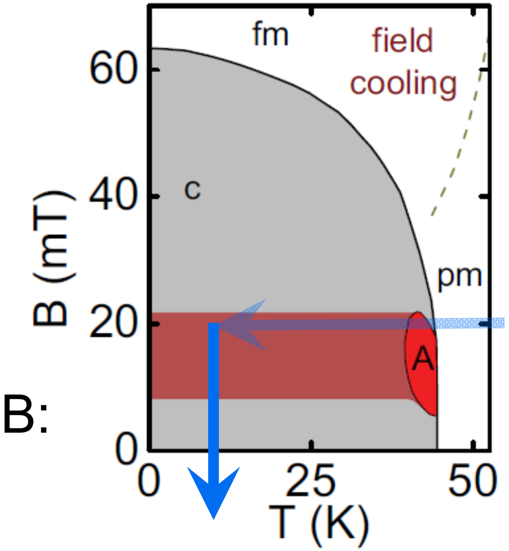
step 2: destroy skyrmion lattice by reducing B-field





observation:
neighboring skyrmions merge, forming elongated objects



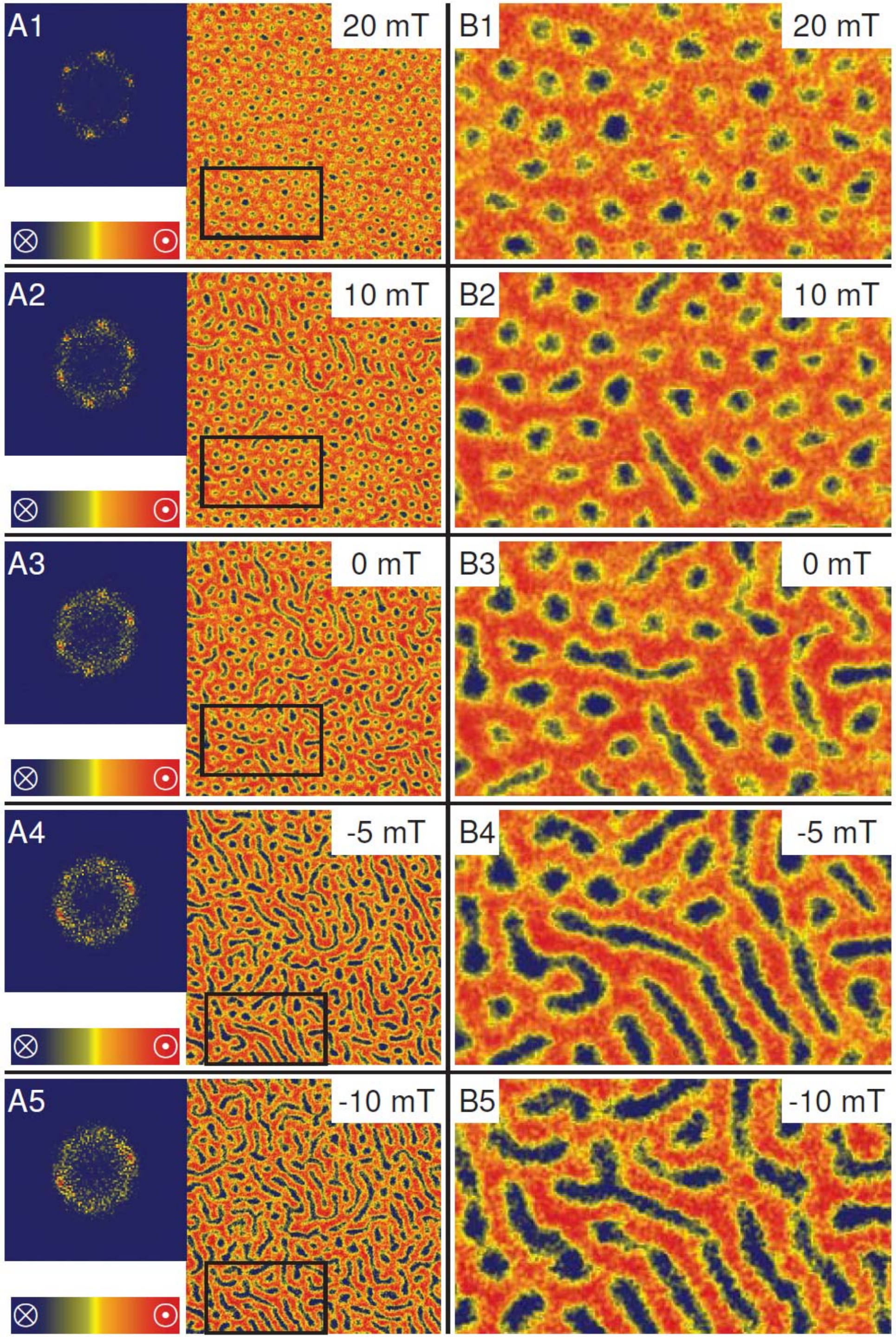


further reducing B:

longer and longer linear structures form by combining skyrmions

realizing helical state with large number of defects

← local helix

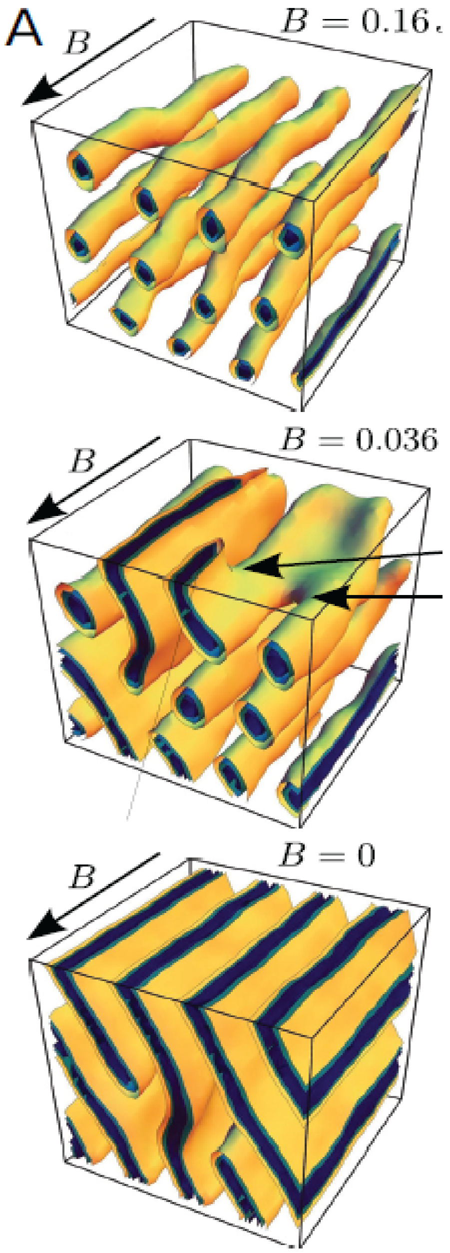


What happens in the bulk?

How does topology change ?

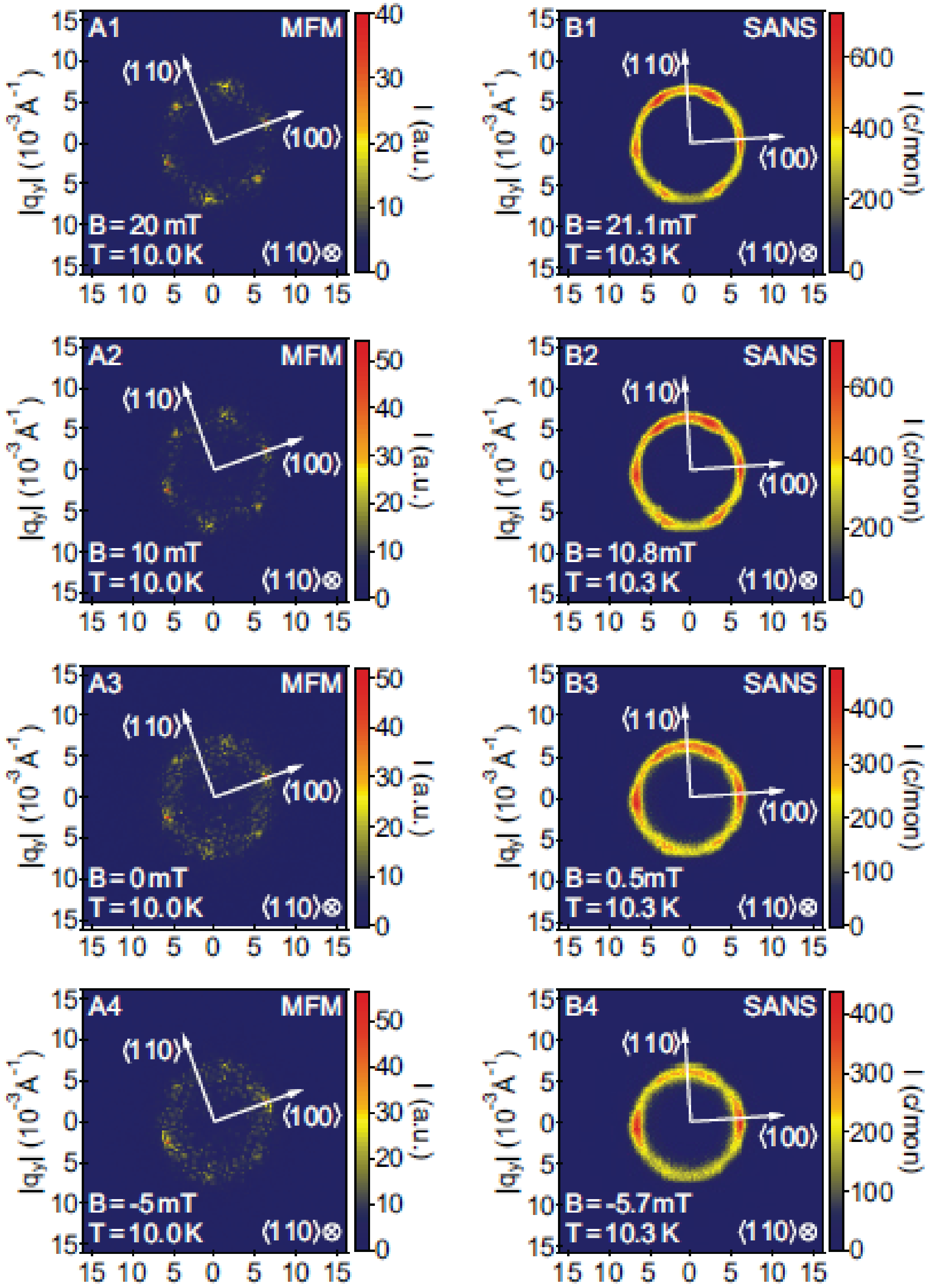
Surface reflects bulk behavior

numerical simulations:



surface: FT of MFM

bulk: neutrons



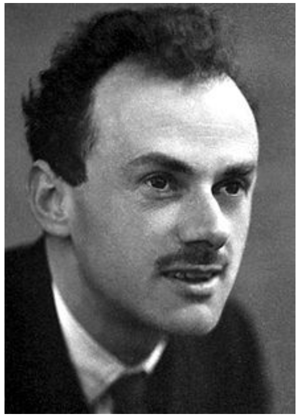
emergent electrodynamics:

winding number of skyrmions
=
one flux quantum of emergent magnetic field

needed to change winding number:

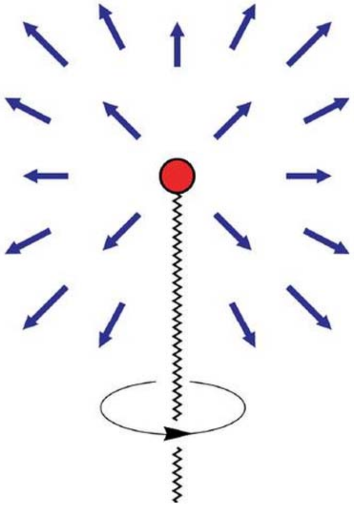
sources and sinks of emergent magnetic field
=
quantized magnetic charges
=
emergent magnetic monopoles and antimonopoles

Historical remarks on magnetic monopoles



Paul Dirac (1931):

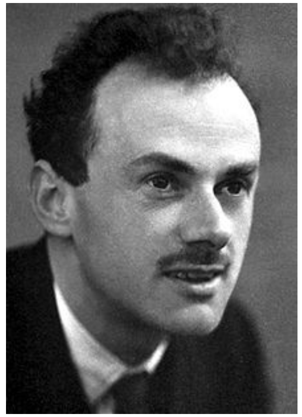
why is charge quantized?
magnetic charge = magnetic monopole
would enforces charge quantization



„Dirac string" invisible only if both electric and magnetic charge are quantized

$$\mathbf{q_m} = n\frac{2\pi\hbar}{\mathbf{e}} \quad \Leftrightarrow \quad \mathbf{e} = n\frac{2\pi\hbar}{\mathbf{q_m}}$$

Historical remarks on magnetic monopoles



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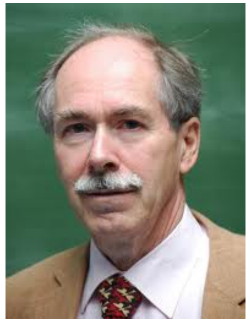
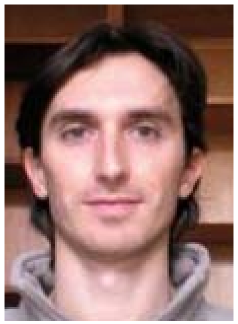


t'Hooft (1974), Polyakov (1974):

magnetic monopoles occur naturally
in certain gauge theories

example: SO(3) gauge theory generalization of
of Heisenberg ferromagnet
QED at low energies with hedgehog=monopole

Ryzhkin (2005) ; Castelnovo, Moessner, Sondhi (2008)



emergent deconfined monopoles in spin ice

previous talk

sources of H-field but **not** quantized

Fennell *et al.*,15 (2009), Morris *et al.*, (2009)

merging of two skyrmions

- winding number changes from -2 to -1 (top to bottom)

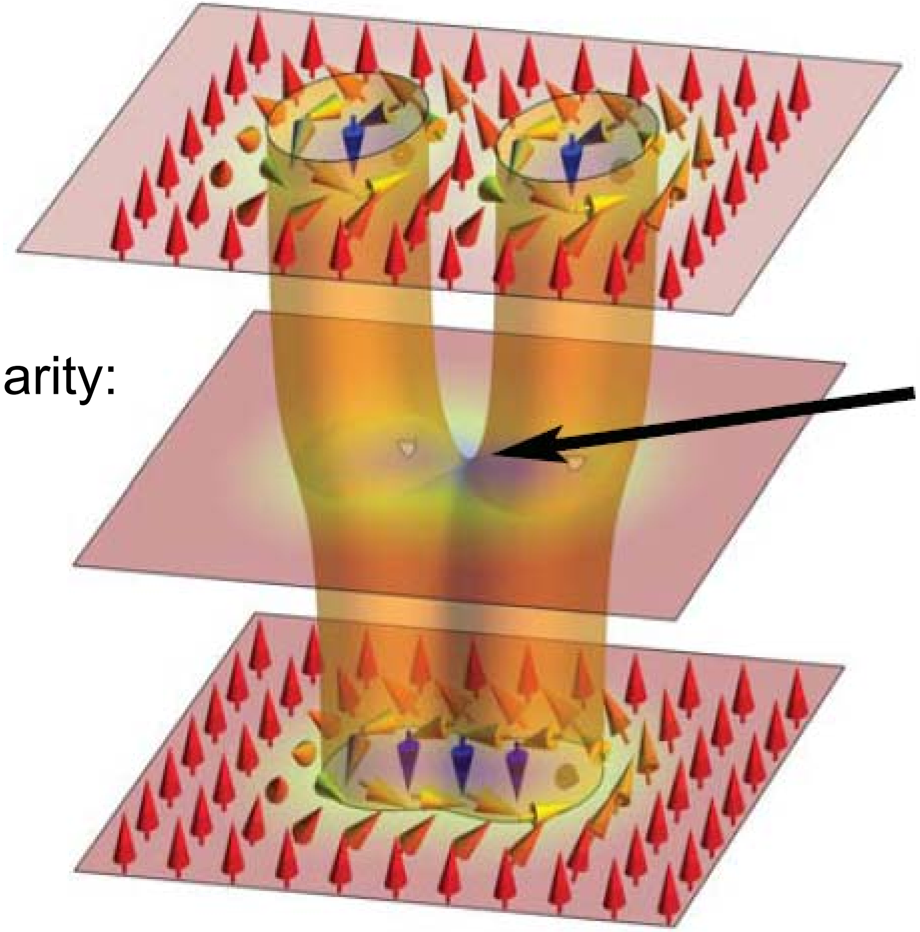
⇨ singularity with **vanishing magnetization**, ***M=0***

- for closed surface encircling singularity: calculate total flux

$$\oint_{\partial\Omega} \mathbf{B_e} d\mathbf{S} = \int_{\mathbf{\Omega}} \nabla \mathbf{B_e}$$

- incoming: two flux quanta
  outgoing: one flux quantum

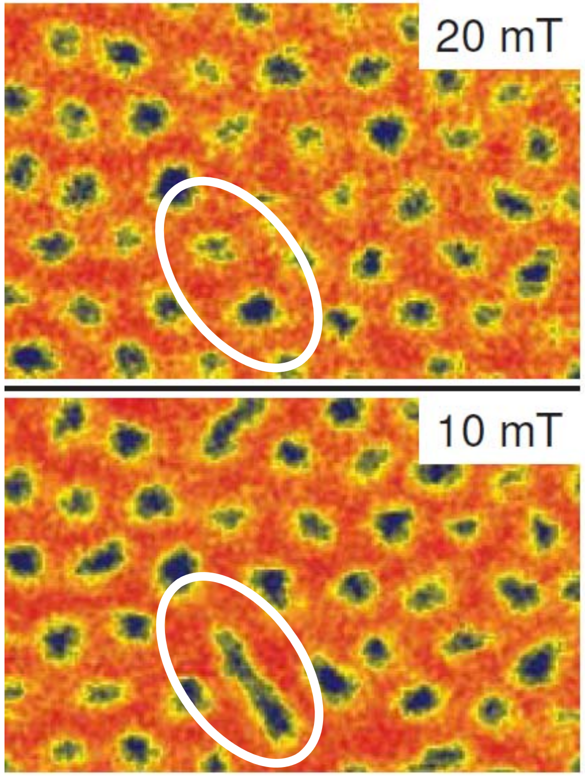
⇨ **singularity = emergent magnetic antimonopoles**

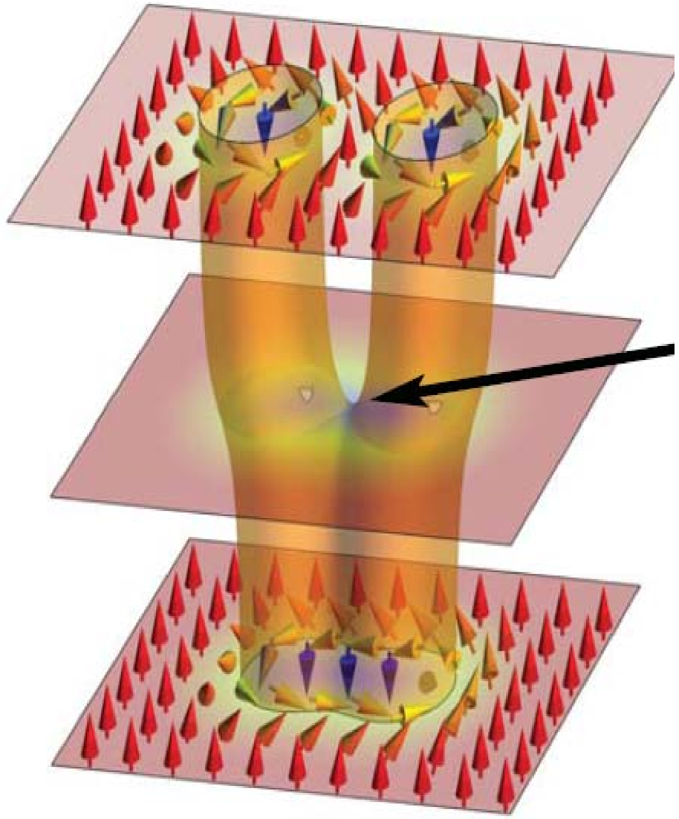


follows Dirac‘s quantization rule but generically NOT deconfined

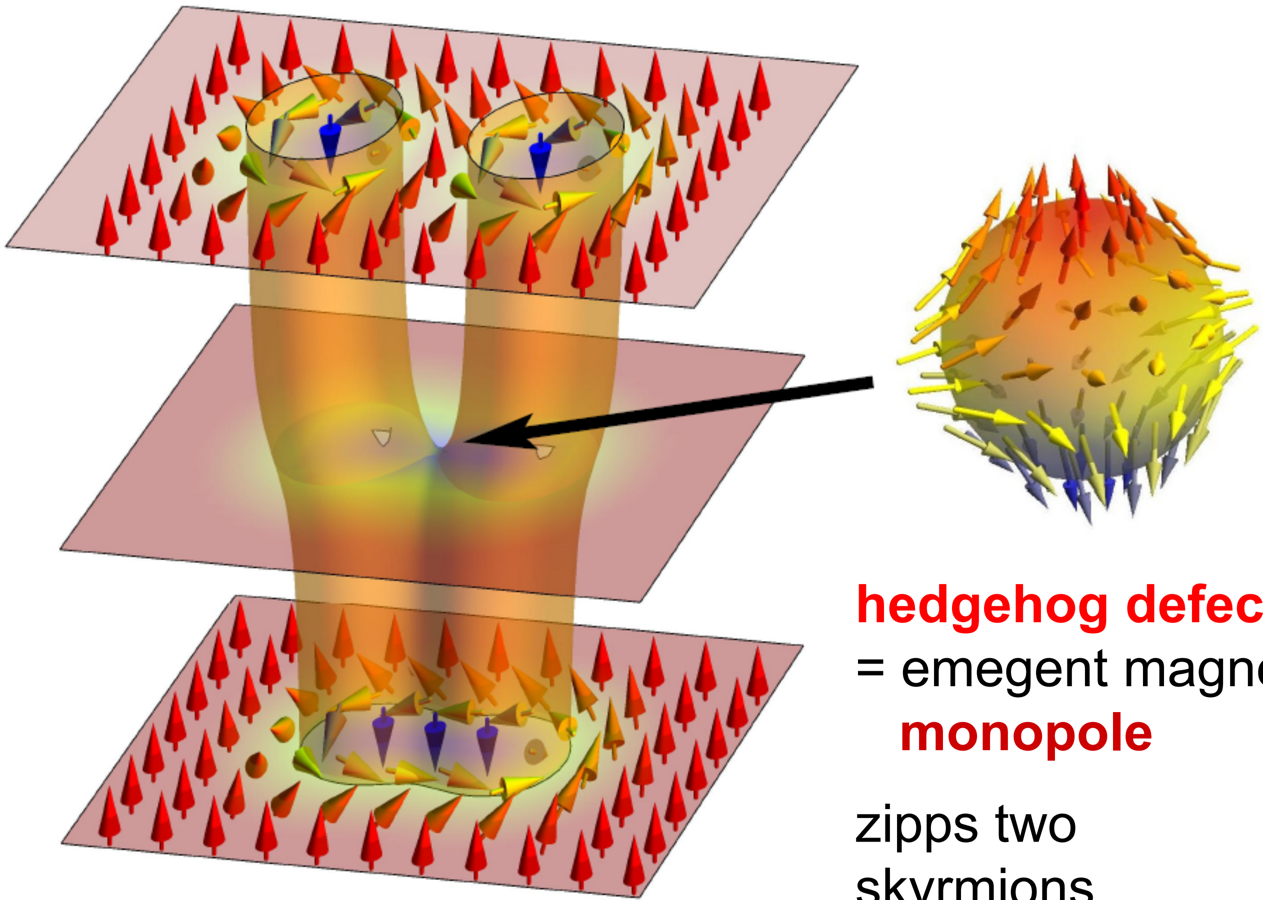
merging of two skyrmions

either **antimonopole** flying out of the surface or **monopole** flying into the surface



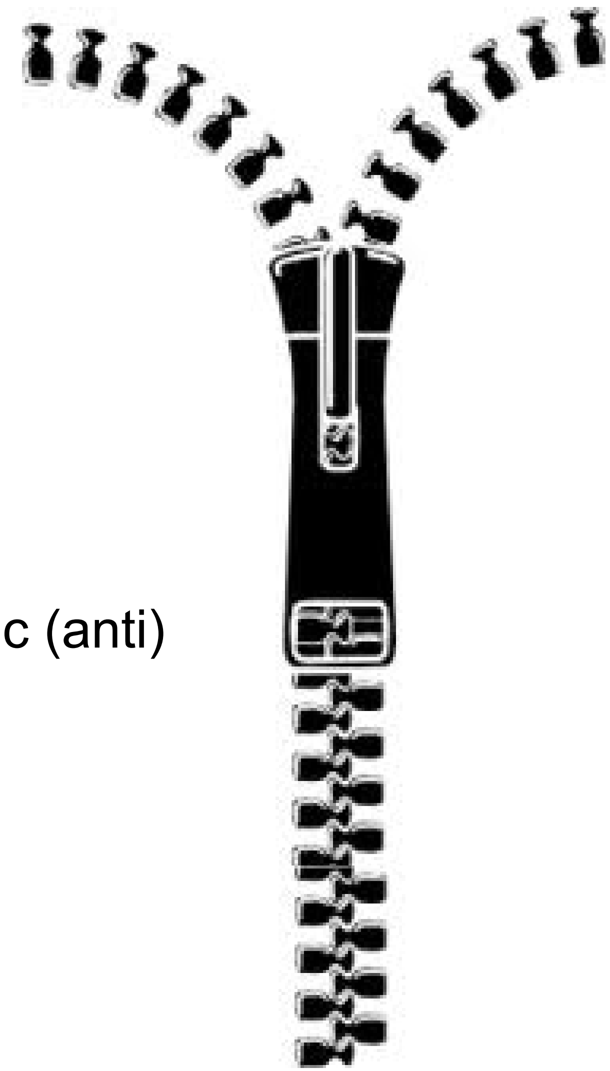


merging of two skyrmions



hedgehog defect
= emegent magnetic (anti)
monopole

zipps two
skyrmions
together



Energetics & dynamics of monopoles

Landau Lifshitz Gilbert equation including thermal fluctuations:

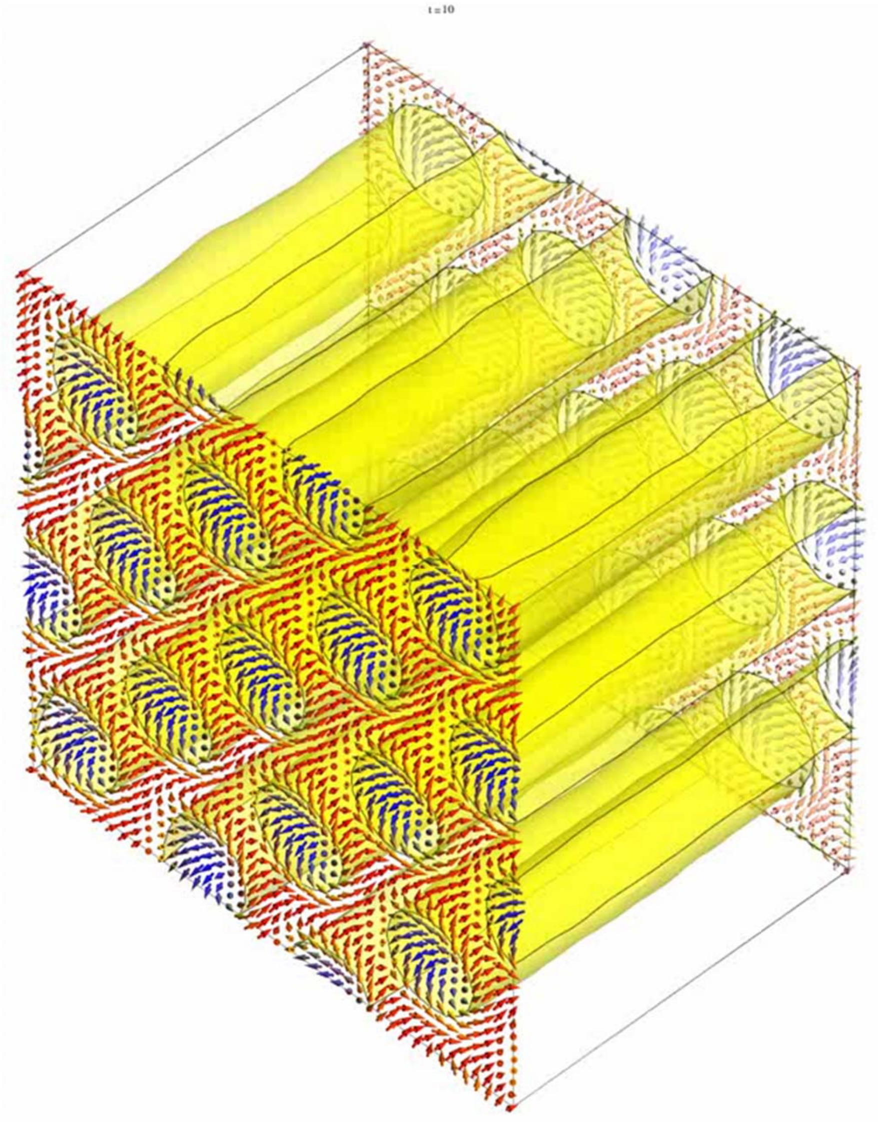
$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times [\mathbf{B}_{eff} + \mathbf{b}_{fl}(t)] - \gamma \frac{\lambda}{M} \mathbf{M} \times (\mathbf{M} \times [\mathbf{B}_{eff} + \mathbf{b}_{fl}(t)])$$

with $< b_{fl,i}(t) >= 0 \qquad < b_{fl,i}(t) b_{fl,j}(t') >= 2D\delta_{i,j}\delta(t-t')$

$$D = \frac{\lambda}{1+\lambda^2} \frac{k_B T}{\gamma M}$$

[1] Garcia-Palacios, Lazaro, PRB **58**, 14940 (1998)

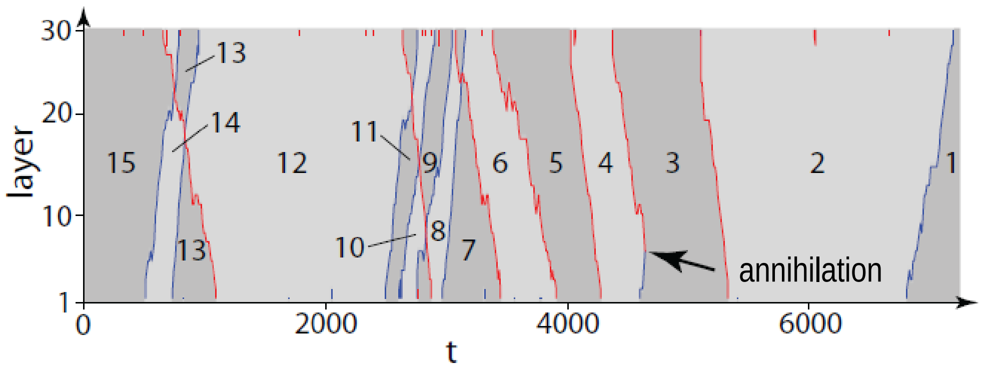
phase conversion with **monopoles** and **antimonopoles**



- stochastic Landau-Lifshitz-Gilbert dynamics (including thermal noise)
- spheres: single monopoles

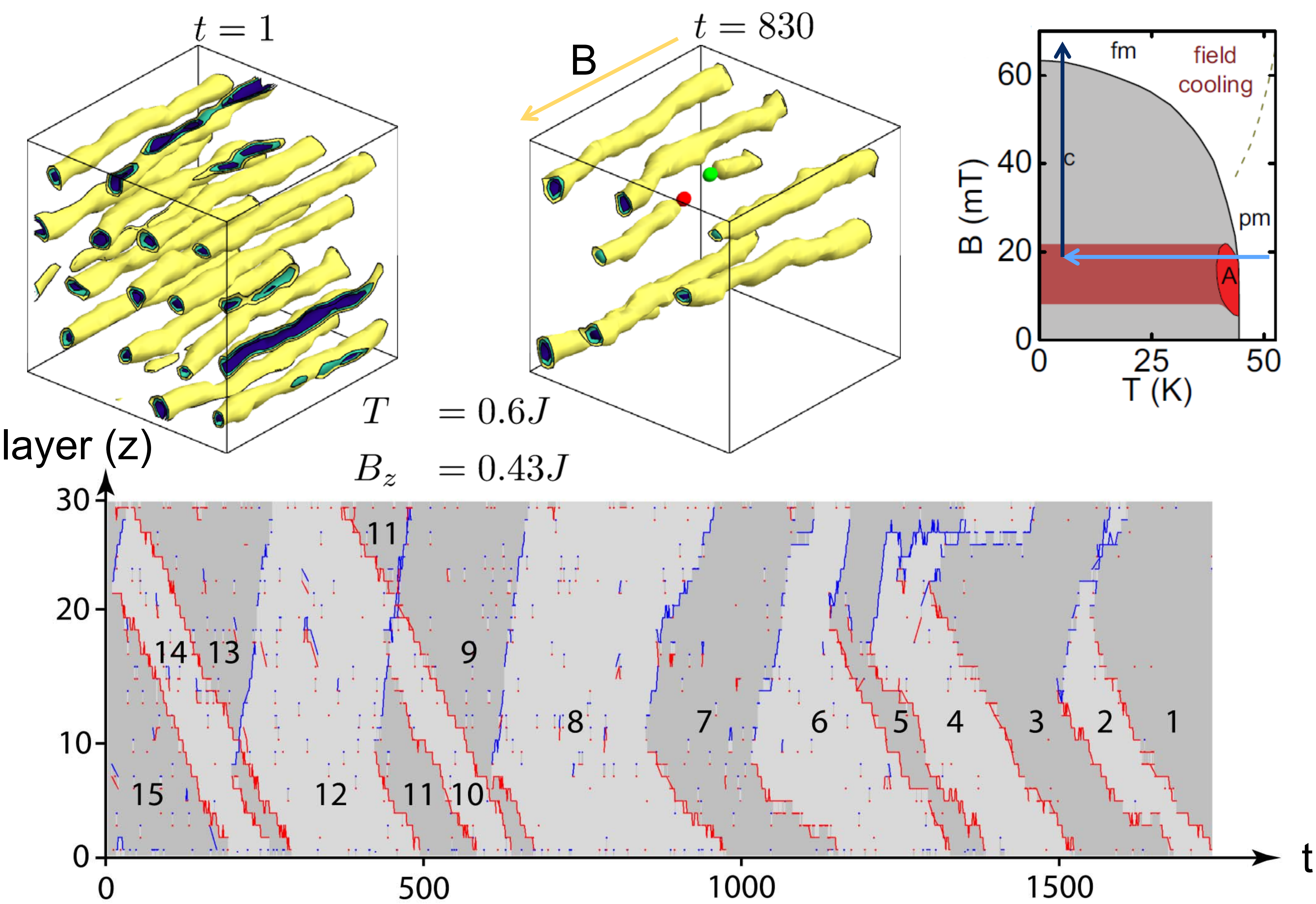
- **antimonopoles** move up
  monopoles move down

Milde, Köhler, Seidel, Eng, Bauer, Chacon, Pfleiderer, Buhrandt, Schütte, A. R., Science, 2013

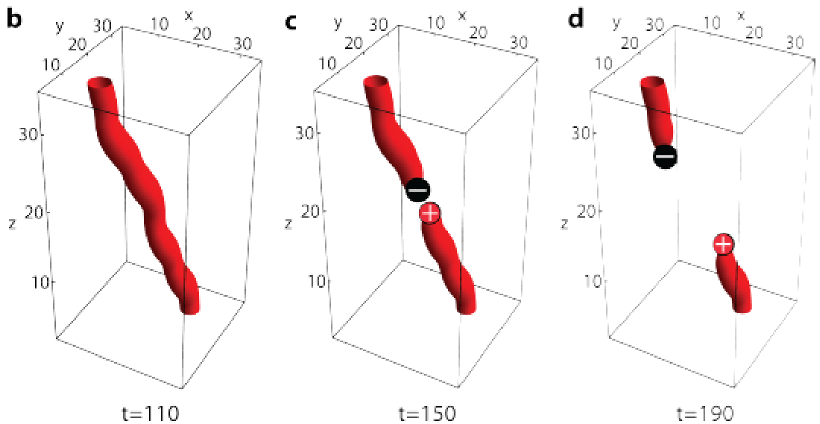


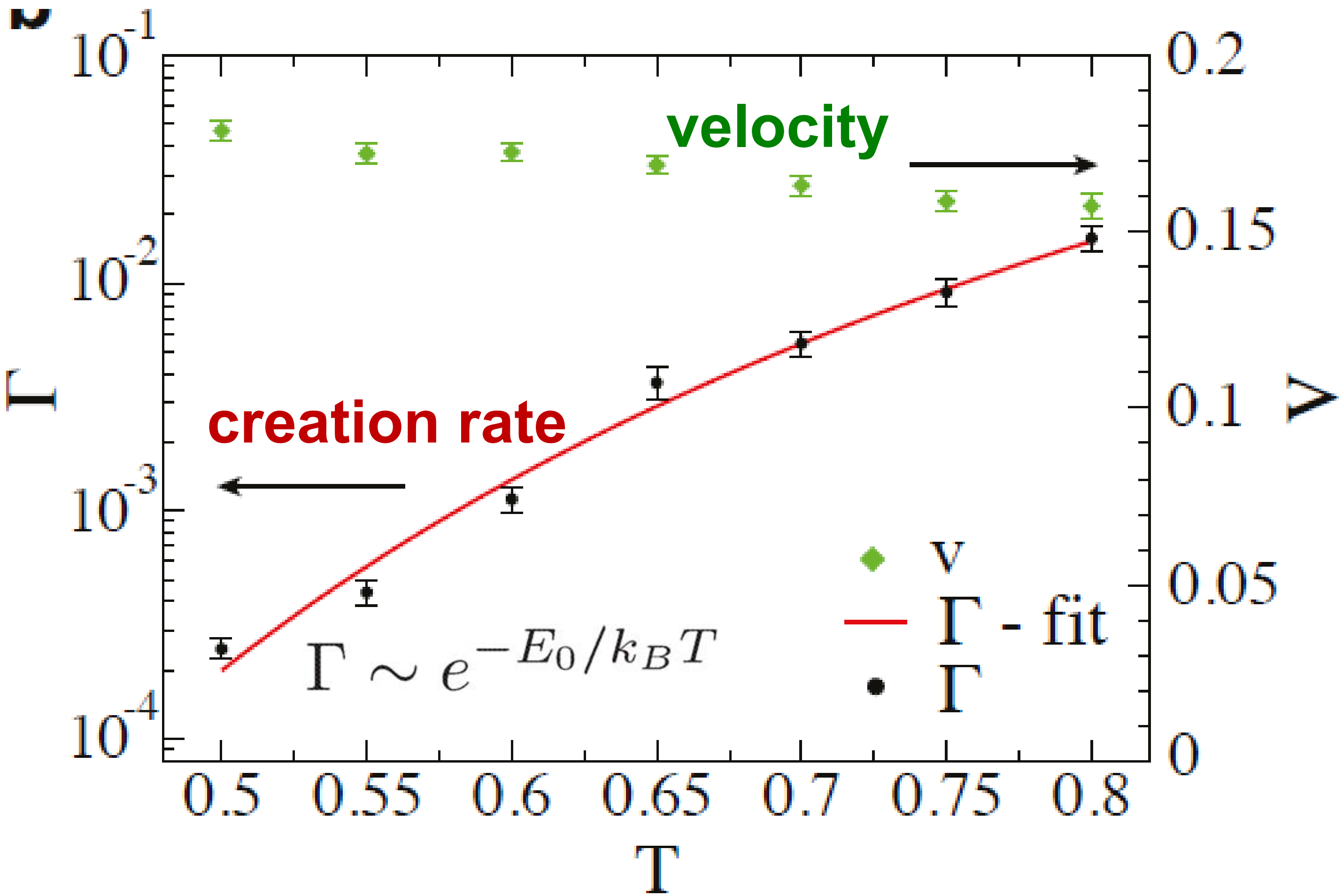
^

Phase conversion to ferromagnetic phase



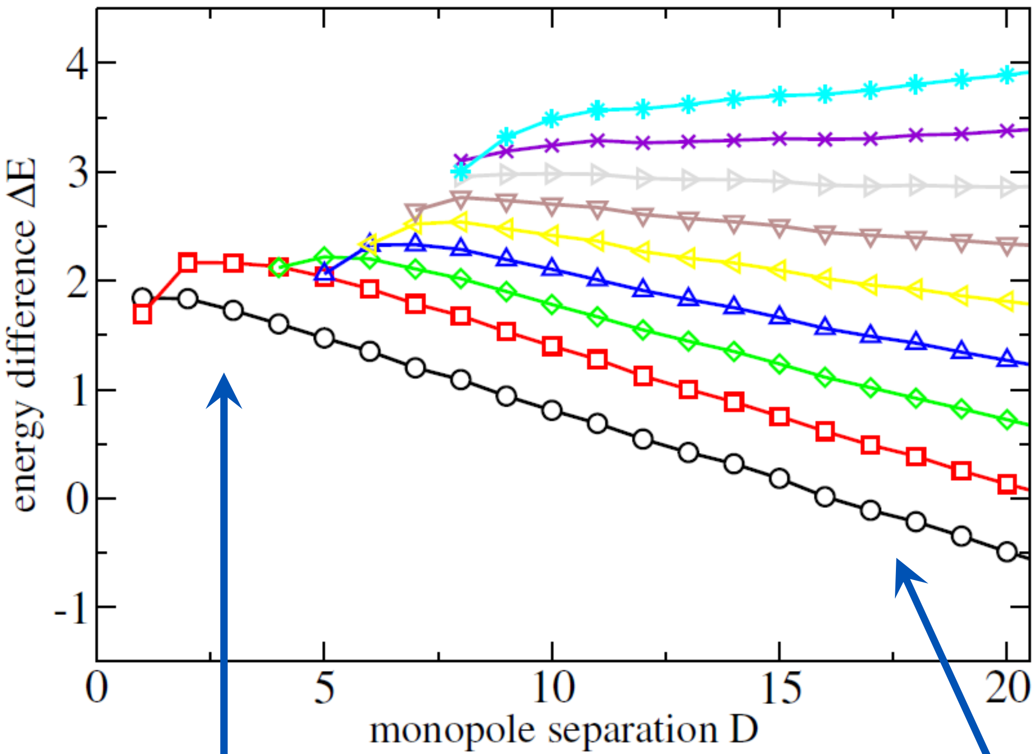
Monopole dynamics





energy scale?
velocity ?

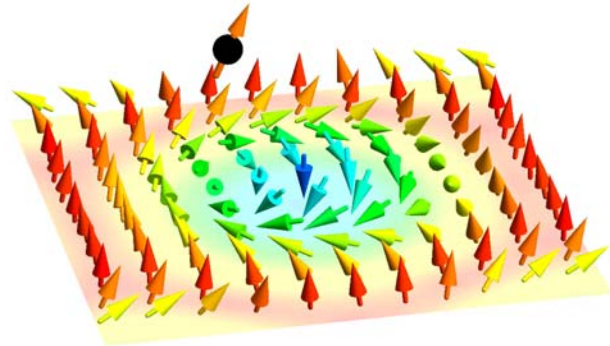
Monopole-antimonopole potentials



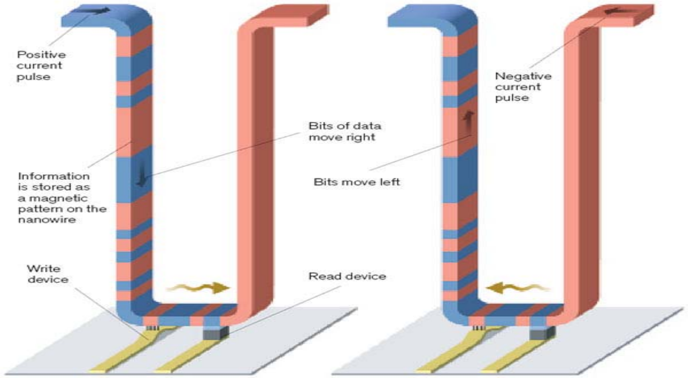
energy scale controlling creation
rates of monopole/antimonople pairs

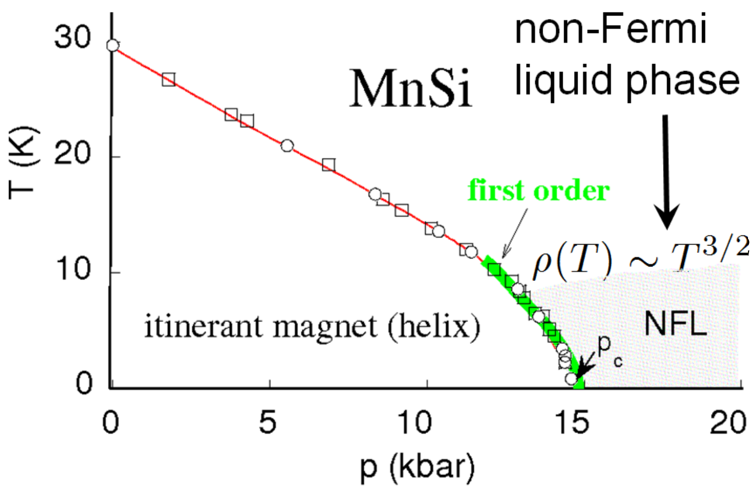
finite slope =
line tension changes
sign as function of
B-field (skyrmions stable
or unstable)

areas of future research

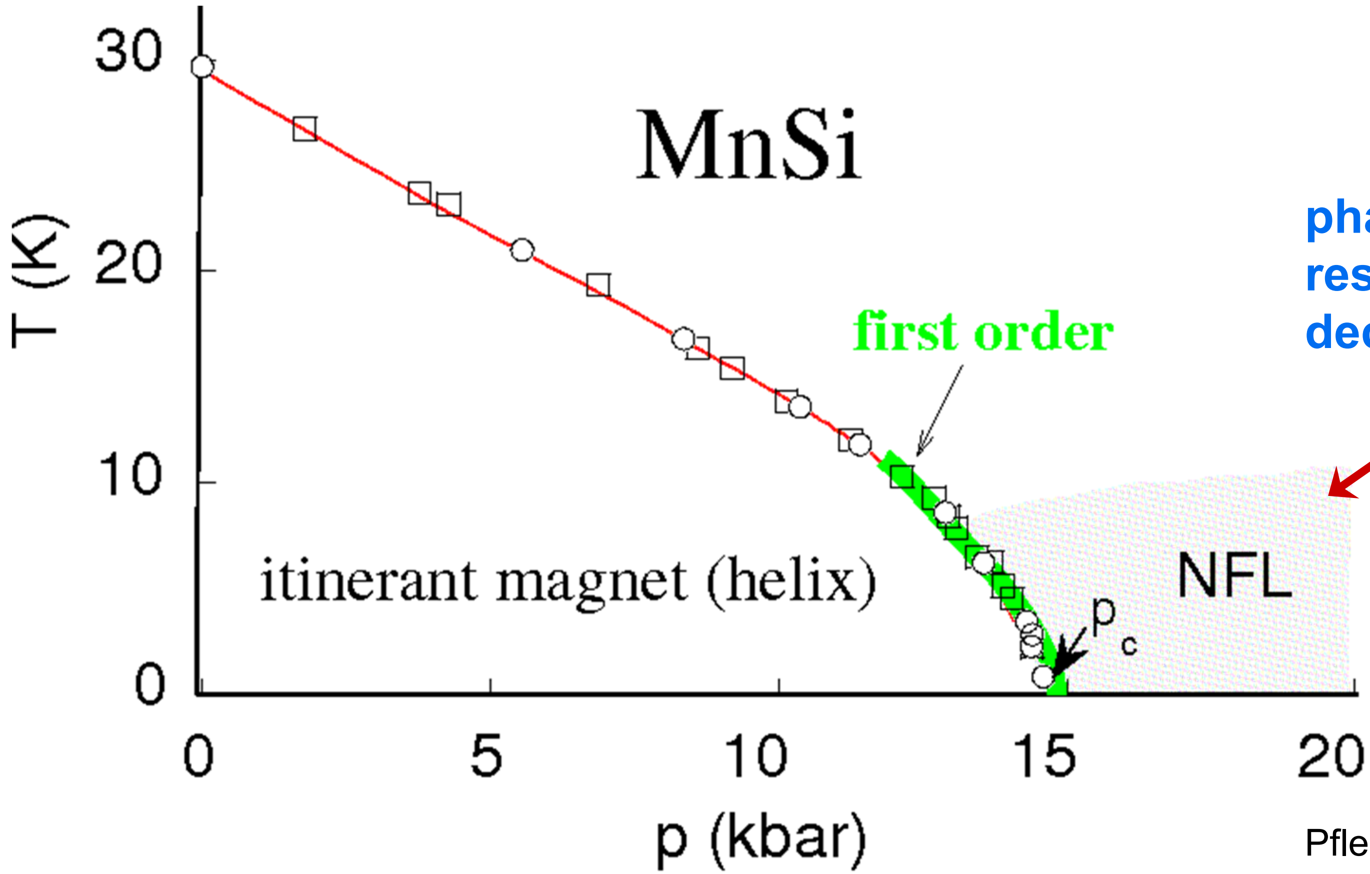


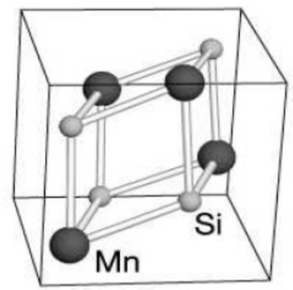
- potential for applications? „**skyrmionics**“
  memory: race-track devices build from skyrmions
  logic: computing with skyrmions
- experimental challenge:
  controlling skyrmions in nano-devices
  controlled writing & deleting of skyrmions
  drive & detect electrically
  using Berry phases
- Berry phases in real-space & momentum space
  e.g. skyrmions in topological insulators
- quantum coherent motion of skyrmions
  in insulators: edge states
- skyrmions & disorder (depinning transition)
- deconfinded skyrmion/monopole liquids:
  exotic high-pressure state in MnSi





MnSi: back to the roots





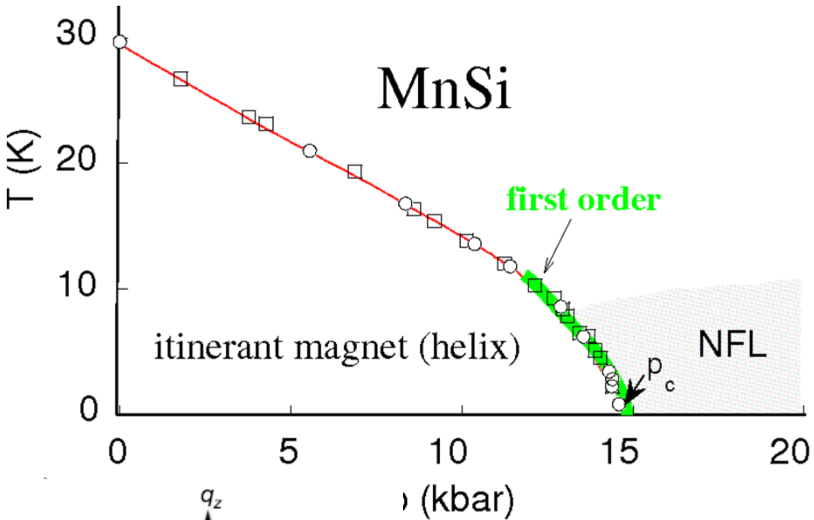
phase with anomalous resistivity over almost 3 decades in temperature:

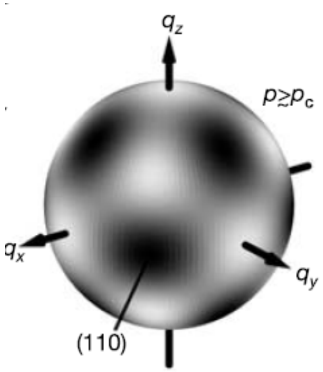
$$\rho(T) \sim T^{3/2}$$

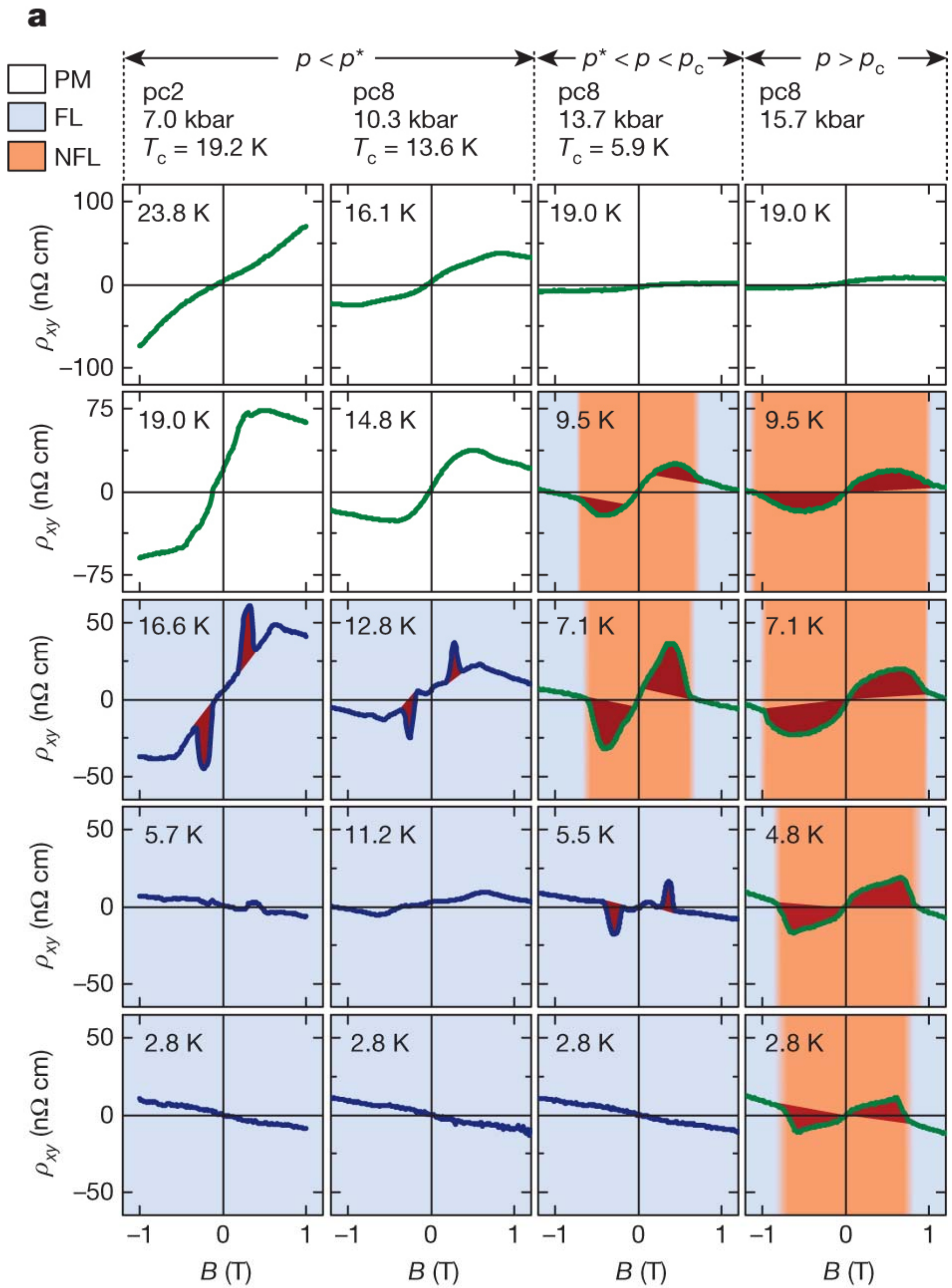
Pfleiderer, A.R. et al. Nature 2000

speculation:
phase of deconfined monopoles/skyrmions/….?

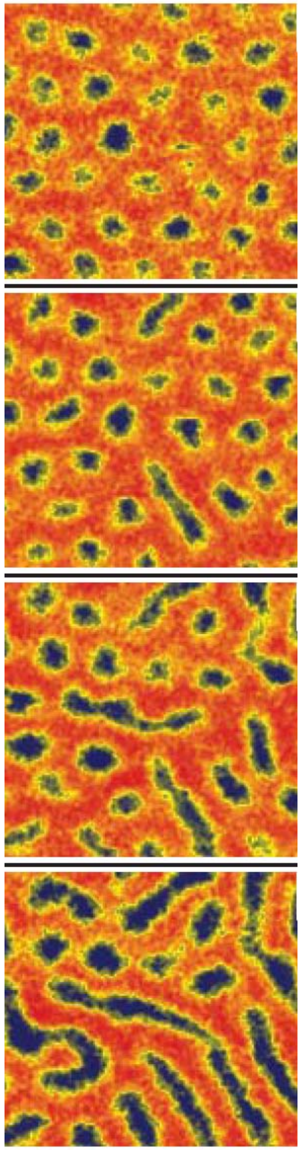
MnSi: back to the roots







Pfleiderer et al. Nature 2013



conclusions

- skyrmions universal in chiral magnets
- easy to move around with ultrasmall currents
- Berry phase coupling: emergent electromagnetism
- phase conversion: emergent magnetic monopoles
- potential for future applications