

Moire Physics in Semiconductors

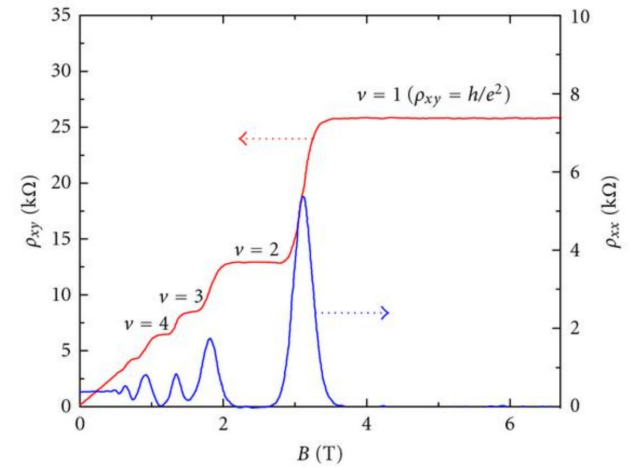
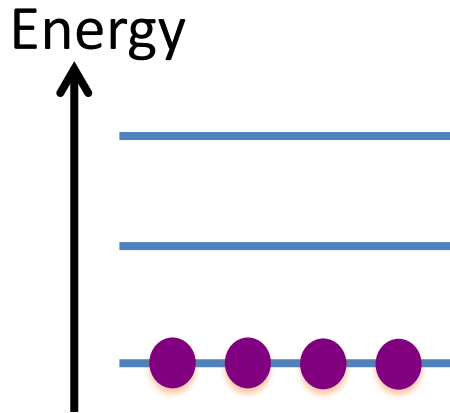
Topology with and without Flat Bands

Liang Fu

2023 Theory Winter School at MagLab

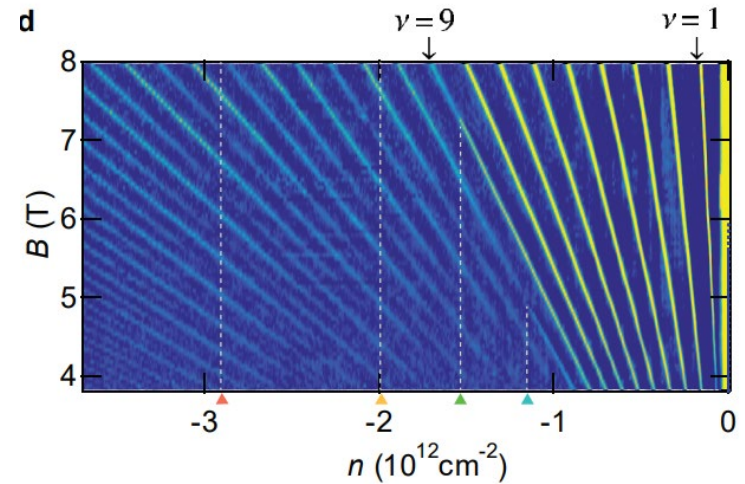


Landau Levels



$$\sigma_{xy} = N \frac{e^2}{h}$$

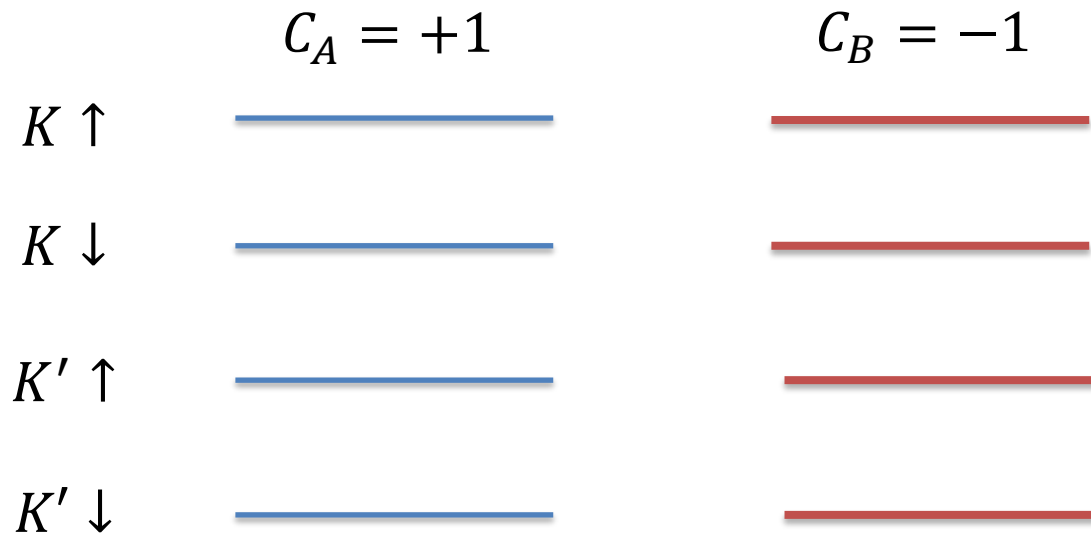
$$\frac{dn}{dB} = N \frac{e}{h}$$



Magic-Angle Twisted Bilayer Graphene

flat band from Dirac fermion with spatially varying interlayer tunneling (AA, BB, AB, BA)

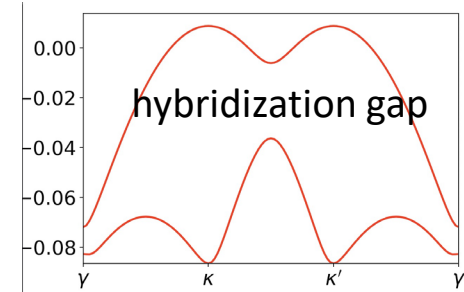
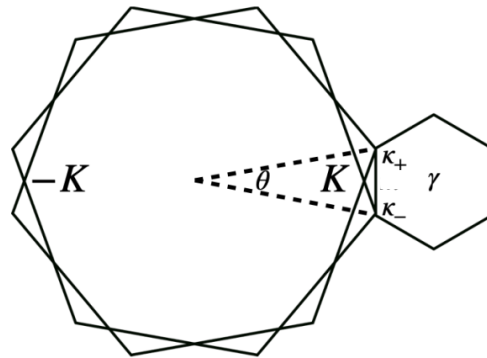
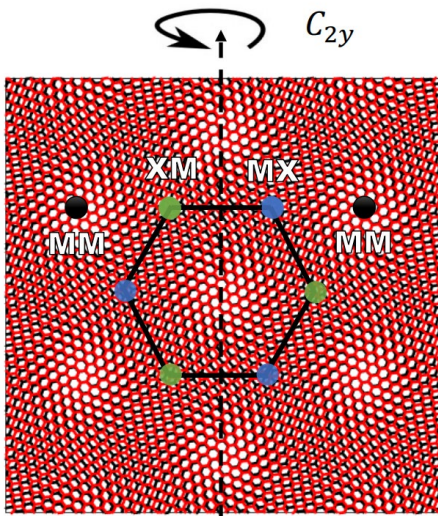
chiral limit: 8 flat Chern bands at $E = 0$



Topological Insulators in Twisted Transition Metal Dichalcogenide Homobilayers

Fengcheng Wu,^{1,2} Timothy Lovorn,³ Emanuel Tutuc,⁴ Ivar Martin,¹ and A. H. MacDonald³

$\theta \sim 0$ (AA stacking)



$$\mathcal{H}_{\uparrow} = \begin{pmatrix} -\frac{\hbar^2(\mathbf{k}-\boldsymbol{\kappa}_+)^2}{2m^*} + \Delta_b(\mathbf{r}) & \Delta_T(\mathbf{r}) \\ \Delta_T^\dagger(\mathbf{r}) & -\frac{\hbar^2(\mathbf{k}-\boldsymbol{\kappa}_-)^2}{2m^*} + \Delta_t(\mathbf{r}) \end{pmatrix}$$

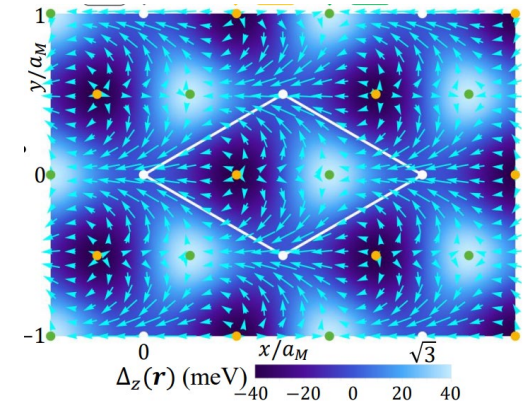
$$\Delta(\mathbf{r}) = (\Delta_x, \Delta_y, \Delta_z) \equiv (\text{Re}\Delta_T^\dagger, \text{Im}\Delta_T^\dagger, \frac{\Delta_b - \Delta_t}{2})$$

- moire potential $V(\mathbf{r})$ and complex-valued interlayer tunneling $w(\mathbf{r})$
- $w(\mathbf{r})$ vanishes at XM & MX due to atomic-scale interference of K-point w.f.

Band Topology from Skrymion Lattice

$$\mathcal{H}_\uparrow = \begin{pmatrix} -\frac{\hbar^2(\mathbf{k}-\boldsymbol{\kappa}_+)^2}{2m^*} + \Delta_b(\mathbf{r}) & \Delta_T(\mathbf{r}) \\ \Delta_T^\dagger(\mathbf{r}) & -\frac{\hbar^2(\mathbf{k}-\boldsymbol{\kappa}_-)^2}{2m^*} + \Delta_t(\mathbf{r}) \end{pmatrix}$$

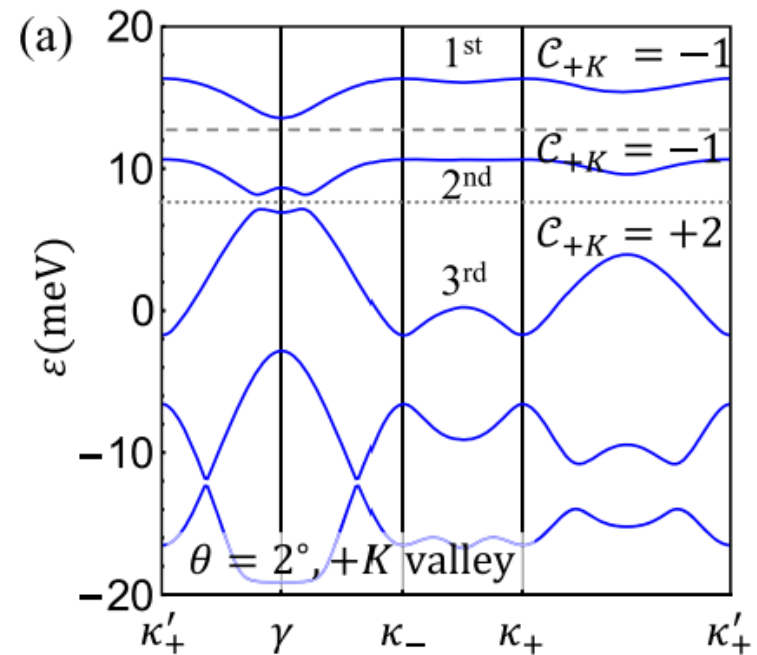
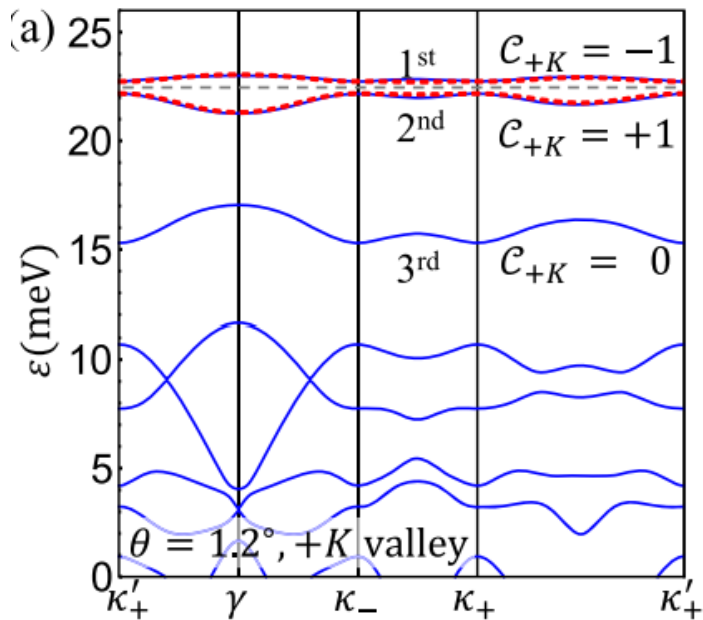
$$\boldsymbol{\Delta}(\mathbf{r}) = (\Delta_x, \Delta_y, \Delta_z) \equiv (\text{Re}\Delta_T^\dagger, \text{Im}\Delta_T^\dagger, \frac{\Delta_b - \Delta_t}{2})$$



Layer-pseudospin Zeeman field forms skrymion lattice!

Wu, Lovorn, Tutuc, Martin and MacDonald, PRL (2019)

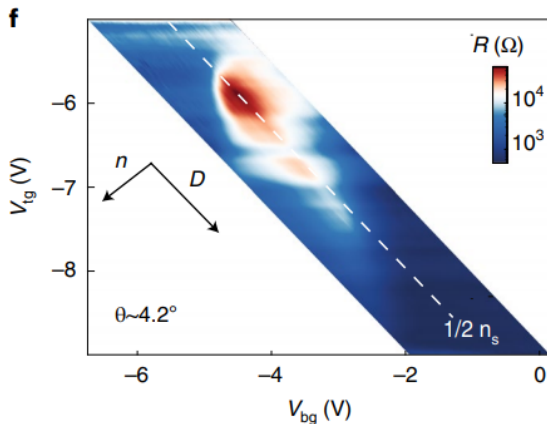
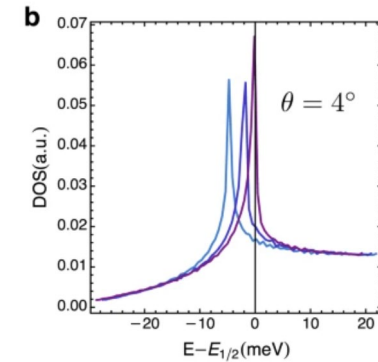
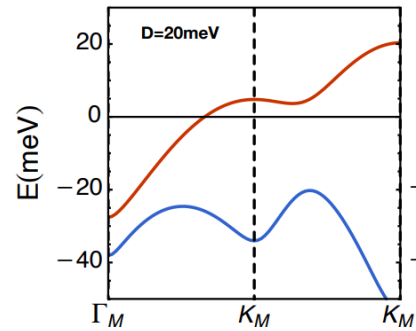
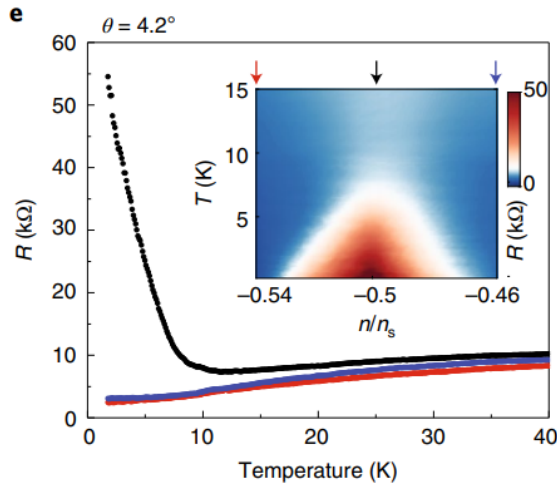
Band Topology from Skrymion Lattice



Wu, Lovorn, Tutuc, Martin and MacDonald, PRL (2019)

Twisted WSe_2 at $\theta \sim 4^\circ$

Metal-insulator transition tuned by E field

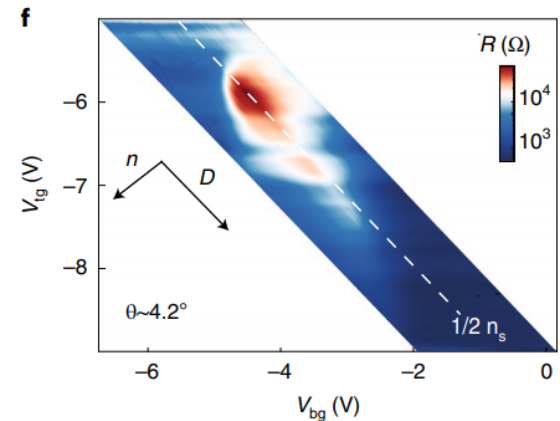
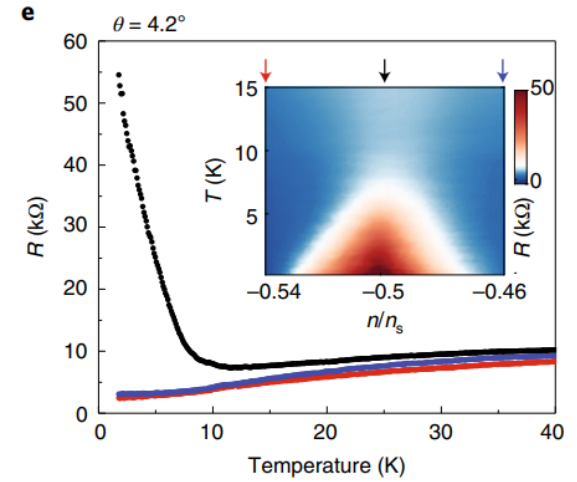
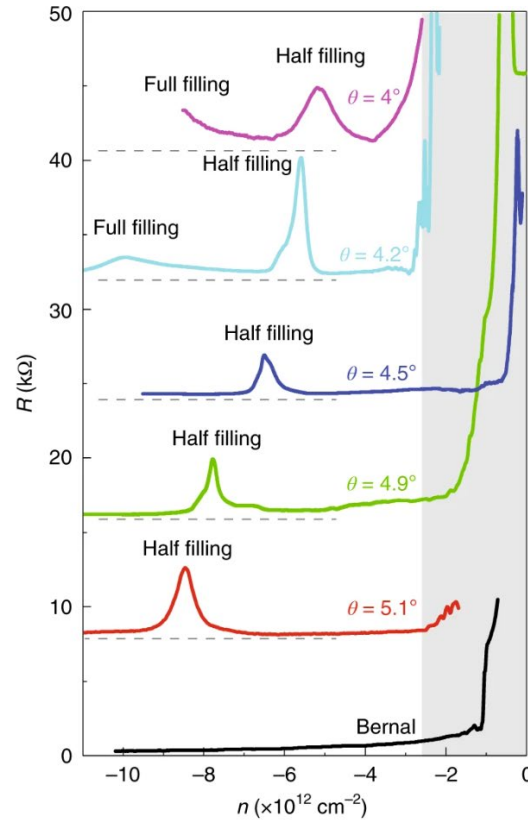
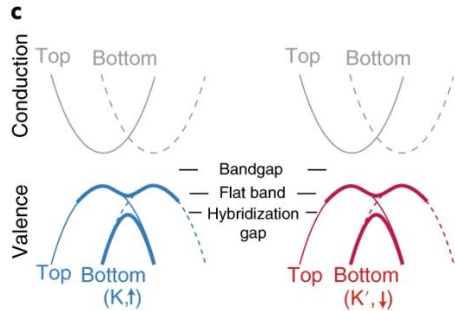
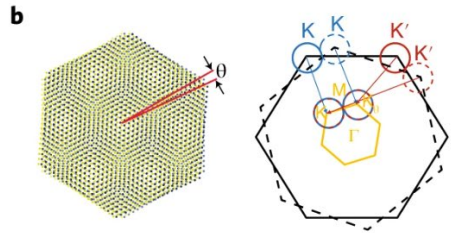
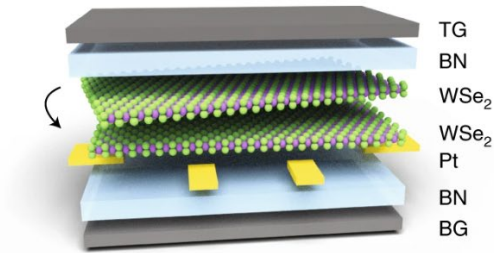


Correlated insulator at $n=1$:

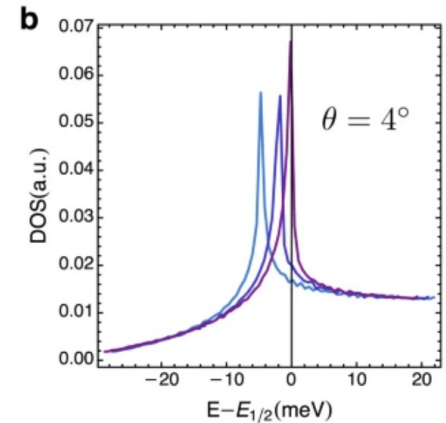
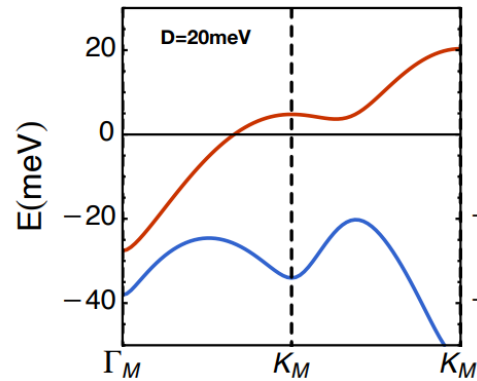
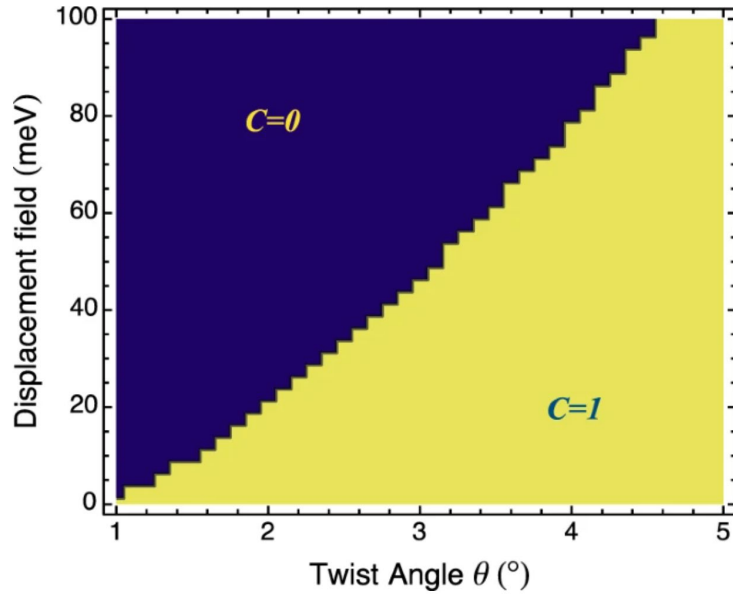
- valley-polarized (Ising) ferromagnet
- intervalley coherent (xy) antiferromagnet

Twisted WSe₂ at $\theta \sim 4^\circ$

Metal-insulator transition tuned by E field



Twisted WSe₂

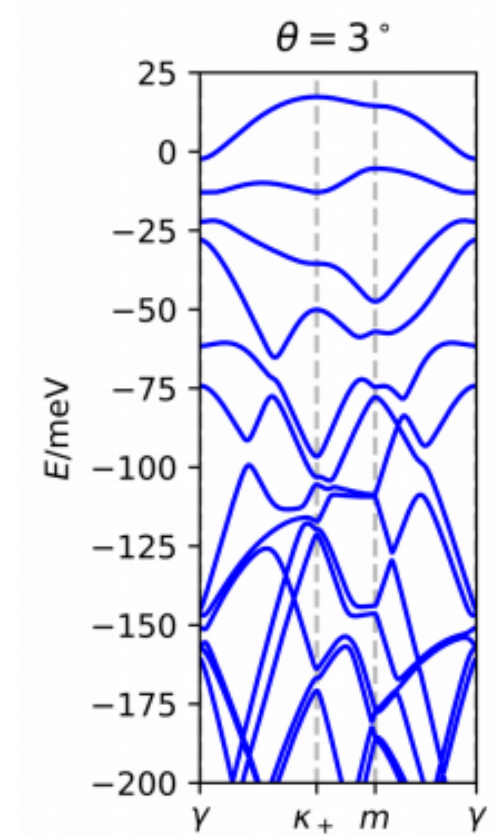
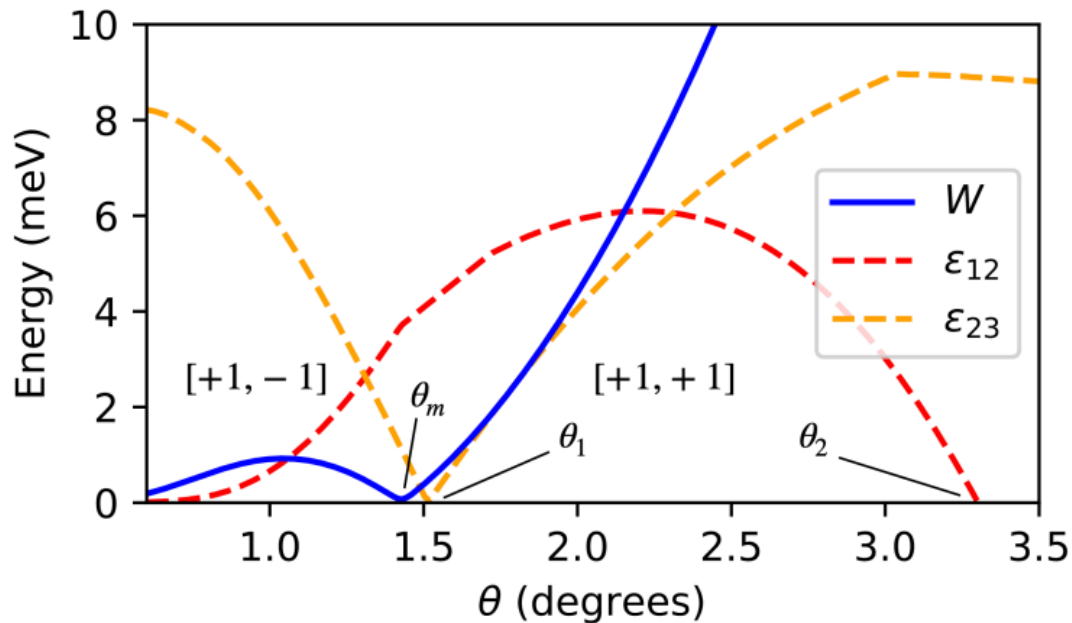


Correlated insulator at $n=1$:

- valley-polarized (Ising) ferromagnet
- intervalley coherent (xy) antiferromagnet

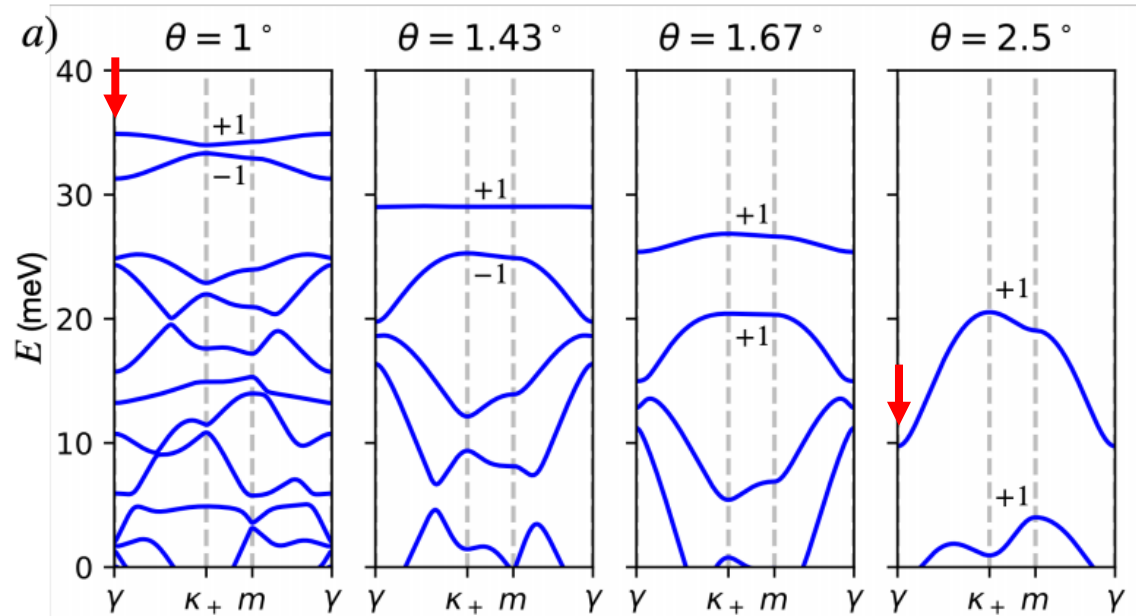
Bi & LF, Nature Communications (2021)
See also Cano & Millis, Das Sarma ...

Isolated Topological Bands



Devakul, Crepel, Zhang & LF, Nature Communications (2021)

$\theta \sim 1.4^\circ$: Magic *Flat Band*



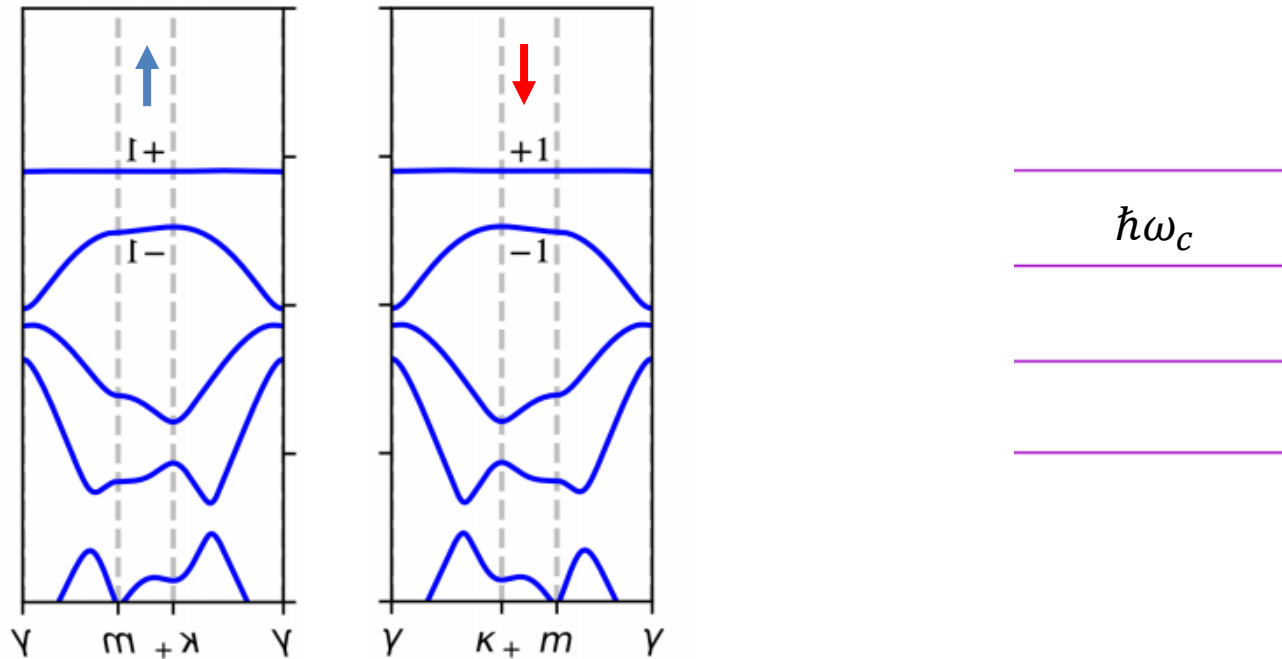
Analytical formula for **magic angle** based on diverging band mass at γ

$$\tilde{\theta}_m^{-2} = \frac{8\pi^2}{9m^*a_0^2} \left(\frac{1}{\mathcal{E}_{n_0} - \mathcal{E}_{n_0+1}} + \frac{1}{\mathcal{E}_{n_0} - \mathcal{E}_{n_0-1}} \right)$$

$$\text{where } \mathcal{E}_n = 2w \cos(\pi n/3) + 2V \cos(2\pi n/3 - \psi),$$

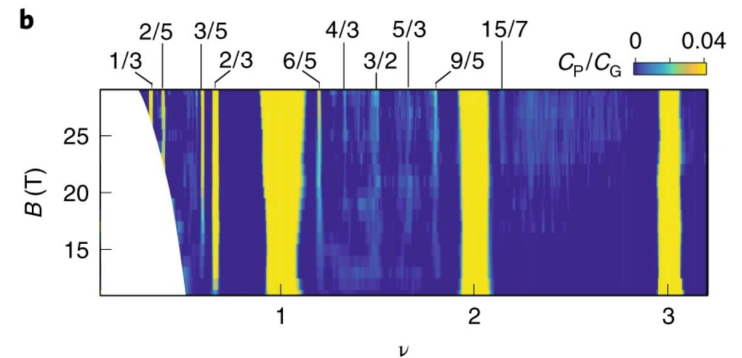
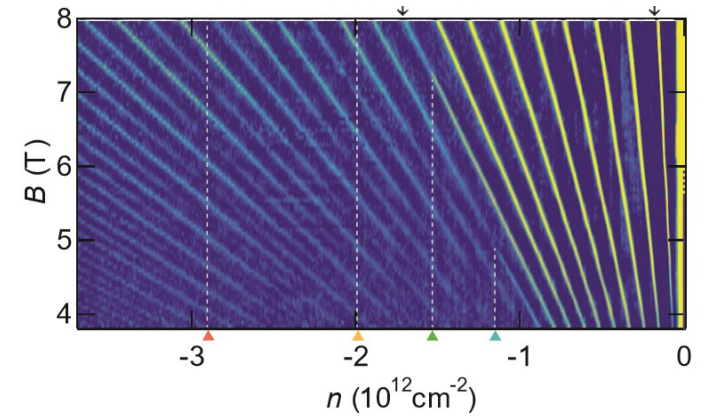
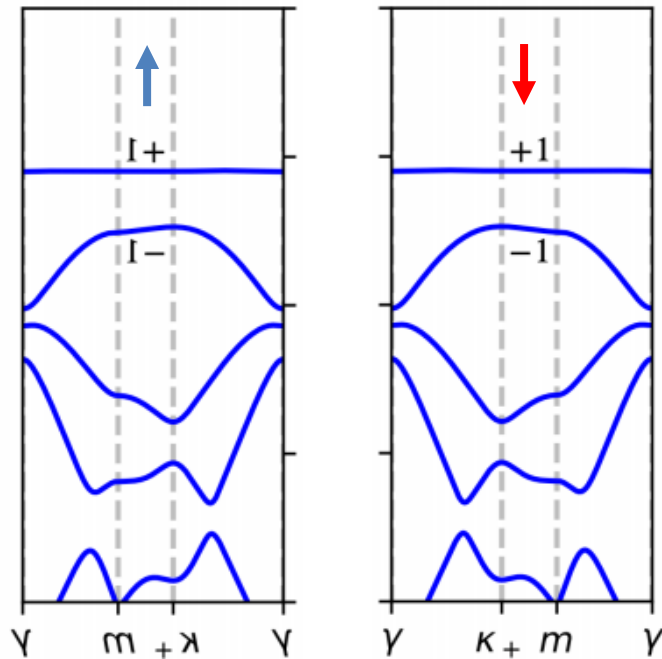
Magic flat band: bandwidth 0.1meV, topological gap 4meV

Comparison with Landau Level



Topological gap 4meV $\Leftrightarrow B = 18\text{T}$ in monolayer WSe₂

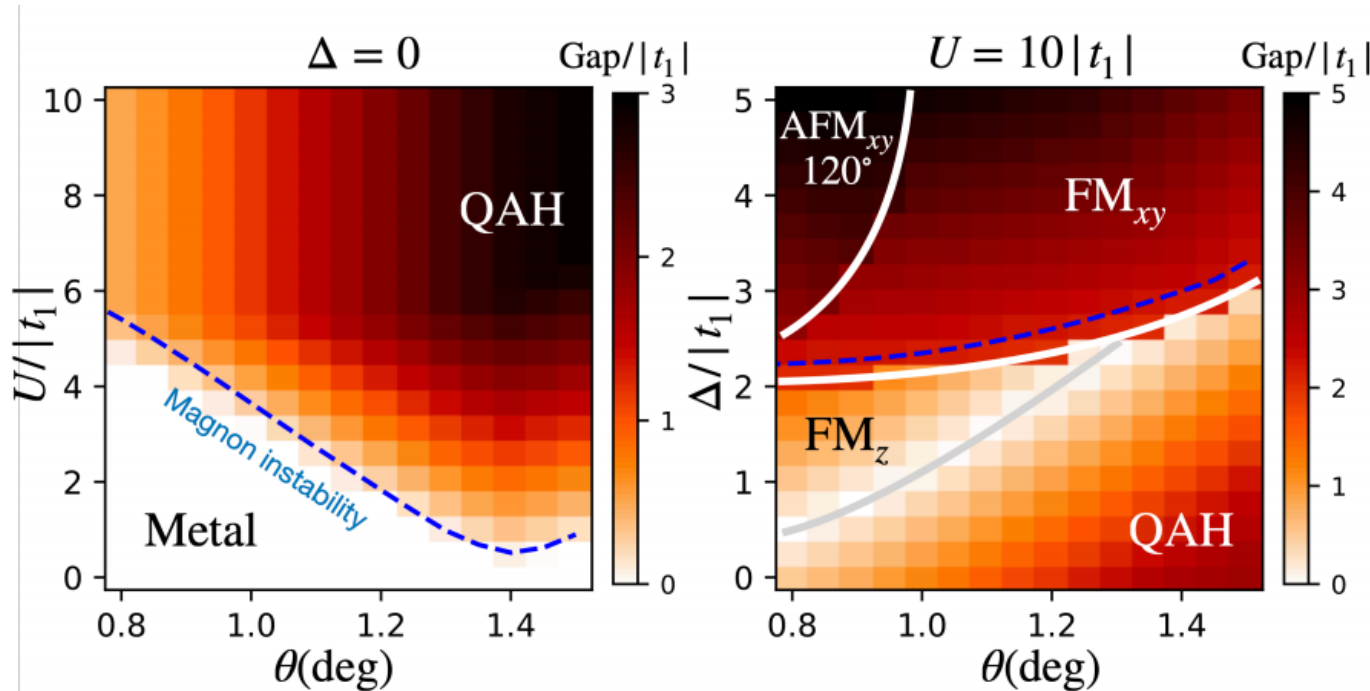
Comparison with Landau Level



Shi et al, Nat. Nano. 2020

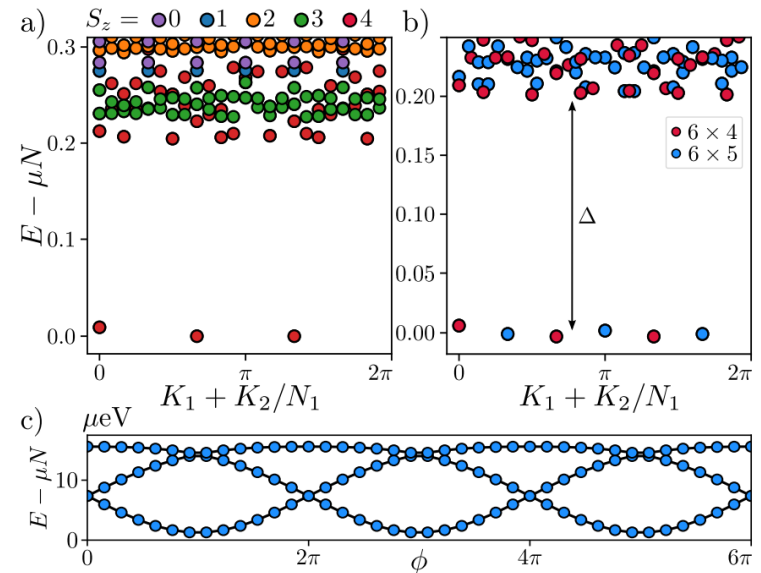
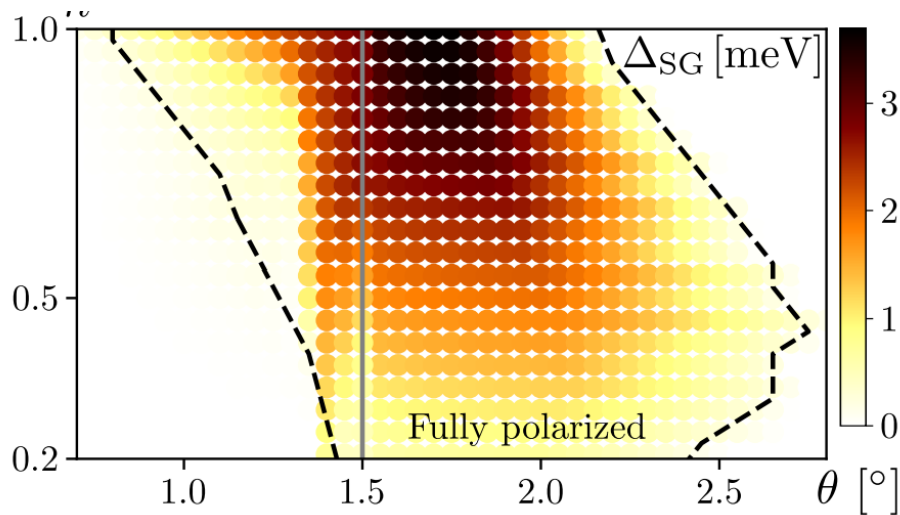
Topological gap 4meV $\Leftrightarrow B = 18\text{T}$ in monolayer WSe_2

Haldane and Mott Insulators at $n=1$ tuned by electric field



Devakul, Crepel, Zhang & LF, Nature Communications (2021)

Fractional Fillings: Anomalous Hall Metal & Fractional Chern Insulator

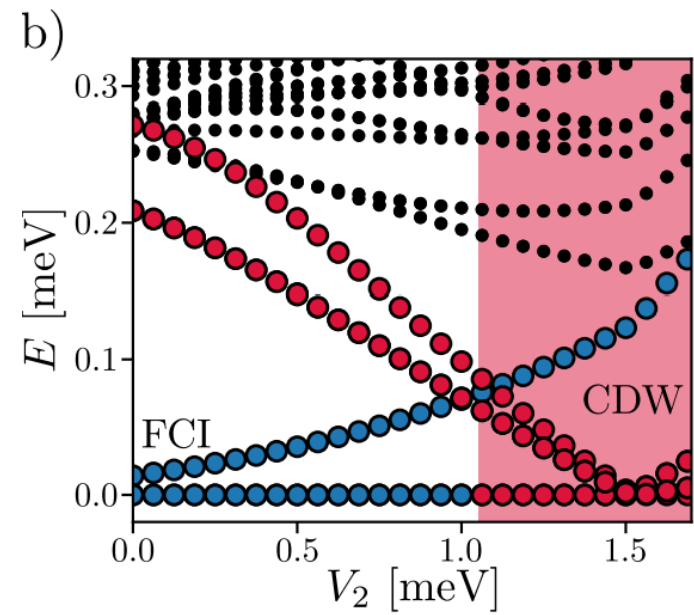
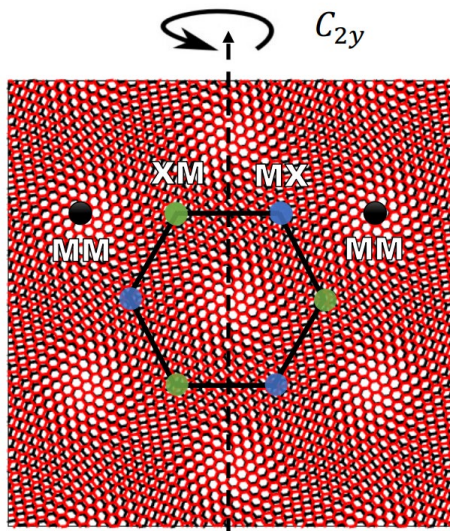


Spontaneous spin-valley polarization
Gapped magnon due to spin-valley locking

Crepel & LF, arXiv:2207.08895

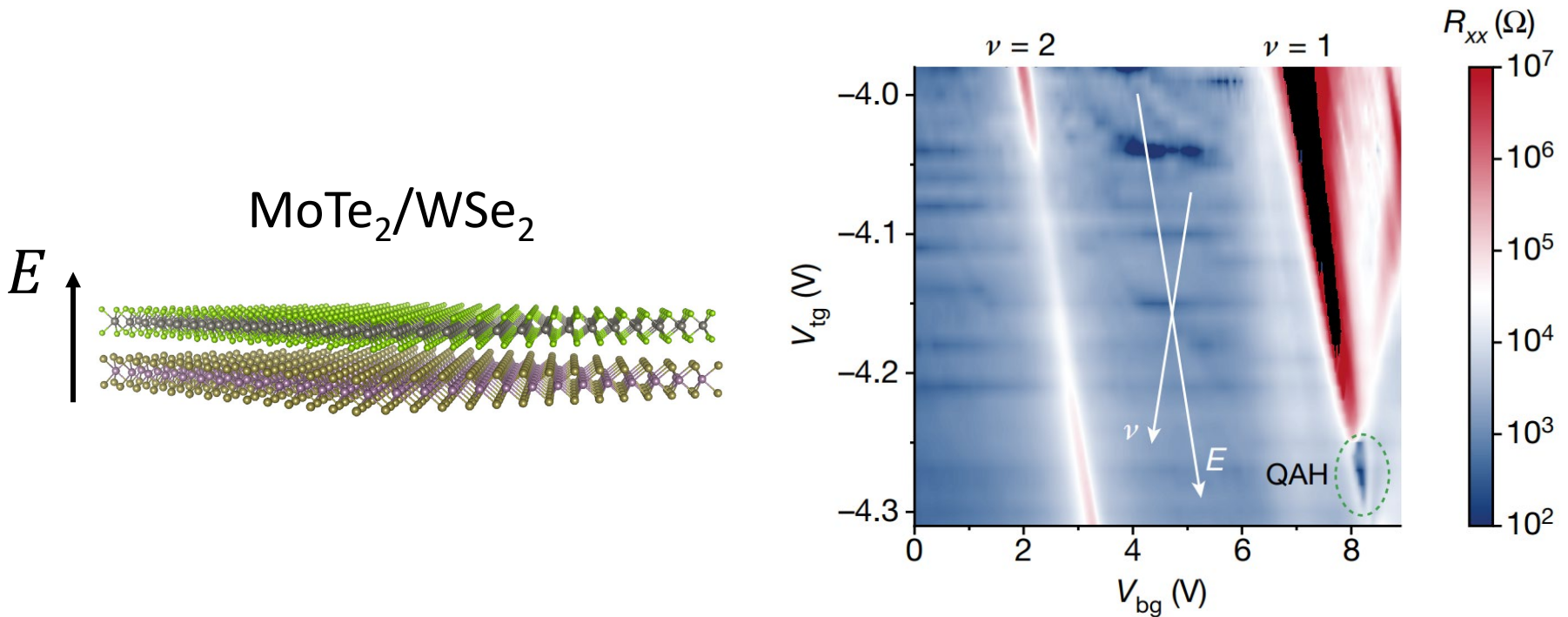
See also Li, Kumar, Sun and Lin, Physical Review Research 3, L032070 (2021).

FCI vs CDW



Quantum anomalous Hall effect from intertwined moiré bands

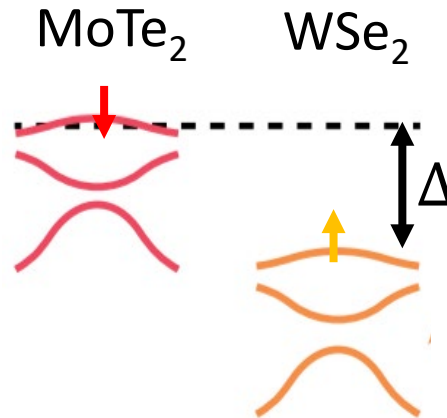
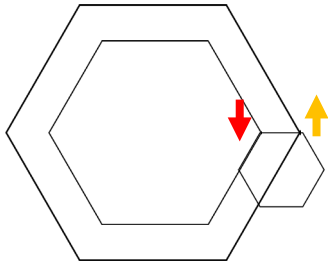
[Tingxin Li](#), [Shengwei Jiang](#), [Bowen Shen](#), [Yang Zhang](#), [Lizhong Li](#), [Zui Tao](#), [Trithep Devakul](#), [Kenji Watanabe](#), [Takashi Taniguchi](#), [Liang Fu](#), [Jie Shan](#)  & [Kin Fai Mak](#) 



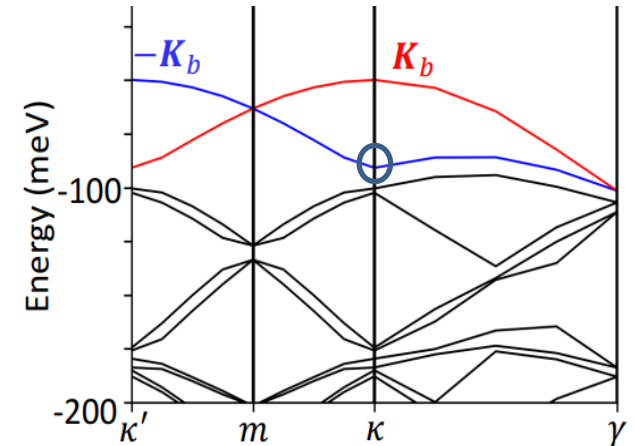
- Electric field tunes interlayer charge transfer

Moire Bands in MoTe₂/WSe₂

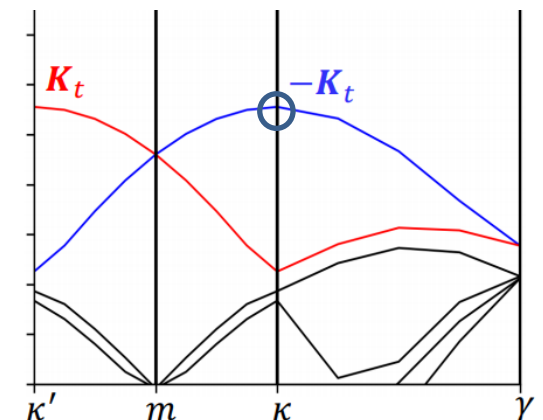
$$a_M = 4.6\text{nm}$$



13 x13 MoTe₂

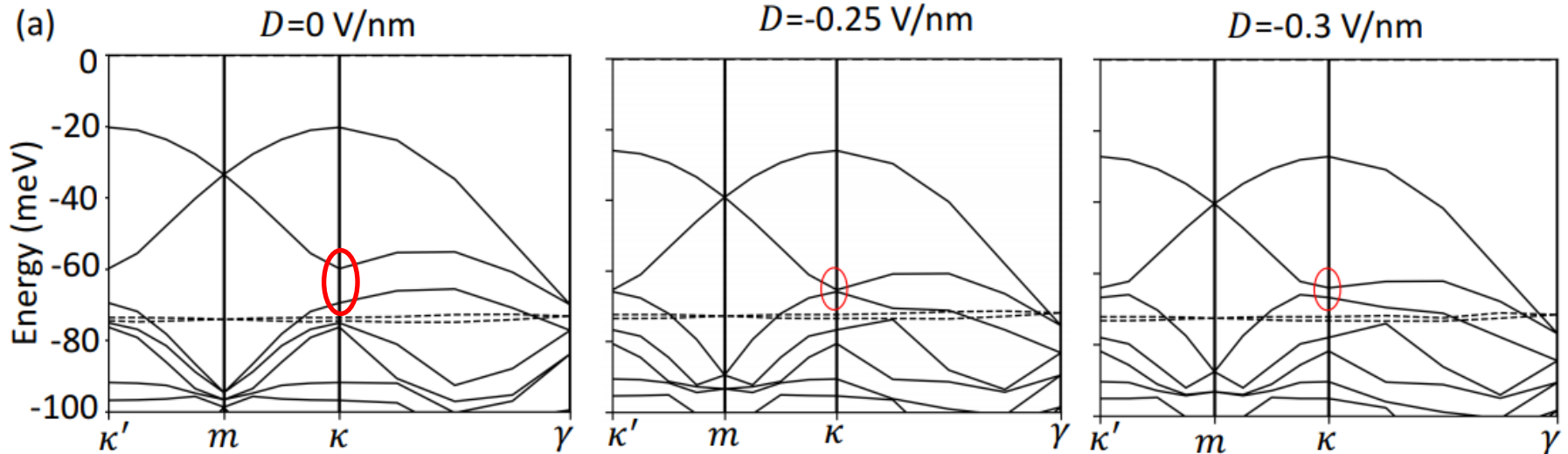


14 x14 WSe₂



- $\Delta = 0.13\text{eV}$ at zero E field
- majority layer: MoTe₂
minority layer: WSe₂
- moire band due to lattice corrugation with bandwidth $\sim 50\text{ meV}$

E Field Tunes Band Inversion

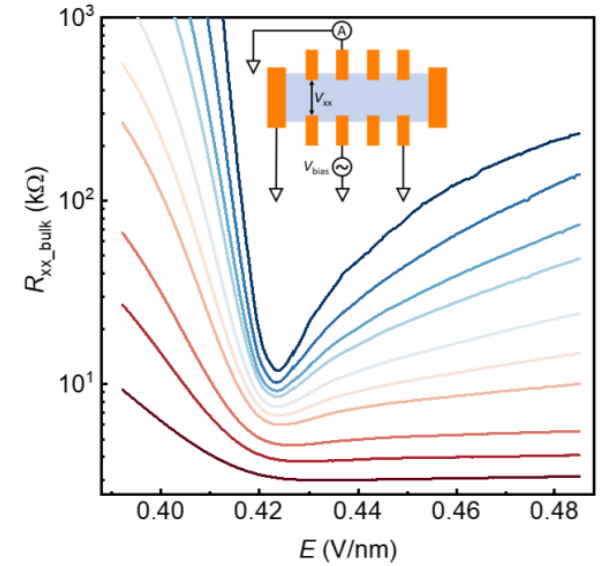
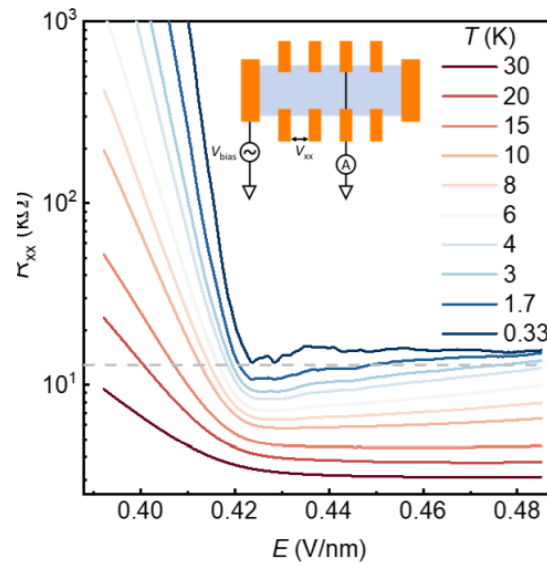
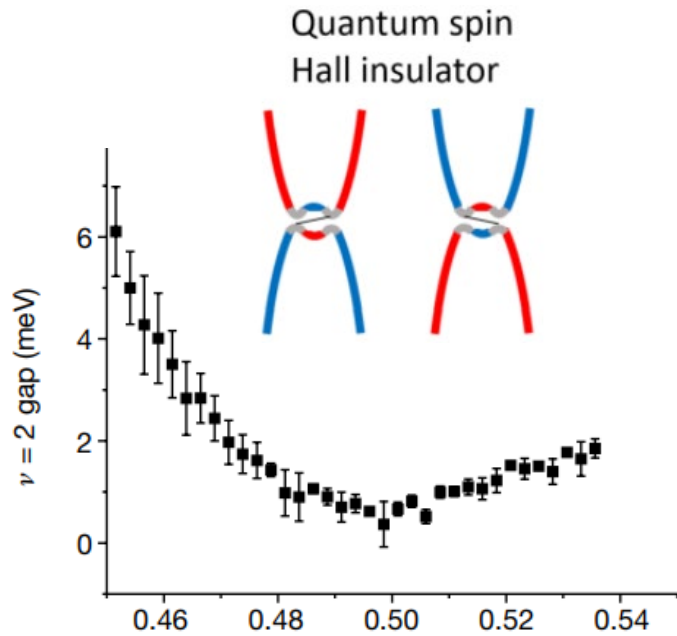


- electric field inverts minibands on two layers
- band inversion + p-wave interlayer tunneling \Rightarrow valley Chern number

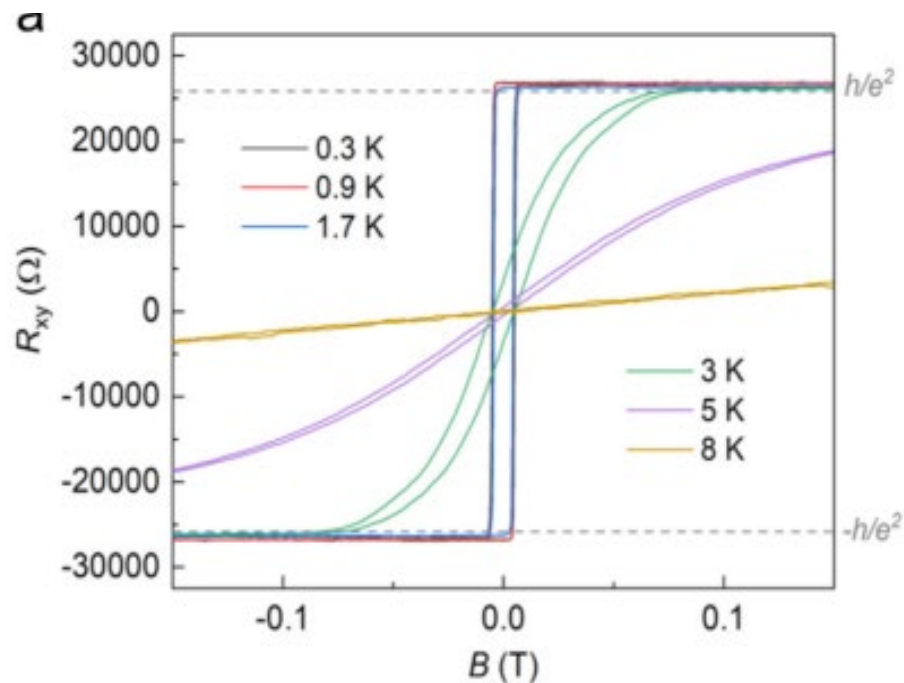
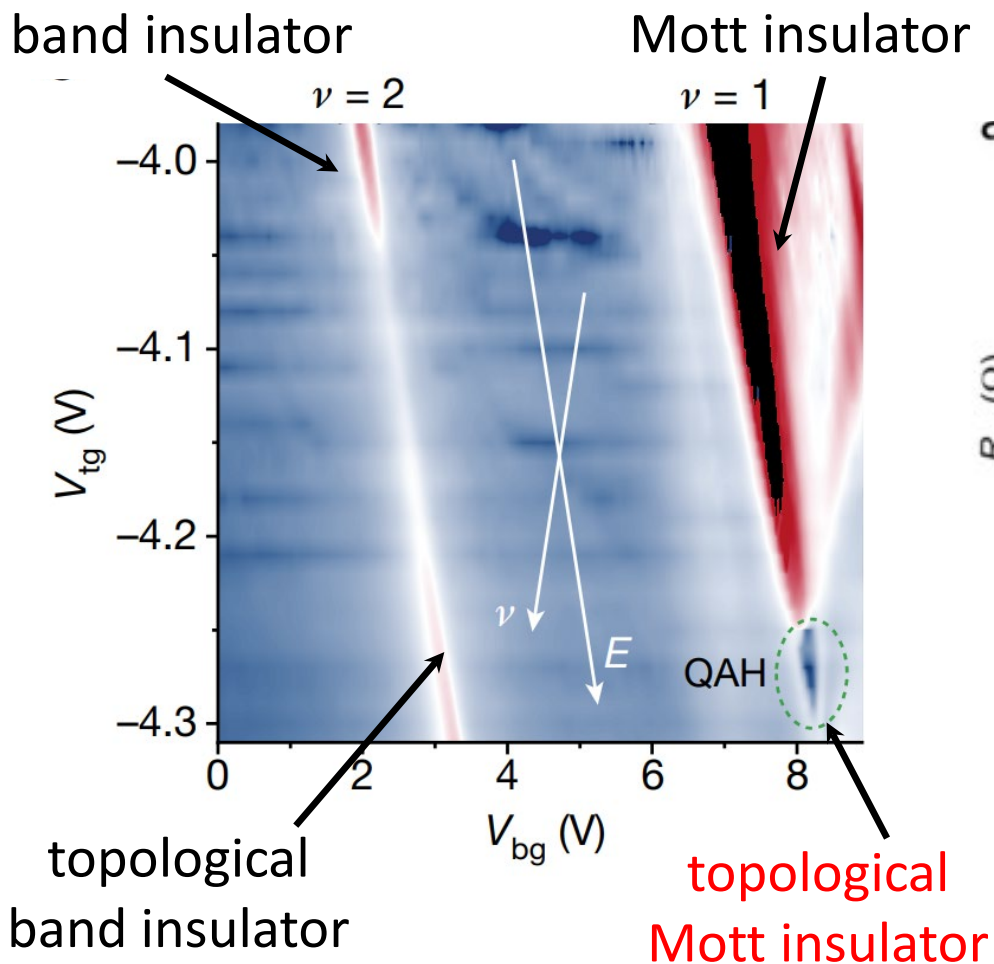
Prediction: E field induced quantum spin Hall insulator at $n=2$

Zhang, Devakul & LF, PNAS (2021)

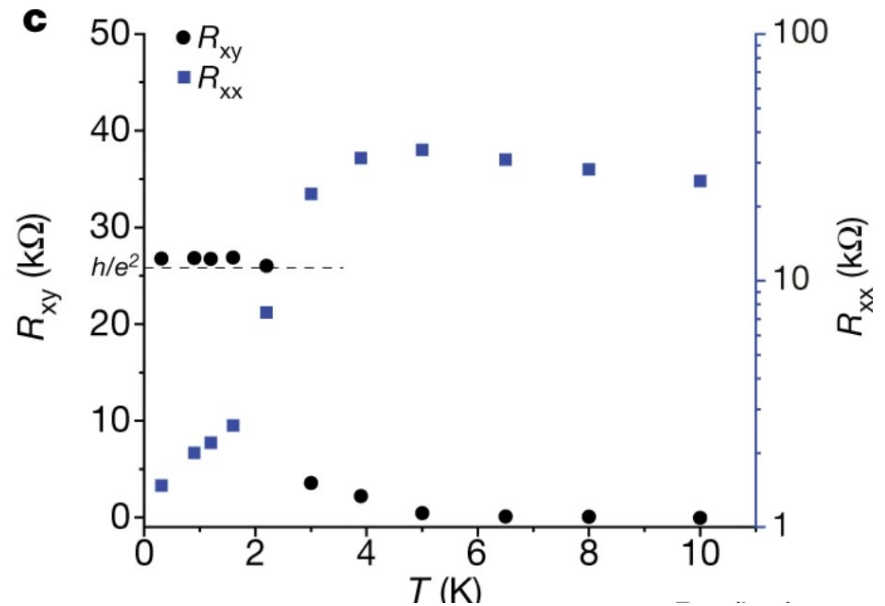
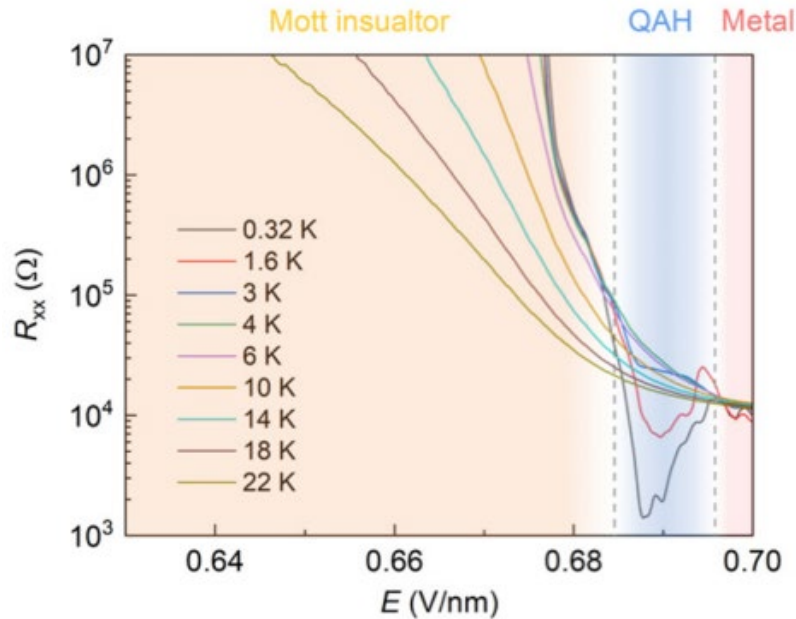
Edge Transport in MoTe₂/WSe₂



Quantum Anomalous Hall Effect at Half Filling



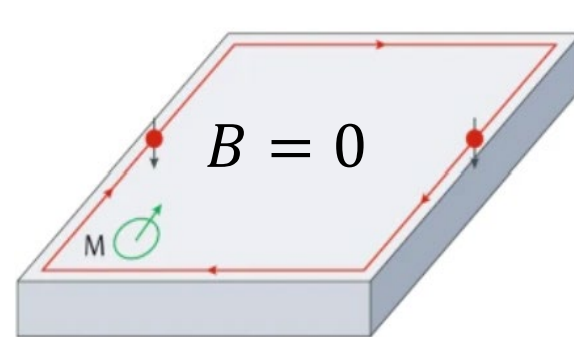
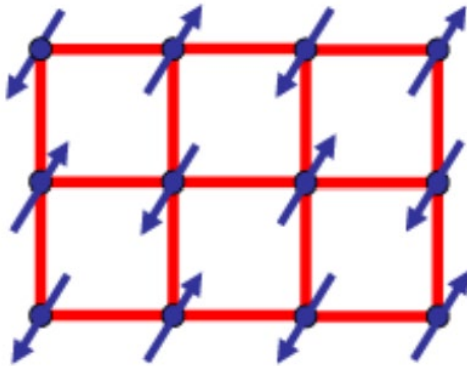
E Field Induced Mott-QAH Transition



- absence of E field hysteresis
- robust and reproducible

Mottness versus Topology

dichotomy between particle and wave



Mott

Topology



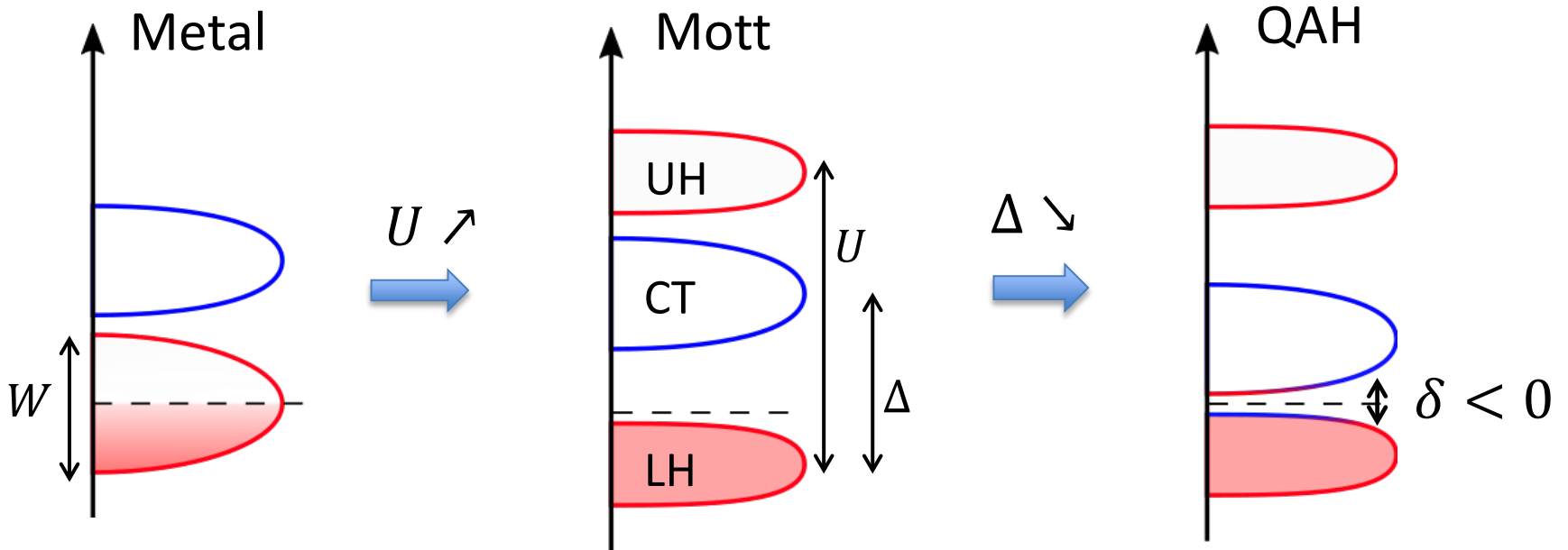
Quantum Anomalous Hall State in Antiferromagnetic Mott Insulators

Devakul & LF, PRX (2022)



Mott to QAH Insulators

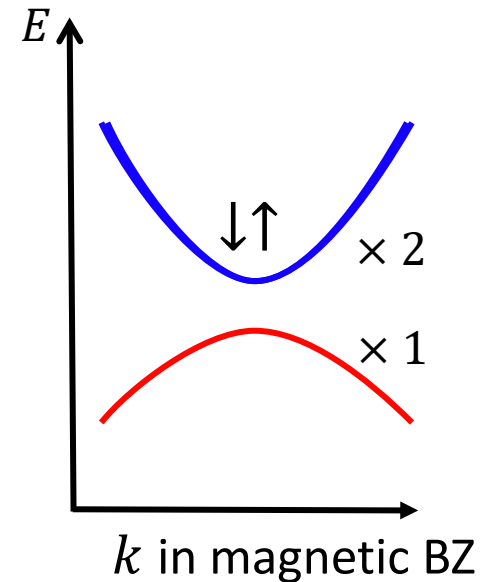
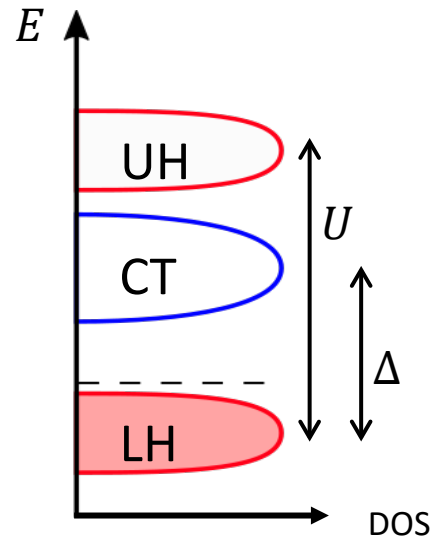
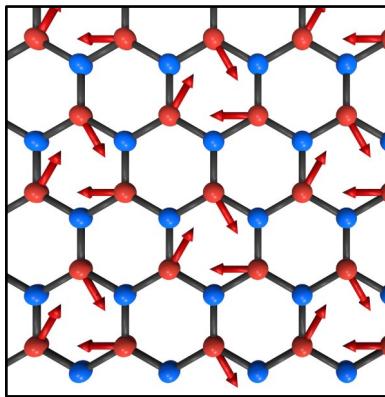
Mott-QAH transition by inverting charge transfer gap



Devakul & LF, PRX (2022)

$$\delta \sim \Delta - W$$

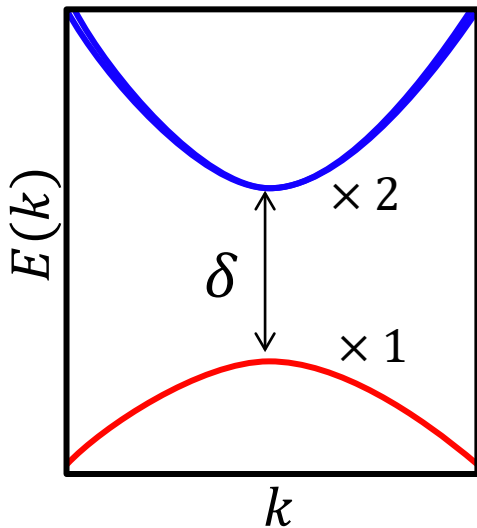
120°-AFM Mott Insulator



- Quasiparticle bands in magnetically ordered insulator are different from noninteracting bands.
- Low-energy states: **spin-polarized holes on majority layer** & **spin-degenerate electrons on minority layer**

Interacting Field Theory

$$\mathcal{H}_{\text{eff}} = \int \psi^\dagger H_{\text{eff}} \psi d\mathbf{k} + g \int n_{B\uparrow}(r)n_{B\downarrow}(r)dr \quad \psi = (\psi_A, \psi_{B\uparrow}, \psi_{B\downarrow})$$

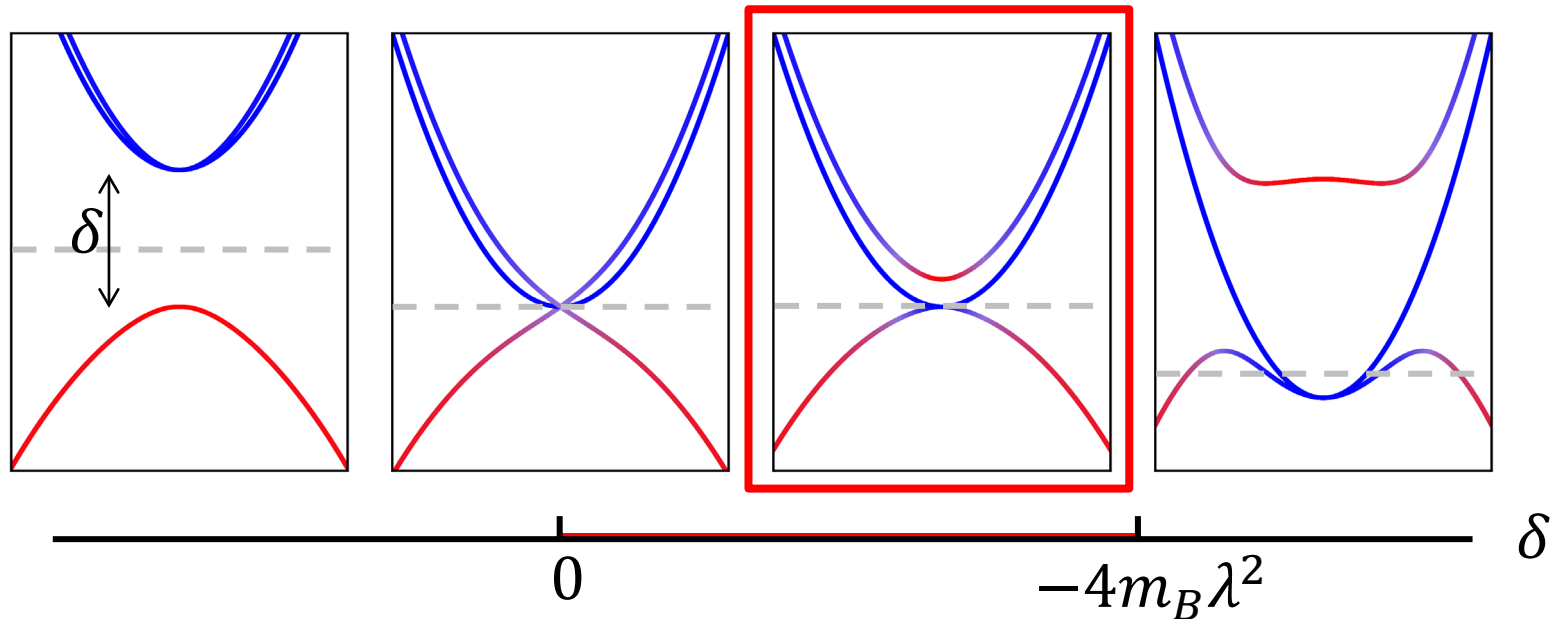


$$H_{\text{eff}} = \begin{pmatrix} -\frac{k^2}{2m_A} & \lambda(k_x + ik_y) & \lambda(k_x - ik_y) \\ " & \frac{k^2}{2m_B} + \delta & 0 \\ " & 0 & \frac{k^2}{2m_B} + \delta \end{pmatrix}$$

- spin degeneracy at $k=0$ on minority layer protected by C_3 & Ts_z
- p-wave hybridization dictated by band symmetry
- g : electron repulsion on minority layer

Interacting Field Theory

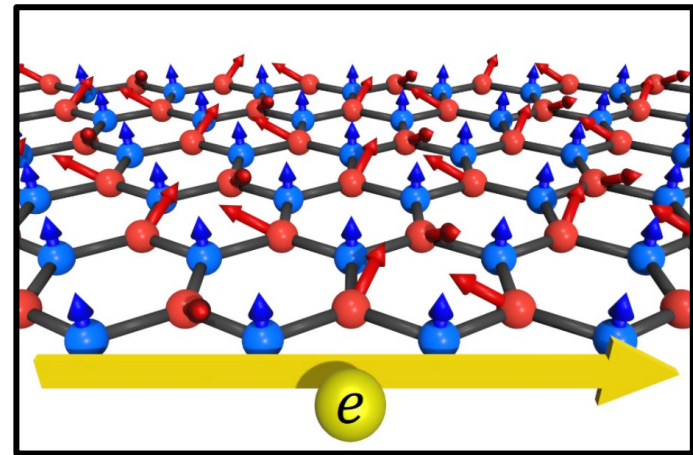
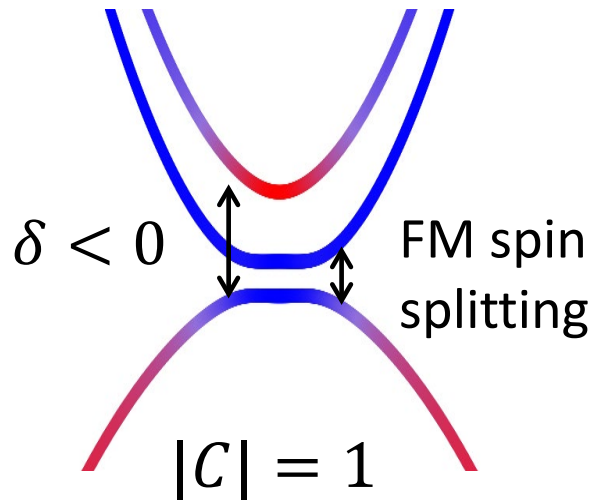
Quasiparticle band at $g = 0$



After band inversion $\delta < 0$, quadratic band touching appears at Fermi level, which is unstable to repulsion g [Sun, Yao, Fradkin, Kivelson, PRL 2009](#)

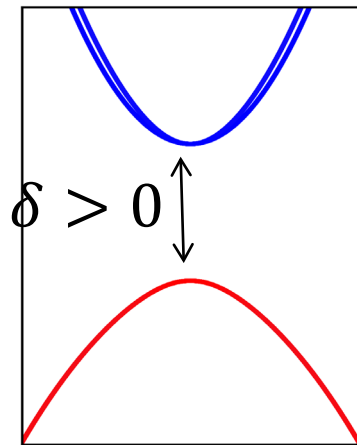
$g > 0$ changes from irrelevant to marginally relevant at band inversion, enabling an unconventional continuous Mott-QAH transition.

QAH with non-coplanar magnetism

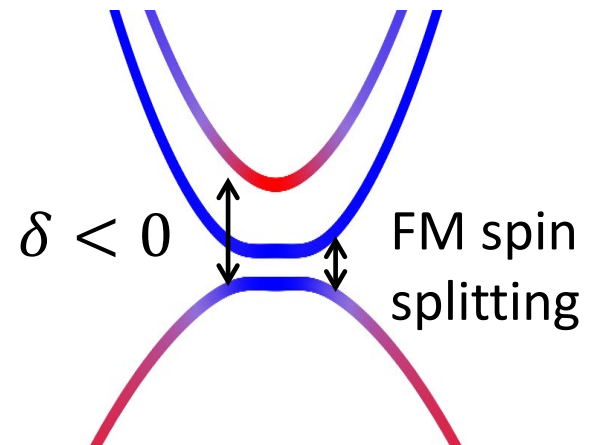
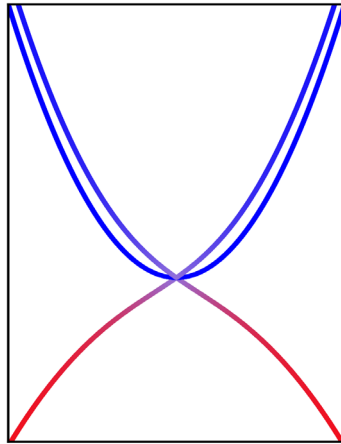


- After quasiparticle band inversion, Ising FM in minority layer opens Chern gap at quadratic band touching
- chiral spin order: xy-AFM in MoTe_2 & Ising FM in WSe_2
- canting of xy-AFM allowed by symmetry

Continuous Mott-Chern Transition



$$C = 0$$



$$|C| = 1$$

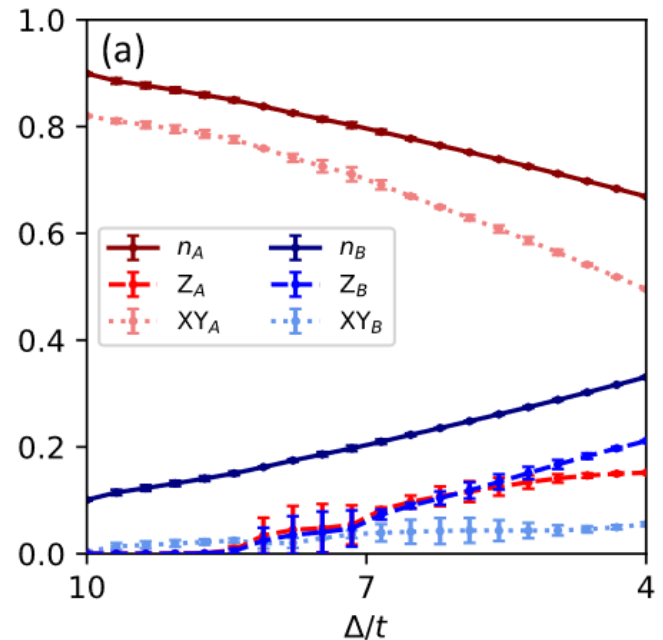
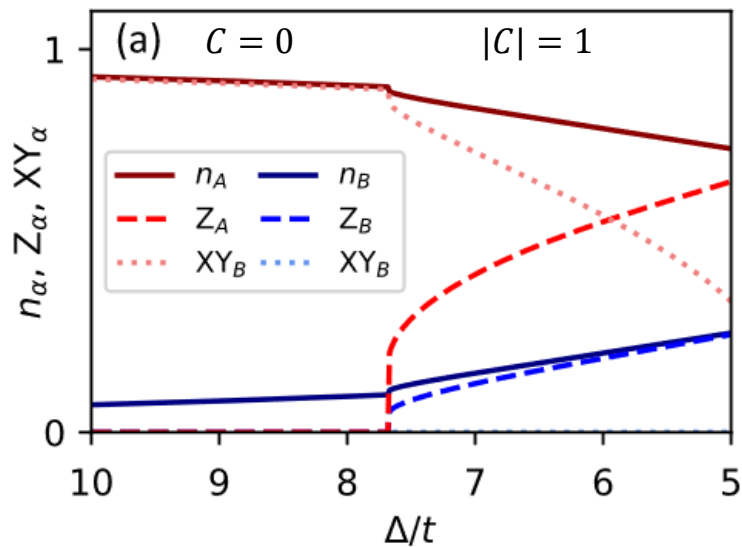
Inverting charge transfer gap induces simultaneous change of magnetism & topology.

Numerical Results



Self-consistent Hartree-Fock

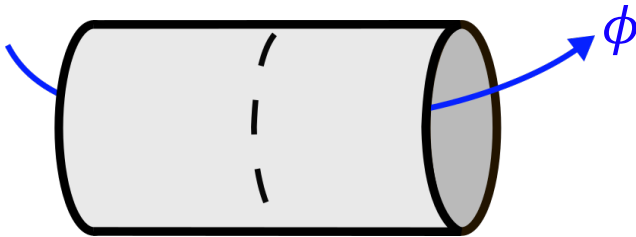
DMRG



Honeycomb Hubbard model

$$H = H_A + H_B + H_{AB} + H_\Delta + H_U$$

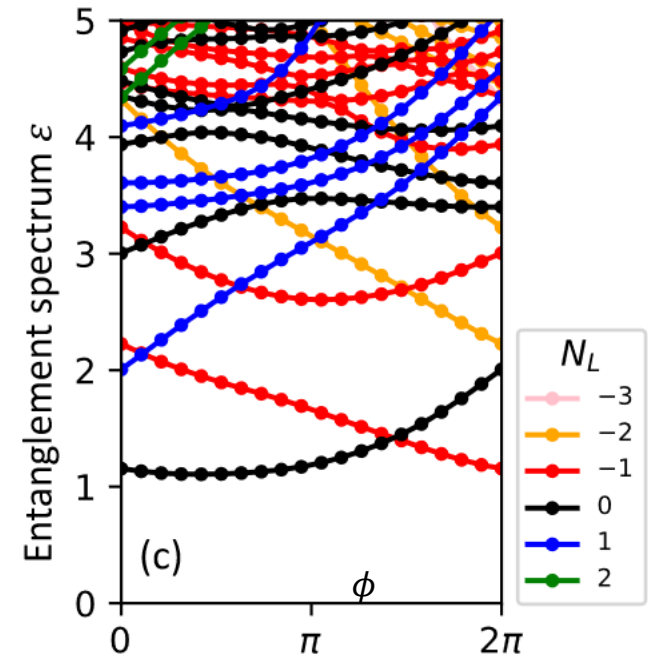
Topology in DMRG



Laughlin 1981.

Charge pumping argument

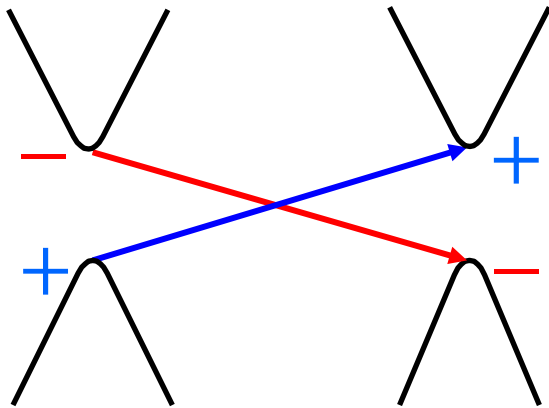
$$\sigma_{xy} = \frac{I_x}{V_y} = \delta_{N_L} \frac{e^2}{h} \Rightarrow |C| = 1$$



$$\begin{aligned} \phi &\rightarrow \phi + 2\pi \\ N_L &\rightarrow N_L - 1 \end{aligned}$$

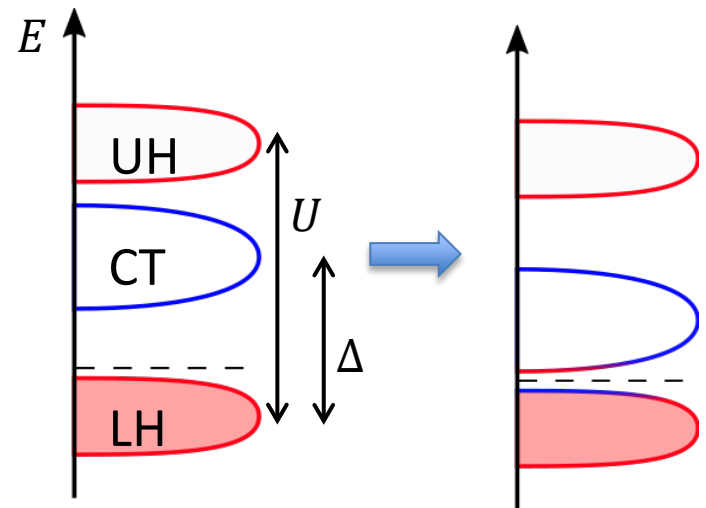
Topological Band & Mott Insulators

Inverting single-particle gap
(even-integer filling)



2007

Inverting many-body gap
(odd-integer filling)



2022

AFM Mott insulators with **negative** charge transfer gap:
potential route to high-temperature QAH

Bridging Mott and Chern




Comparison with Other QAH Systems


Magnetically doped TI film [Chang et al \(2013\)](#)

- FM of dopant opens Chern gap at surface Dirac point
- even integer filling

Magic-angle graphene [Sharpe et al, Serlin et al \(2019\)](#)

- fully valley-polarized flat Chern band

$$C_K = +1$$


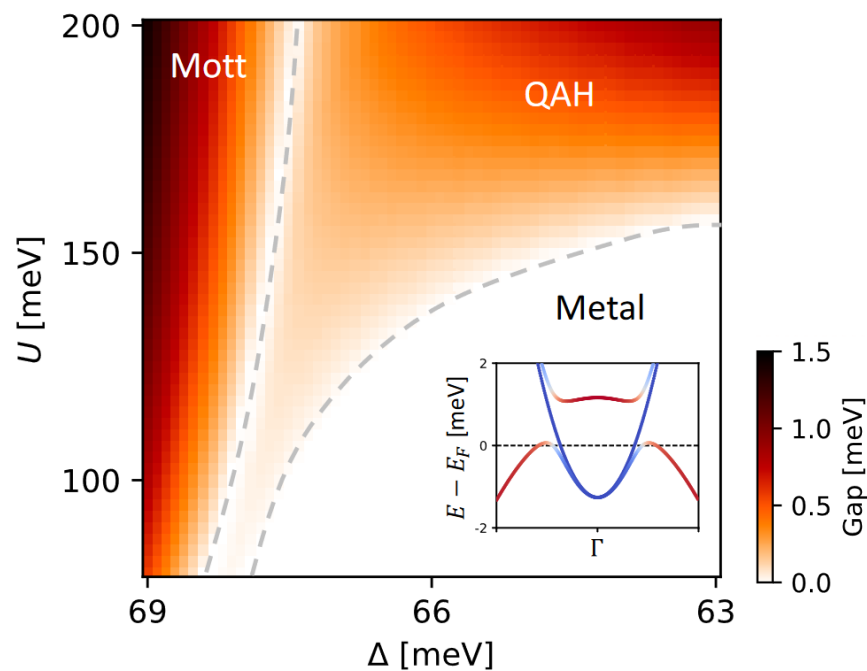
$$C_{K'} = -1$$


[Zhang & Senthil, MacDonald, Xie et al, Pan et al ...](#)

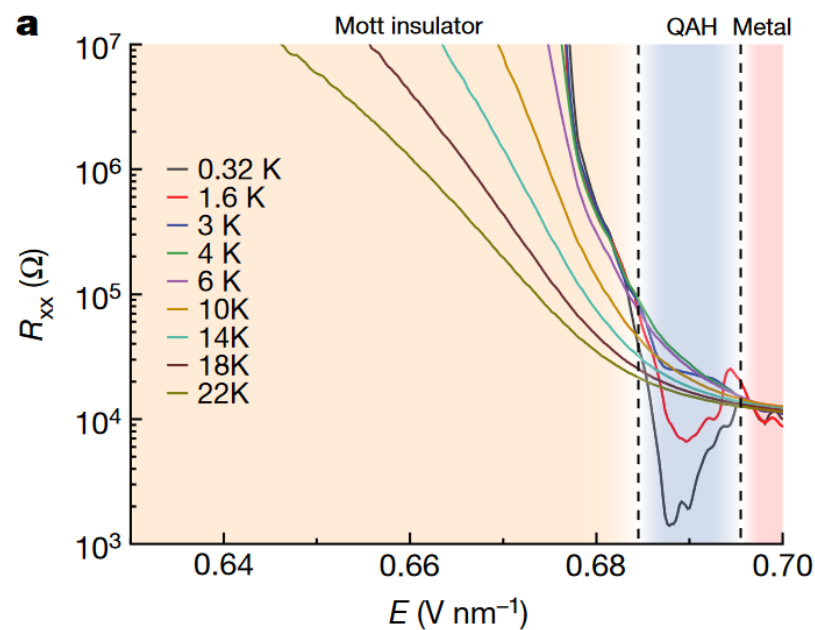
QAH in $\text{MoTe}_2/\text{WSe}_2$ differs fundamentally from flat band FM.

Comparison with Experiment

Hartree-Fock Phase Diagram



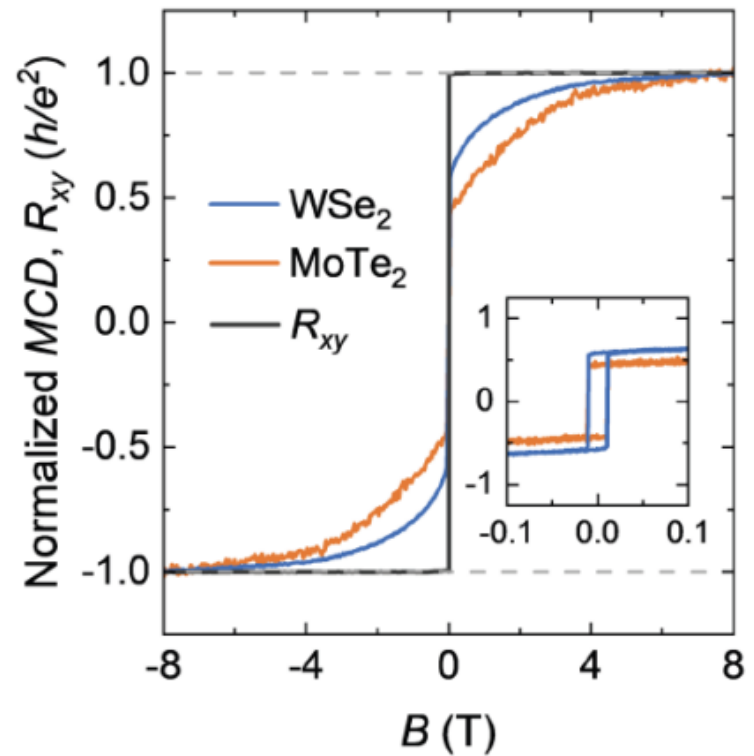
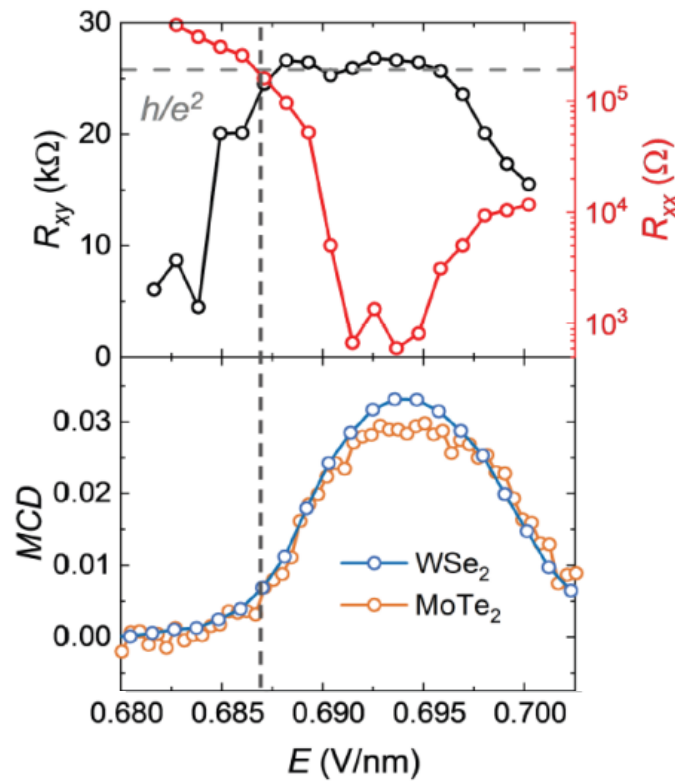
MoTe₂/WSe₂



Prediction for Magnetism

- Mott: **zero** spin S_z polarization
- QAH: **finite but incomplete** spin S_z polarization increasing with B field and E field
- Intervalley XY magnetic order: **gapless** magnon

Evidence for Canted Spin Texture in QAH



Outlook

- charge gap across Mott-Chern transition
- spin superfluidity
- critical exponents
- inverting quantum spin liquid

Theory

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