Majorana Mode in Solid State

Liang Fu (MIT)

Theory Winter School, Tallahassee, 01/06/13

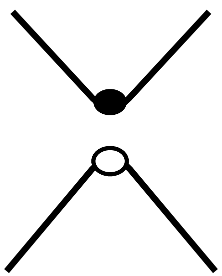
Particles and Symmetries

	Boson	Fermion
Elementary particles:	photon	electron
Emergent particles:	magnon	Landau quasi-particle
Symmetry quantum numbers:	charge 0 spin 1	charge e spin 1/2

Majorana Fermion

Dirac equation: $(i\gamma^\mu \partial_\mu - m)\psi = 0$

Dirac: ψ is complex quantum field

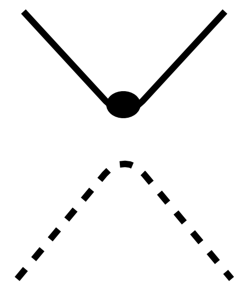


E>0 solution: electron

E<0 solution: positron (hole)

Majorana (1937): let ψ be real

$$\psi(x) = \psi^\dagger(x)$$



Majorana fermion is a neutral <u>fermionic</u> particle that is its own anti-particle.

Whether Majorana fermion exists as elementary particle is currently unknown.

Majorana Mode in Solid State

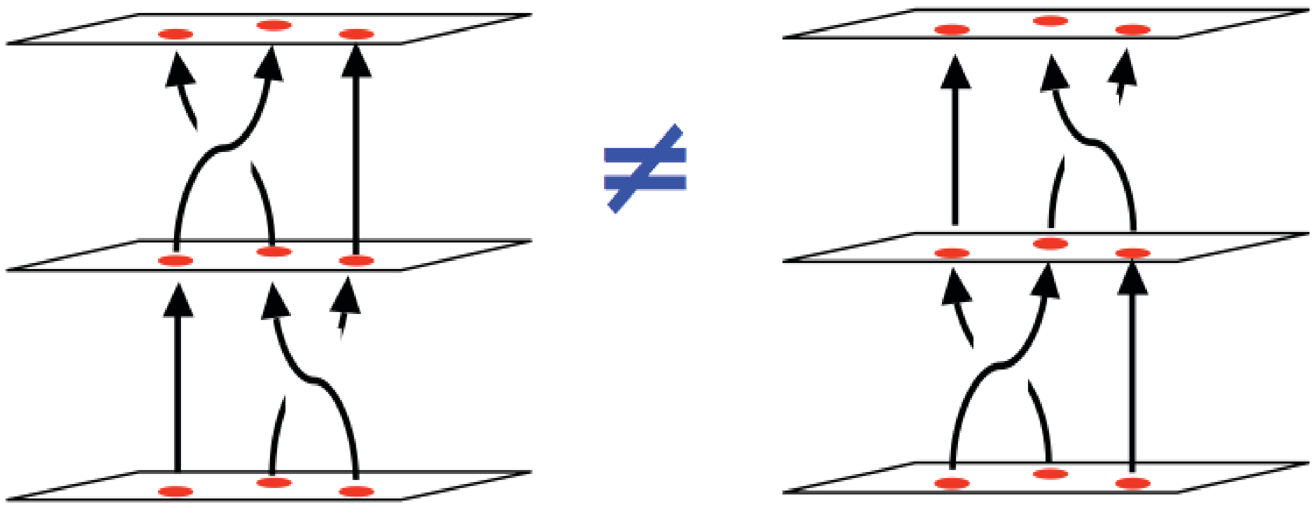
An emergent zero-energy degree of freedom that is localized in space:

- mathematically described by a real operator $\gamma = \gamma^\dagger$
- does not possess any distinctive symmetry quantum number

(analogous to Majorana fermion)

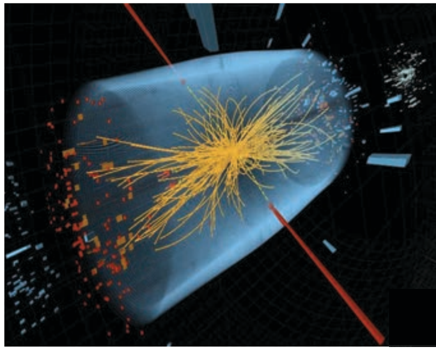
Majorana modes exist in certain topological phases of matter and exhibit universal properties that reflect topological order of the parent phase.

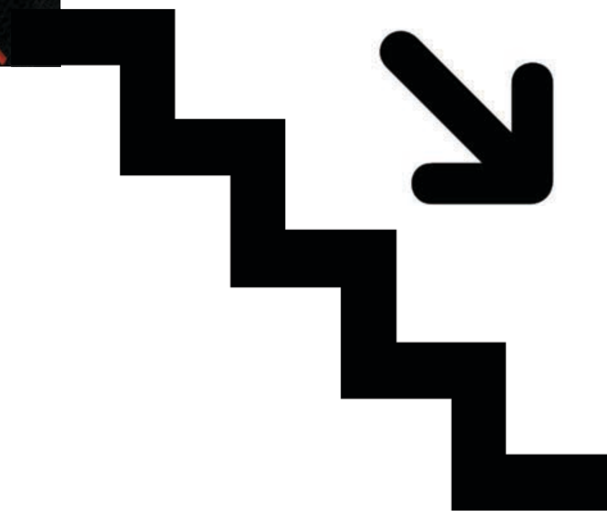
Non-Abelian Statistics of Majorana Mode

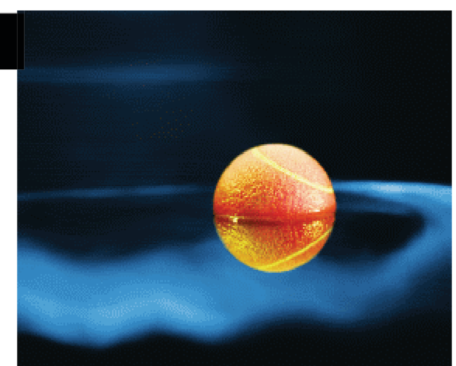


Exchanging Majorana modes leads to a change of ground state in a way that depends (only) on the order of exchange operations.

Topology matters

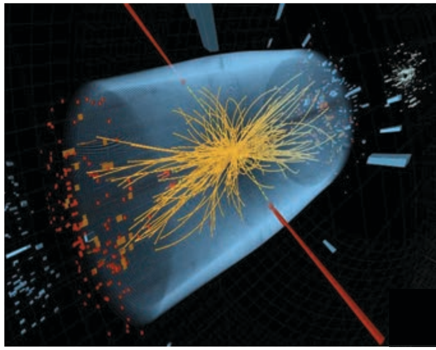


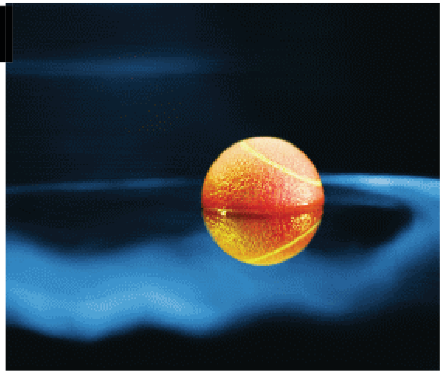




Descent of Majorana

Rise of Topology





Descent of Majorana

Outline

Lecture 1: Physics of Majorana mode in superconductors

Lecture 2: Realizations in spin-orbit-coupled systems (Alicea)

Lecture 3: Striking measurable properties of Majorana mode

Lecture 4: Towards finding Majorana and future directions (Alicea)

Majorana Mode in Superconductor

1D **single-band**, **spinless, p-wave** BCS superconductor (Kitaev 00)

- Majorana mode is localized at the end of “Kitaev wire”
- Its existence is dictated by topological property of bulk

Majorana Mode in Superconductor

1D **single-band**, **spinless, p-wave** BCS superconductor

Hamiltonian for infinite wire:

$$H = \sum_k \underbrace{c_k^\dagger (E_k - \mu) c_k}_{\text{kinetic energy}} + \Delta(k) \underbrace{(c_k^\dagger c_{-k}^\dagger + c_{-k} c_k)}_{\text{pairing}}$$

Majorana Mode in Superconductor

1D **single-band**, **spinless, p-wave** BCS superconductor

Hamiltonian for infinite wire:

$$H = \sum_k \underbrace{c_k^\dagger (E_k - \mu) c_k}_{\text{kinetic energy}} + \Delta(k) \underbrace{(c_k^\dagger c_{-k}^\dagger + c_{-k} c_k)}_{\text{pairing}}$$

- Fermi-Dirac statistics dictates p-wave pairing: $\Delta(-k) = -\Delta(k)$
- Node at band edge: $\Delta(k=0) = 0$

1D Spinless Superconductor

Hamiltonian for infinite wire:

$$H = \sum_k c_k^\dagger (E_k - \mu) c_k + \Delta(k)(c_k^\dagger c_{-k}^\dagger + c_{-k} c_k)$$

$$= \sum_k (c_k^\dagger, c_{-k}) \underbrace{\begin{pmatrix} E_k - \mu & \Delta(k) \\ \Delta(k) & \mu - E_k \end{pmatrix}}_{\text{BdG Hamiltonian } H(k) \text{ in Nambu space}} \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}$$

For single-band spinless SC:

$$H(k) = \epsilon(k) \left[n_x(k)\sigma_x + n_z(k)\sigma_z \right] \qquad \epsilon(k) = \sqrt{(E_k - \mu)^2 + \Delta^2(k)}$$

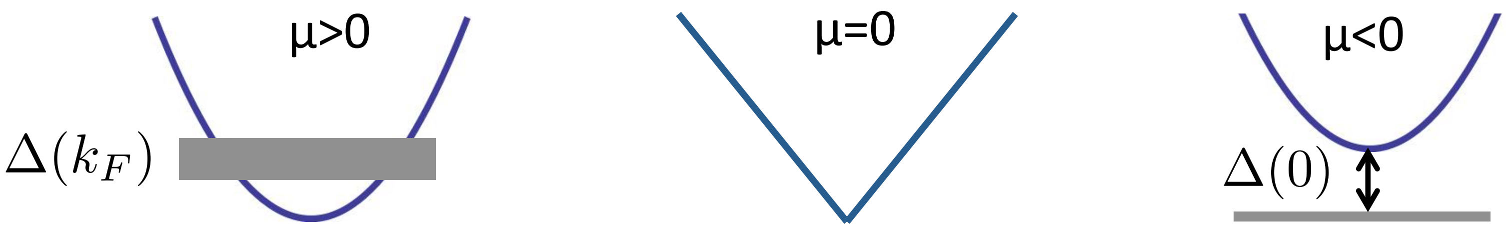
- (n_x, n_z) is a unit vector if $\varepsilon(k) \neq 0$, i.e,. there is a gap

1D Spinless Superconductor

Hamiltonian for infinite wire:

$$H = \sum_k c_k^\dagger (E_k - \mu) c_k + \Delta(k)(c_k^\dagger c_{-k}^\dagger + c_{-k} c_k)$$

Energy spectrum:



fully gapped
weak pairing BCS

gapless at k=0
$\Delta(k) \propto k$

fully gapped
strong pairing BEC

- two gapped phases, separated by a gap-closing transition

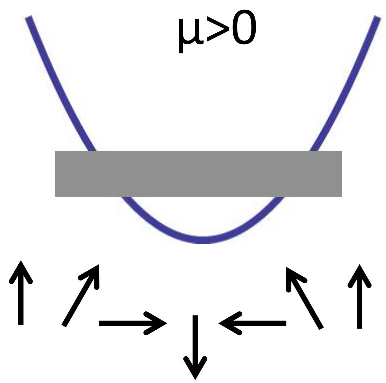
c.f. Read & Green, 00

Topology of 1D Superconductor

$$H(k) = \epsilon(k)\left[n_x(k)\sigma_x + n_z(k)\sigma_z\right]$$

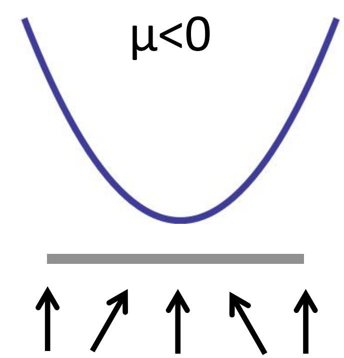
- winding number of (n_x,n_z) as a function of k in 1D Brillouin zone defines a topological invariant N for gapped 1D superconductor

Weak pairing BCS



N = 1:

Strong pairing BEC



N = 0:

- two phases are topologically distinct: weak pairing is nontrivial
- generalization to multi-band: Z_2 topological invariant (N mod 2)

(Kitaev 00)

Majorana Mode

Semi-infinite wire: 

- end of wire = domain wall between weak and strong pairing (vacuum) phase
- change of topology across the domain leads to a zero-energy localized mode
- BdG equation = 1D Dirac equation with a mass twist

$$H = \begin{pmatrix} E_k - \mu & \Delta(k) \\ \Delta(k) & \mu - E_k \end{pmatrix} \to \begin{pmatrix} -\mu(x) & -iv\partial_x \\ iv\partial_x & \mu(x) \end{pmatrix}$$ for small μ

- zero-mode corresponds to a real, local operator

$$\gamma = \int dx\, u(x)c^\dagger(x) + v(x)c(x)$$ where $u(x) = v^*(x) = e^{-\mu x/v}$

$\gamma = \gamma^\dagger$: Majorana zero mode

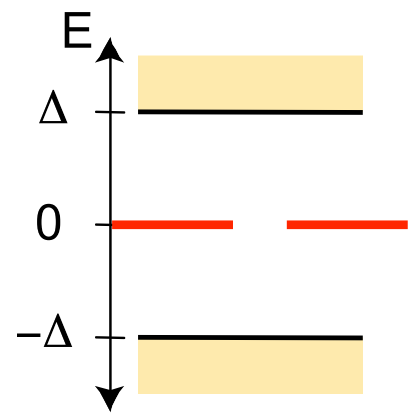
in contrast, ε>0 solutions correspond to ordinary fermions (quasi-particles).

General Properties of Majorana Modes

- zero-energy, real solution to BdG equation: protected by symmetry of BdG
- localized at boundary between two topologically distinct SCs

General Properties of Majorana Modes

- zero-energy, real solution to BdG equation: protected by symmetry of BdG
- localized at boundary between two topologically distinct SCs



- there is always an even number of Majorana modes in a closed system
- splitting of zero modes decays exponentially as their separation

Majorana Qubits

Presence of Majorana modes leads to degenerate superconducting ground states.

- ground state of superconductor is a non-Slater state & corresponds to quasi-particle vacuum $|G\rangle = |0\rangle_1 \otimes |0\rangle_2 ...$
- two Majorana modes make up Fock space of a single fermion degree of freedom

$$\Gamma^\dagger = \gamma_1 + i\gamma_2,\ \Gamma = \gamma_1 - i\gamma_2$$

$$\{\Gamma, \Gamma^\dagger\} = 1$$

$$\Gamma^\dagger |1\rangle_M = 0,\ \Gamma|0\rangle_M = 0$$

|0> and |1> form a Majorana qubit

Majorana Qubits

Presence of Majorana modes leads to degenerate superconducting ground states.

- ground state of superconductor is a non-Slater state & corresponds to quasi-particle vacuum $|G\rangle = |0\rangle_1 \otimes |0\rangle_2 ...$
- two Majorana modes make up Fock space of a single fermion degree of freedom

$$|G\rangle = |0\rangle_1 \otimes |0\rangle_2 ... \otimes |0\rangle_M$$

$$|G'\rangle = |0\rangle_1 \otimes |0\rangle_2 ... \otimes |1\rangle_M$$

Majorana qubit

finite-energy quasi-particles

- 2M Majorana modes => M Majorana qubits => 2^M-fold degeneracy

Topological Degeneracy

Majorana as a bookkeeper:

- Hilbert space of Majorana modes is *isomorphic* to Hilbert space of degenerate many-body ground states.

Topological Degeneracy

Majorana as a bookkeeper:

- Hilbert space of Majorana modes is *isomorphic* to Hilbert space of degenerate many-body ground states.
- Different ground states cannot be distinguished by local observables*, because low-energy operators involve product of different Majorana operators and hence are nonlocal.

Topological Degeneracy

Majorana as a bookkeeper:

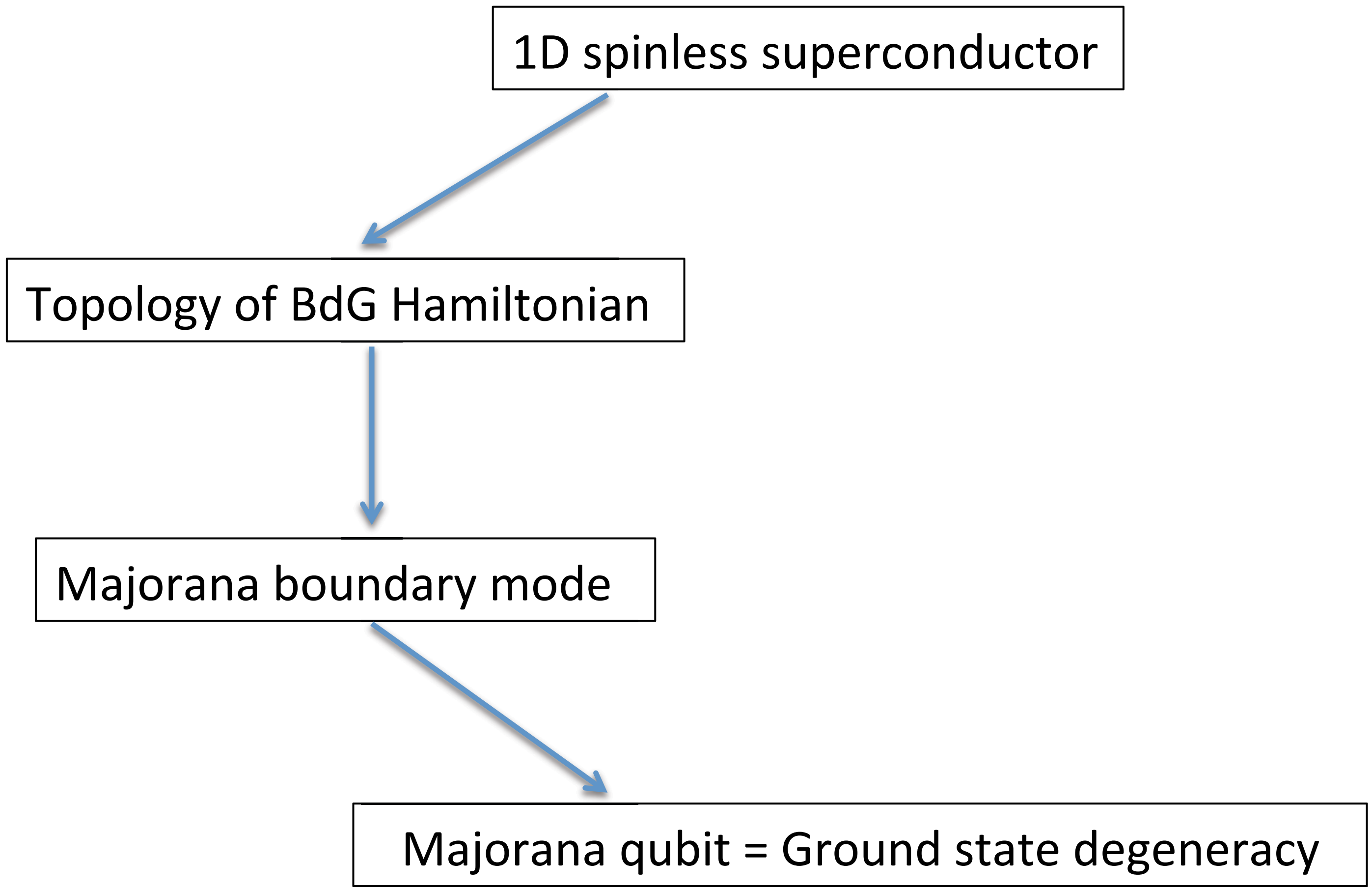
- Hilbert space of Majorana modes is *isomorphic* to Hilbert space of degenerate many-body ground states.
- Different ground states cannot be distinguished by local observables*, because low-energy operators involve product of different Majorana operators and hence are nonlocal.
- Ground state degeneracy is robust against local perturbations*: topological protection.

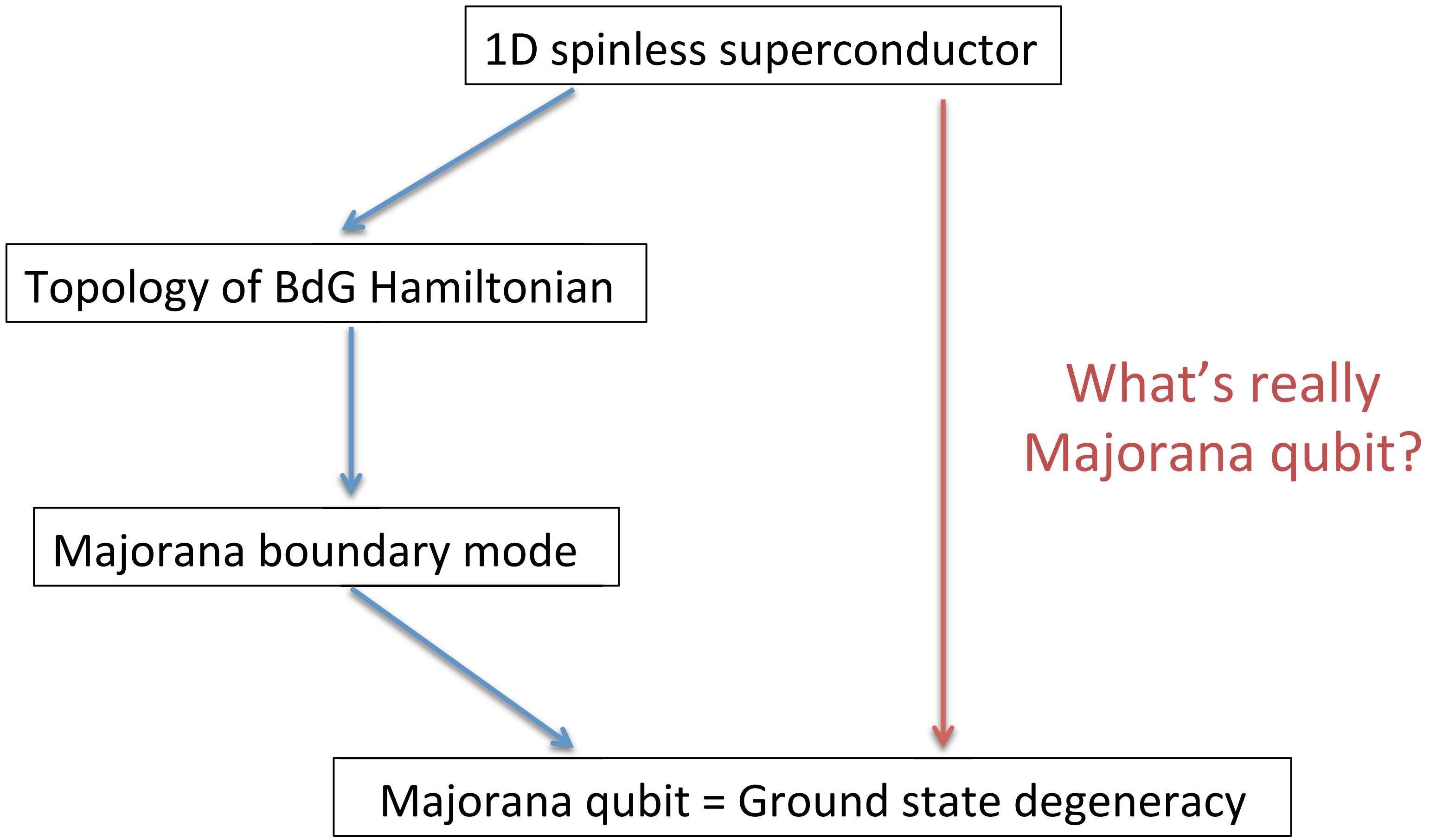
Topological Degeneracy

Majorana as a bookkeeper:

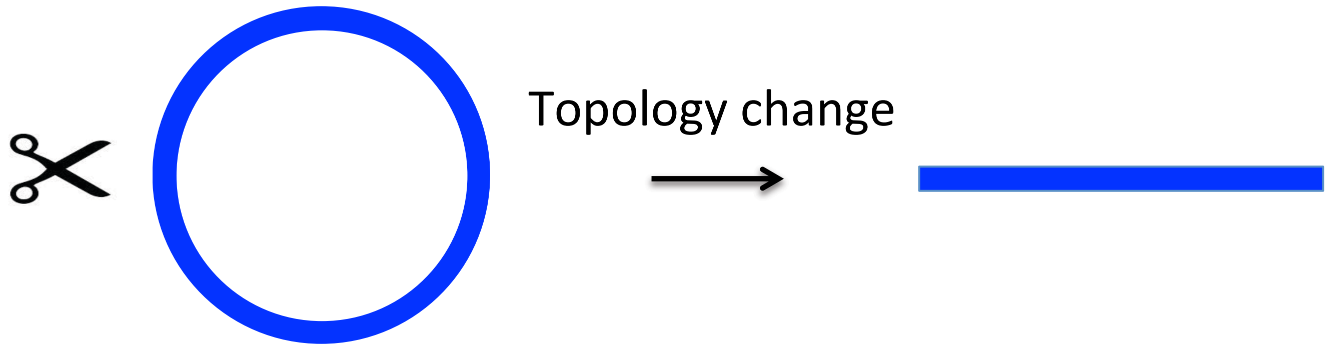
- Hilbert space of Majorana modes is *isomorphic* to Hilbert space of degenerate many-body ground states.
- Different ground states cannot be distinguished by local observables*, because low-energy operators involve product of different Majorana operators and hence are nonlocal.
- Ground state degeneracy is robust against local perturbations*: topological protection.

Majorana qubit can be used as ideal quantum memory: basis for topological quantum computing

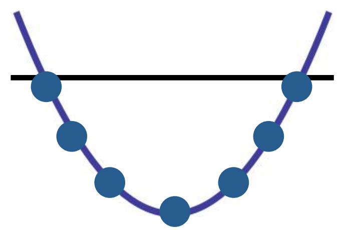

1D spinless superconductor
Topology of BdG Hamiltonian
Majorana boundary mode
Majorana qubit = Ground state degeneracy


1D spinless superconductor
Topology of BdG Hamiltonian
Majorana boundary mode
Majorana qubit = Ground state degeneracy
What's really
Majorana qubit?

Origin of Ground State Degeneracy



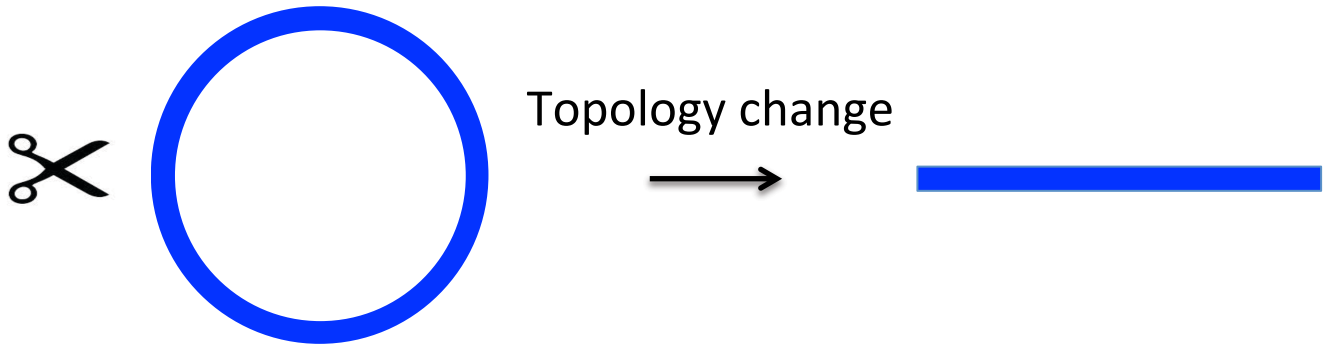
1. periodic bc:



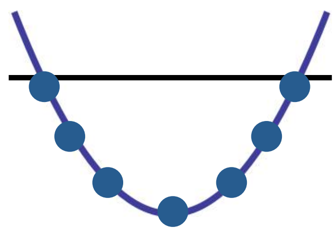
- quantized momenta k=2πn/L
- an unpaired electron at k=0

=> BCS ground state has odd # of electrons

Origin of Ground State Degeneracy



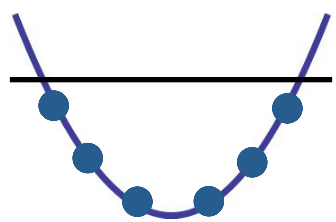
1. periodic bc:



- quantized momenta k=2πn/L
- an unpaired electron at k=0

=> BCS ground state has odd # of electrons

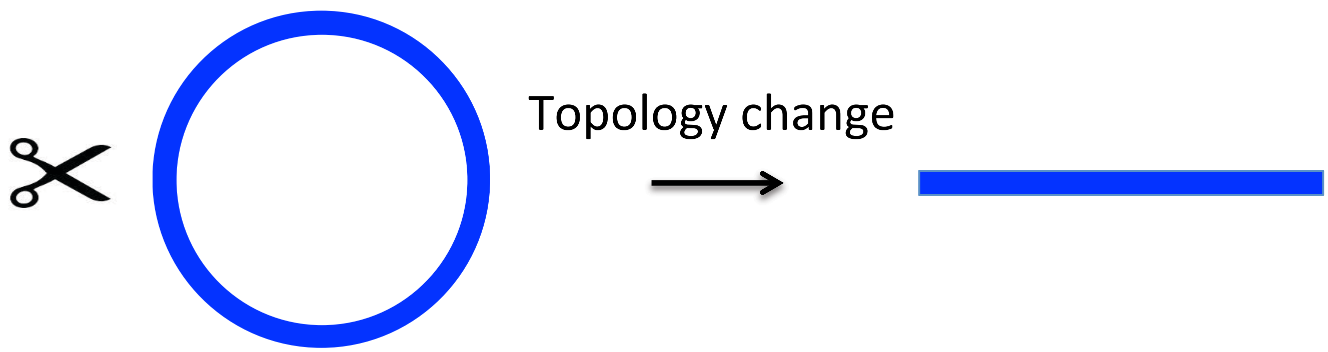
2. anti-periodic bc:



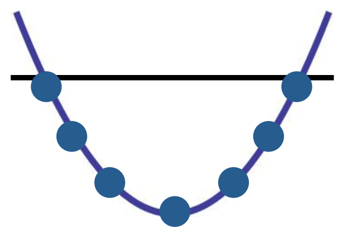
- equivalent to threading a h/(2e) flux
- shifted momenta k=2π(n+1/2)/L

=> BCS ground state has even # of electrons

Origin of Ground State Degeneracy



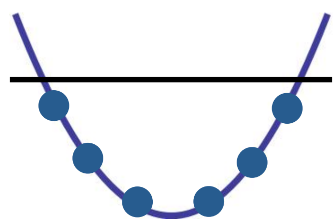
1. periodic bc:



- quantized momenta $k=2\pi n/L$
- an unpaired electron at $k=0$

=> BCS ground state has odd # of electrons

2. anti-periodic bc:



- equivalent to threading a $h/(2e)$ flux
- shifted momenta $k=2\pi(n+1/2)/L$

=> BCS ground state has even # of electrons

3. open bc with two ends: on equal terms with 1 & 2 => even/odd degeneracy

Even-Odd Degeneracy

For M=1: the Majorana qubit states $|0>_M$ and $|1>_M$ correspond to the superconductor ground state with an even and odd number of electrons.

Majorana qubit = electron number parity

The even-parity and odd-parity ground states are locally indistinguishable.

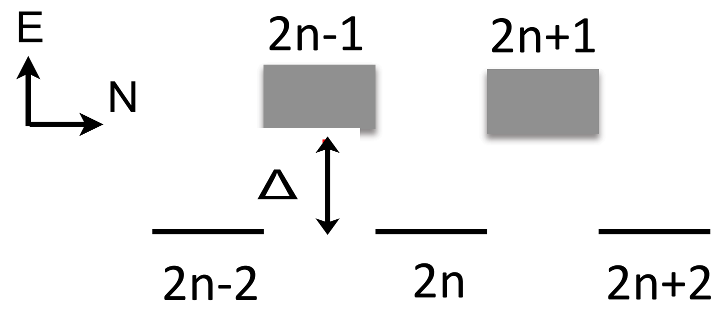
Even-odd degeneracy occurs in BCS phase if there is an *odd* number of bands (including spin) at Fermi energy

Even-Odd Degeneracy

The remarkable fact that in the presence Majorana modes, a topological superconductor can accommodate either an even or odd number of electrons on equal ground is key to understanding exotic properties of Majorana modes.

Number and Phase

Spin-singlet SC



Ground state manifold parameterized by either Cooper pair number n, or superconductor phase θ (2π-periodic)

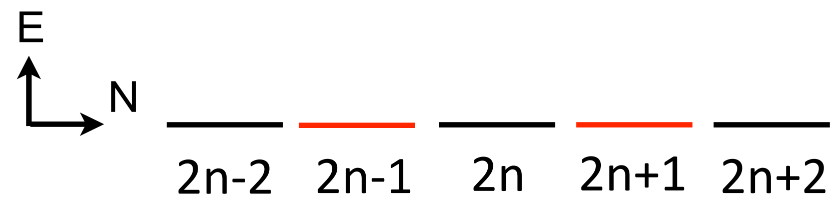
Number-phase relation:

$$|\theta\rangle = \sum_n e^{i\theta n}|n\rangle \qquad [n,\theta] = i$$

Number, Phase and Majorana Qubit

LF, PRL 104, 056402 (2010)

Topological SC



Ground state manifold can be parameterized in two ways:

1. Superconductor phase θ (2π-periodic) AND Majorana qubit ($|0\rangle_M$ or $|1\rangle_M$)
2. Electron number N (integer)

Generalized number-phase relation: $[N, \frac{\theta}{2}] = i$

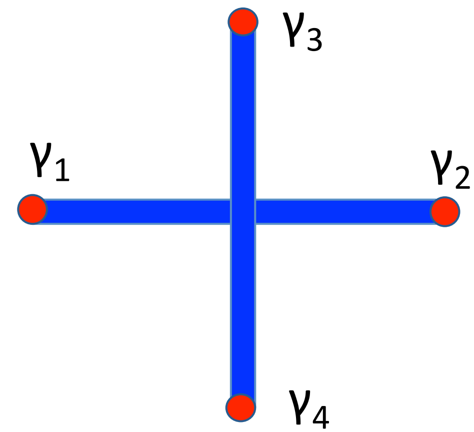
Even-parity state: $|\theta\rangle \otimes |0\rangle_M = \sum_{N=2n} e^{-i\theta N/2} |N\rangle$ invariant under θ => θ + 2π

Odd-parity state: $|\theta\rangle \otimes |1\rangle_M = \sum_{N=2n+1} e^{-i\theta N/2} |N\rangle$ changes sign under θ => θ + 2π

Phase Doubling from 2π to 4π

Two crossed wires forming a Josephson junction: M=2

- Josephson coupling due to Cooper pair tunneling fixes the relative superconductor phase $\theta=\theta_1-\theta_2$
- For a fixed total number of electrons N_t, there is a two-fold degeneracy



Two natural basis states for even N_t:

$$|\theta\rangle \otimes |0_{12}, 0_{34}\rangle_M = \sum_{N_A=2n} e^{i\theta N_A/2}|N_t\rangle \otimes |N_t - N_A\rangle,$$

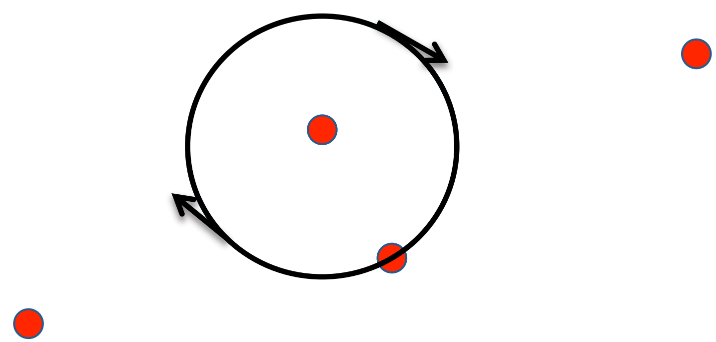
$$|\theta\rangle \otimes |1_{12}, 1_{34}\rangle_M = \sum_{N_A=2n+1} e^{i\theta N_A/2}|N_t\rangle \otimes |N_t - N_A\rangle,$$

Generic state: $\alpha|00\rangle + \beta|11\rangle$ corresponds to superposition of even and odd sectors in different parts of the superconductor

$\xrightarrow{\theta \Rightarrow \theta + 2\pi}$ $\alpha|00\rangle - \beta|11\rangle$ non-Abelian Berry phase

Braiding and Non-Abelian Statistics in 2D

- Majorana mode exists in vortex core of spinless p+ip superconductor



- moving γ_1 around γ_2 (=braiding twice) advances the superconductor phase in the enclosed region by 2π => change the ground state

Basis for topological quantum computation

Majorana Mode in Solid State

Lecture 1: Physics of Majorana mode in superconductors

Lecture 2: Realizations in spin-orbit-coupled systems (Alicea)

Lecture 3: Striking measurable properties of Majorana mode

Lecture 4: Towards finding Majorana and future directions (Alicea)

Exotic Properties of Majorana Modes

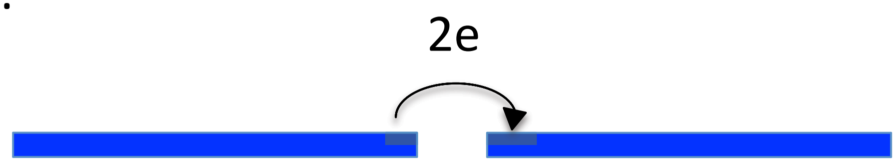
- Particle = Anti-particle: pair annihilation and production
- Non-locality: two spatially separated Majorana modes form one qubit
- Non-Abelian statistics: braiding changes Majorana qubit

Exotic Properties of Majorana Modes

- Particle = Anti-particle: pair annihilation and production
- Non-locality: two spatially separated Majorana modes form one qubit
- Non-Abelian statistics: braiding changes Majorana qubit

Conventional Josephson Effect: 2π Periodic

Spin-singlet SC:



- single-electron tunneling is suppressed by pairing gap
- Cooper-pair tunneling leads to Josephson effect

$$E = -E_J \cos(\theta),\ \theta = \theta_1 - \theta_2$$

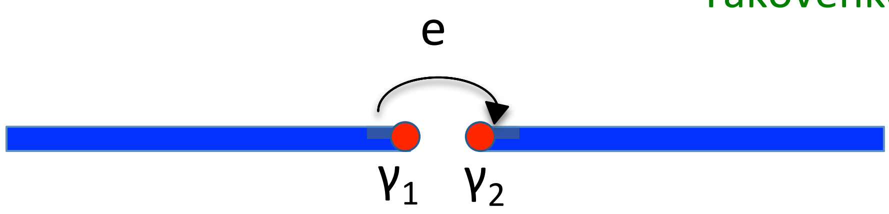
$$I = \frac{\partial E}{\partial \theta} = E_J \sin(\theta)$$

- Current-phase relation is 2π periodic: manifestation of quantized charge 2e of the Cooper pair

4π Josephson Effect via Majorana

Yakovenko et al 04

Topological SC:



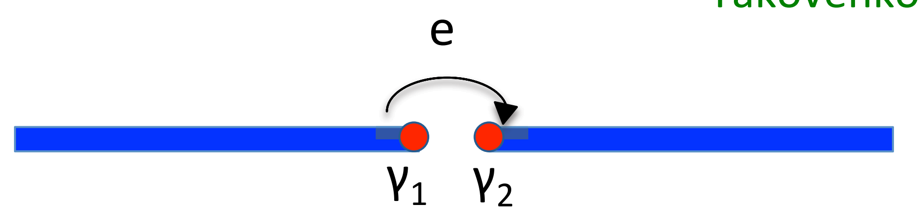
- Majorana mode enables electron to enter/exit without energy cost

=> Josephson effect via single-electron tunneling

4π Josephson Effect via Majorana

Yakovenko et al 04

Topological SC:



- Majorana mode enables electron to enter/exit without energy cost

=> Josephson effect via single-electron tunneling

tunneling Hamiltonian: $$H_T = t c_L^\dagger(0) c_R(0) + h.c.$$

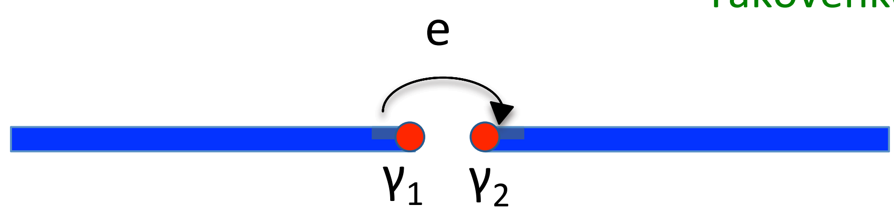
mode expansion: $$c_L^\dagger(0) = u_1(0)\gamma_1 e^{i\theta_L/2} + \ldots \quad c_L(0) = u_1^*(0)\gamma_1 e^{-i\theta_L/2} + \ldots$$

projecting H_T to low-energy: $$H_T = i\lambda\gamma_1\gamma_2\cos(\theta/2), \quad \lambda = \mathrm{Im}[t u_1(0) u_2^*(0)]$$

4π Josephson Effect via Majorana

Yakovenko et al 04

Topological SC:



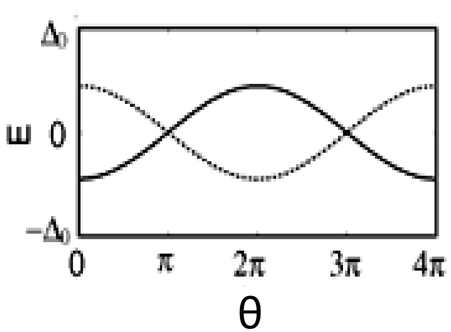
- Majorana mode enables electron to enter/exit without energy cost

=> Josephson effect via single-electron tunneling

low-energy Hamiltonian:

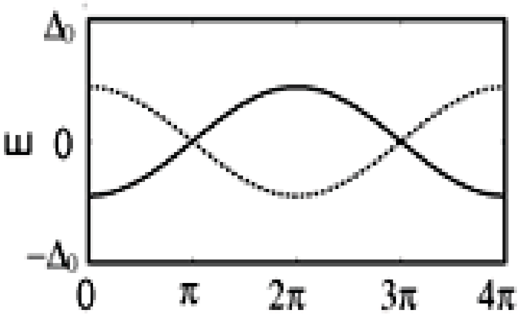
$$H_T = i\lambda\gamma_1\gamma_2\cos(\theta/2), \qquad i\gamma_1\gamma_2 = 2n_f - 1 = \begin{cases} +1 \text{ for } |0\rangle_M \\ -1 \text{ for } |1\rangle_M \end{cases}$$

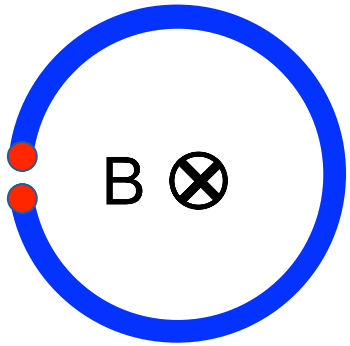
energy spectrum:



- coupling of two Majoranas leads to level splitting of $|0\rangle_M$ and $|1\rangle_M$: pair annihilation
- energy splitting depends on θ with 4π periodicity
- level crossing is protected by local parity conservation in a gapped superconductor

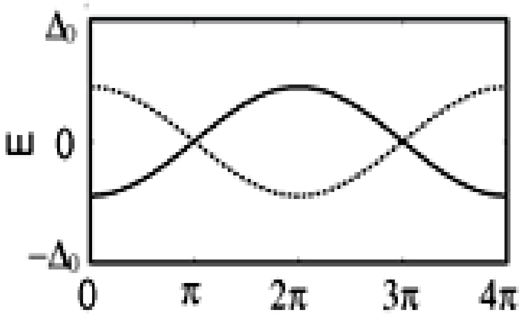
4π Josephson Effect in a Ring





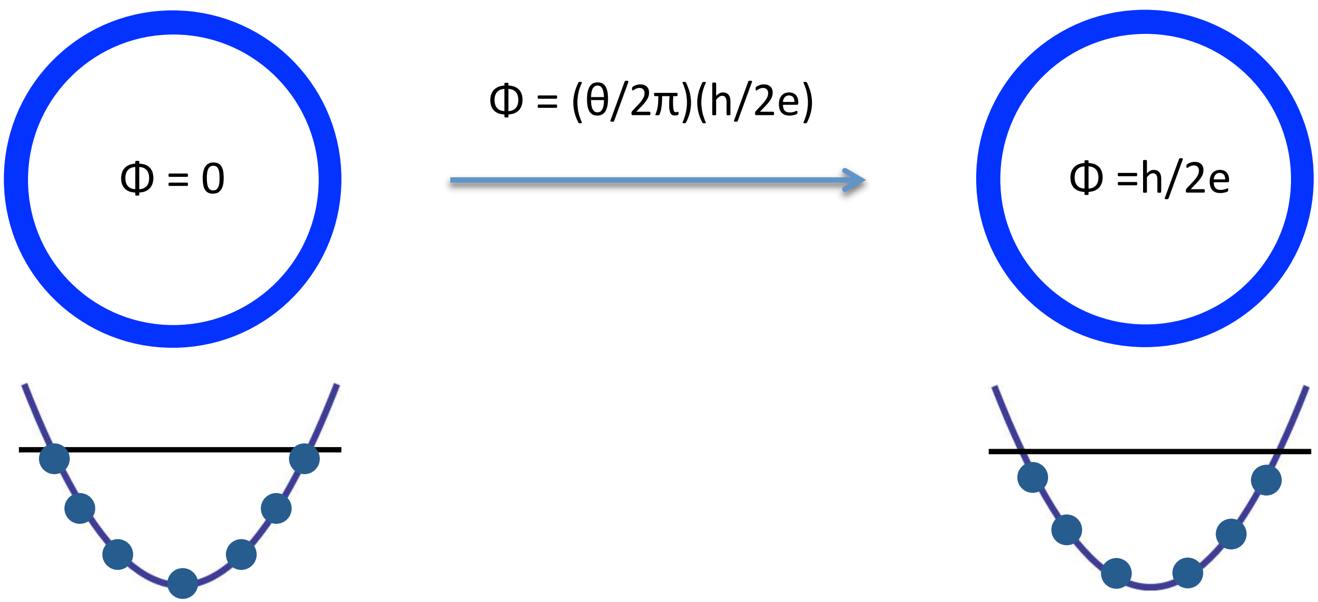
$\Phi = (\theta/2\pi)(h/2e)$

4π Josephson Effect in a Ring



- energy of even- and odd-parity states flips
- for a closed and gapped system, parity conservation forbids switching “branches”:

=> 4π-periodic Josephson effect



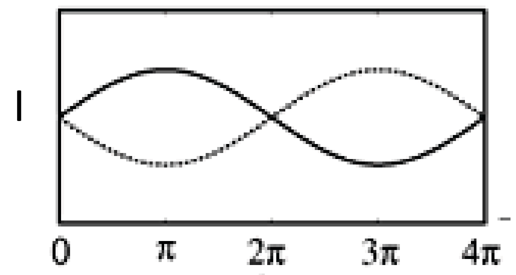
ground state is odd-parity

ground state is even-parity

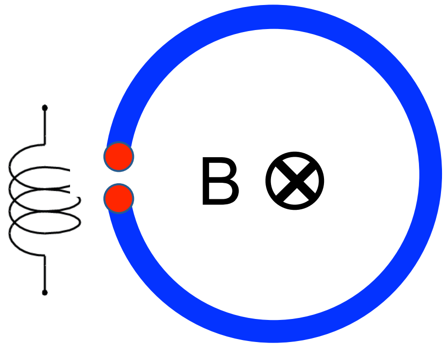
How to Measure 4π Josephson Effect?

Current-phase relation:

$$I = \frac{2e}{\hbar}\frac{\partial E}{\partial \theta} \propto \pm \sin\frac{\theta}{2}$$



Measure current-phase relation in RF SQUID LF & Kane, PRB 08

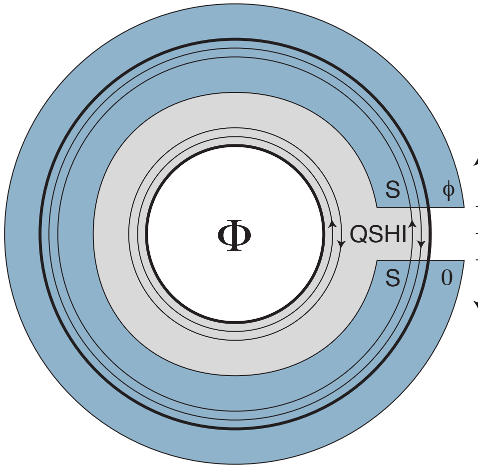


Φ = (θ/2π)(h/2e)

How to Measure 4π Josephson Effect?

Measure current-phase relation in RF SQUID: LF & Kane, PRB 08

- promising realization in quantum spin Hall system HgTe or InAs/GaSb

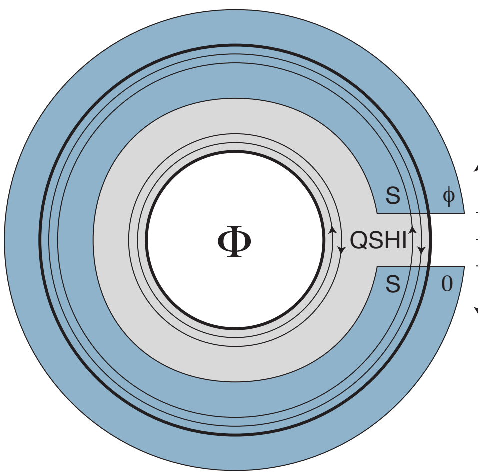


How to Measure 4π Josephson Effect?

Measure current-phase relation in RF SQUID: LF & Kane, PRB 08

- promising realization in quantum spin Hall system HgTe or InAs/GaSb

Harmful effect of quasi-particle poisoning: changes Majorana qubit and thus switches branch without violating parity conservation

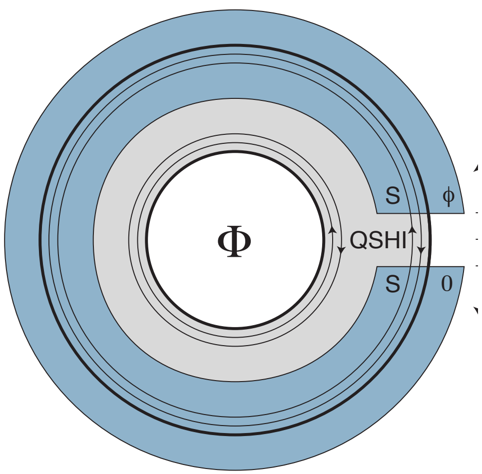


How to Measure 4π Josephson Effect?

Measure current-phase relation in RF SQUID: LF & Kane, PRB 08

- promising realization in quantum spin Hall system HgTe or InAs/GaSb

Harmful effect of quasi-particle poisoning: changes Majorana qubit and thus switches branch without violating parity conservation



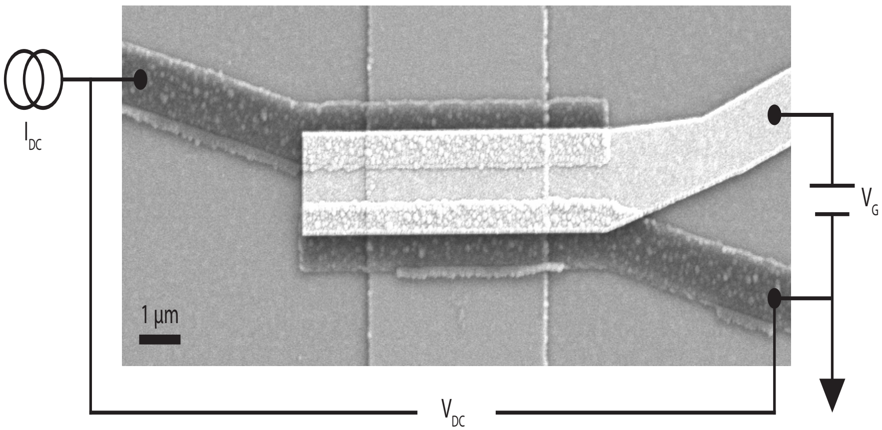
Measurement time scale vs. Majorana qubit lifetime

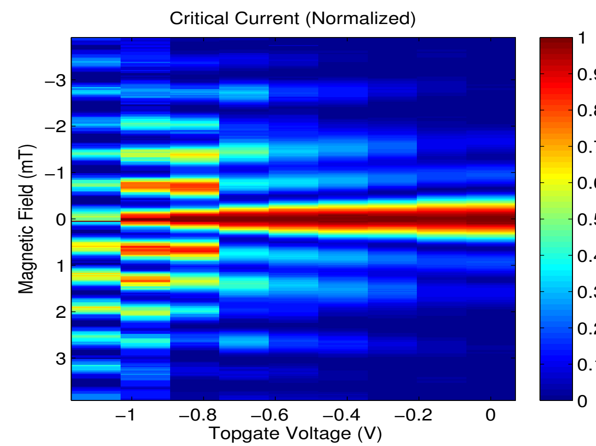
- fast measurement: 4π-periodic Josephson effect
- slow measurement: monitor noise in supercurrent to extract the lifetime and its temperature dependence

$$\tau^{-1} \propto \begin{cases} e^{-\Delta_0/T} & \text{quasiparticles,} \\ e^{-(T_0/T)^{1/3}} & \text{hopping.} \end{cases}$$

Induced Superconductivity in the Quantum Spin Hall Edge

Sean Hart[†1], Hechen Ren[†1], Timo Wagner[1], Philipp Leubner[2], Mathias Mühlbauer[2], Christoph Brüne[2], Hartmut Buhmann[2], Laurens W. Molenkamp[2], Amir Yacoby[1] (arXiv 13)

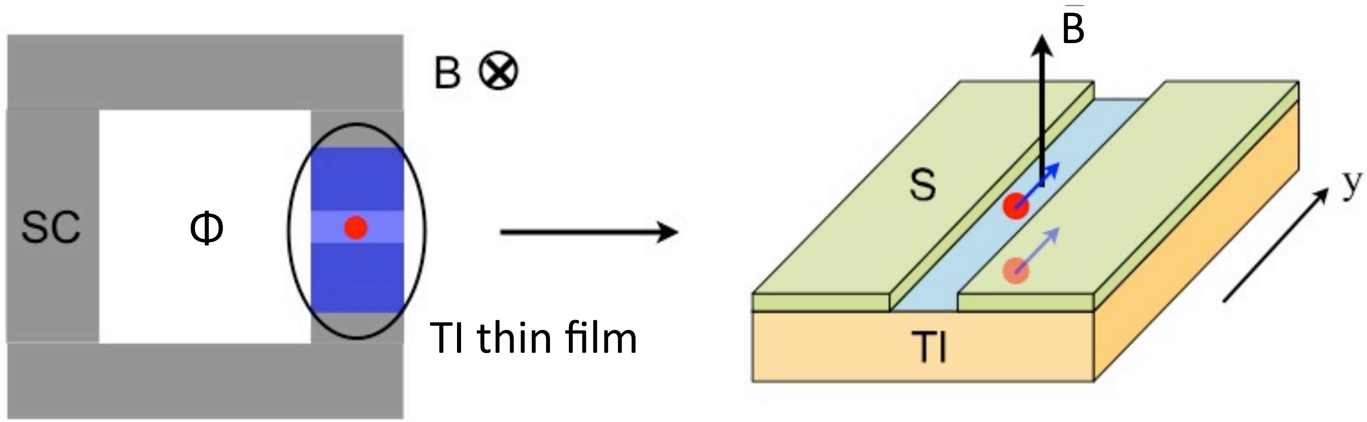




- magnetic interference pattern reveals supercurrent flowing at HgTe edge
- towards current-phase measurement

Majorana Pair Creation & Annihilation

LF & Potter, PRB 13

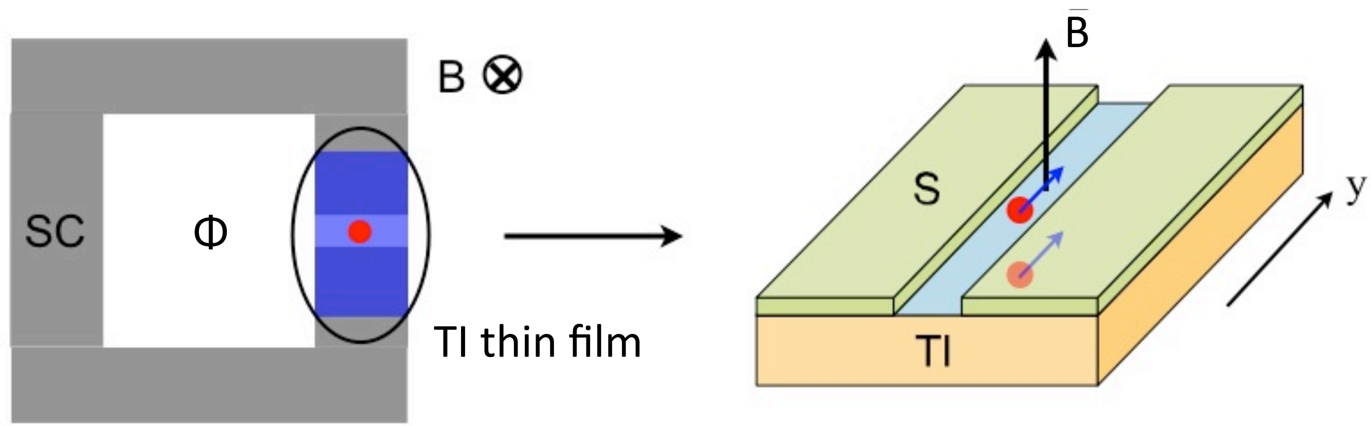


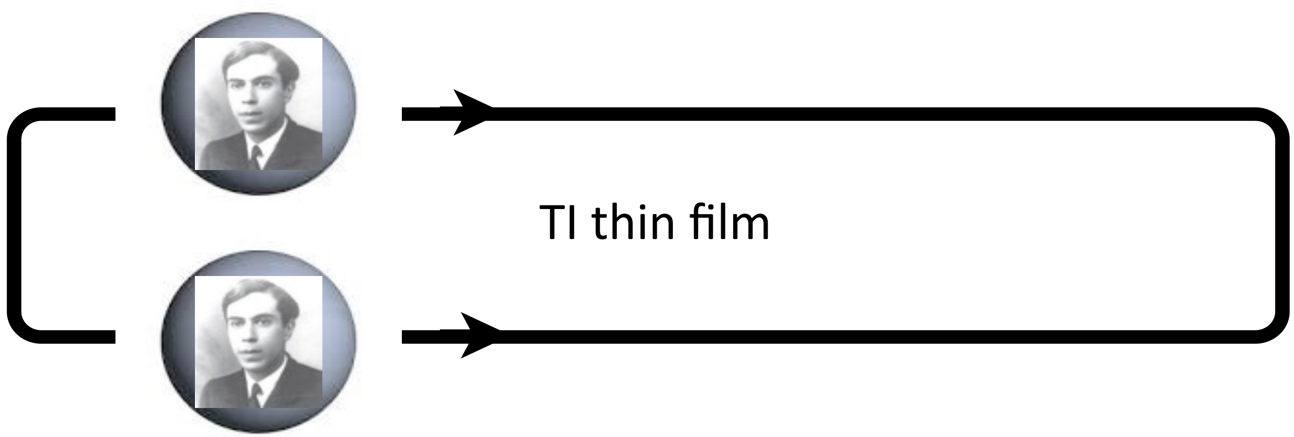
- Consider 1 flux quanta in SC-TI-SC junction:
  creates 1 Jonsephson vortex that traps 1 Majorana on *top and bottom* surface

- position of Majorana (y) is proportional to flux through SC loop Φ:
  advance Φ by 2π transports Majorana from one edge to the other

Majorana Pair Creation & Annihilation

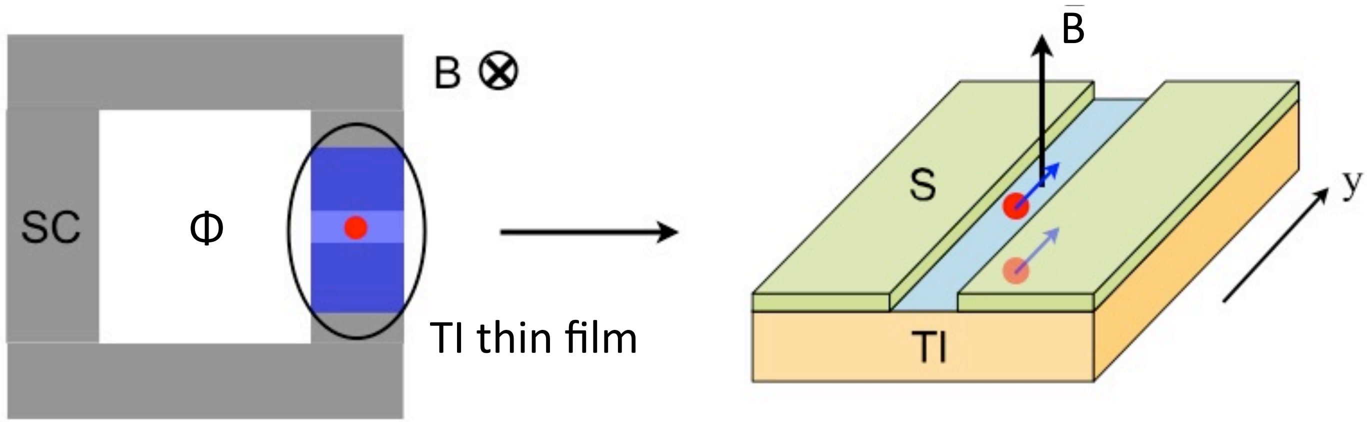
LF & Potter, PRB 13

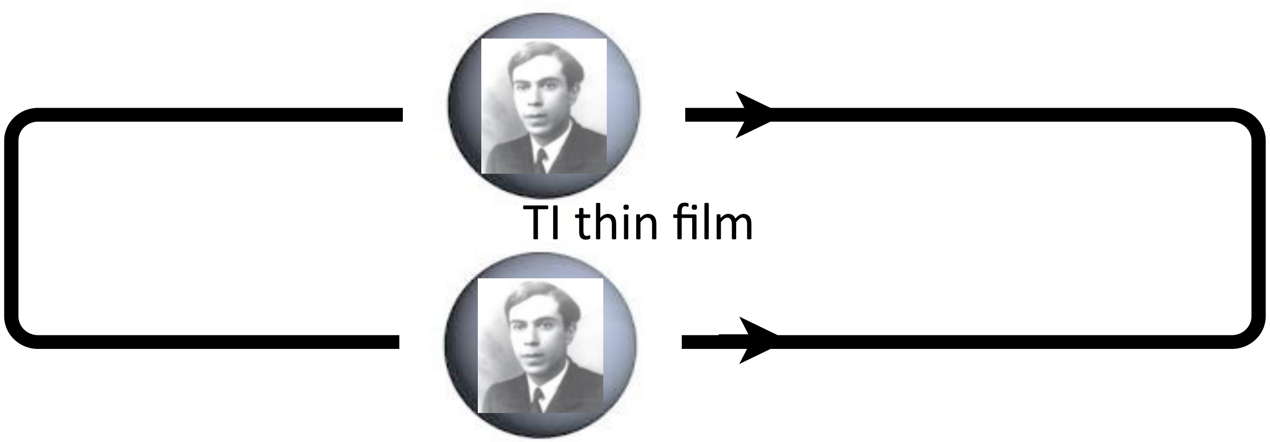




Majorana Pair Creation & Annihilation

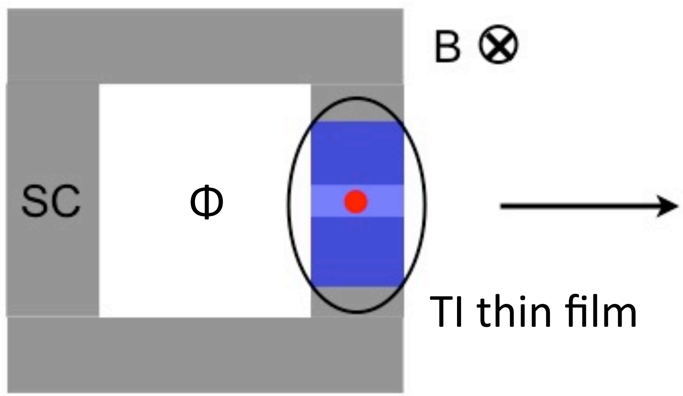
LF & Potter, PRB 13

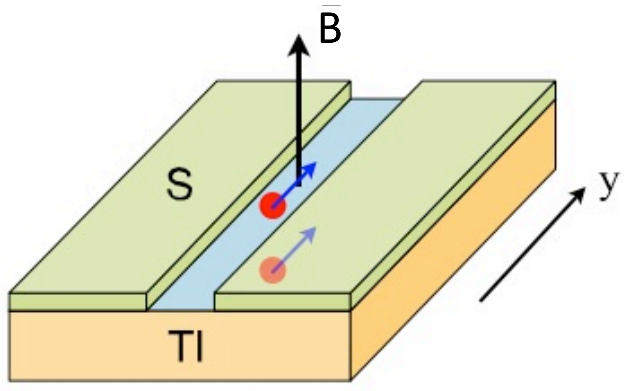


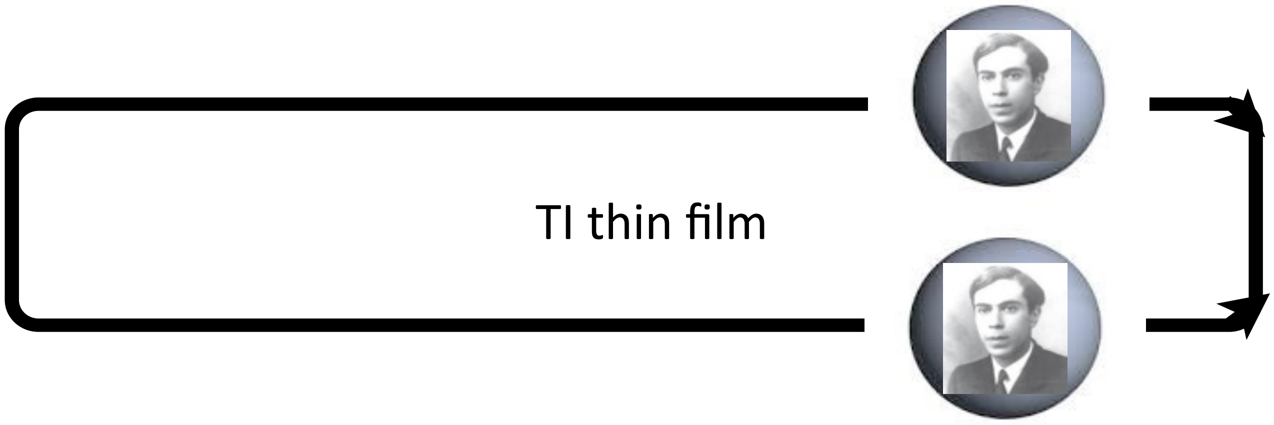


Majorana Pair Creation & Annihilation

LF & Potter, PRB 13

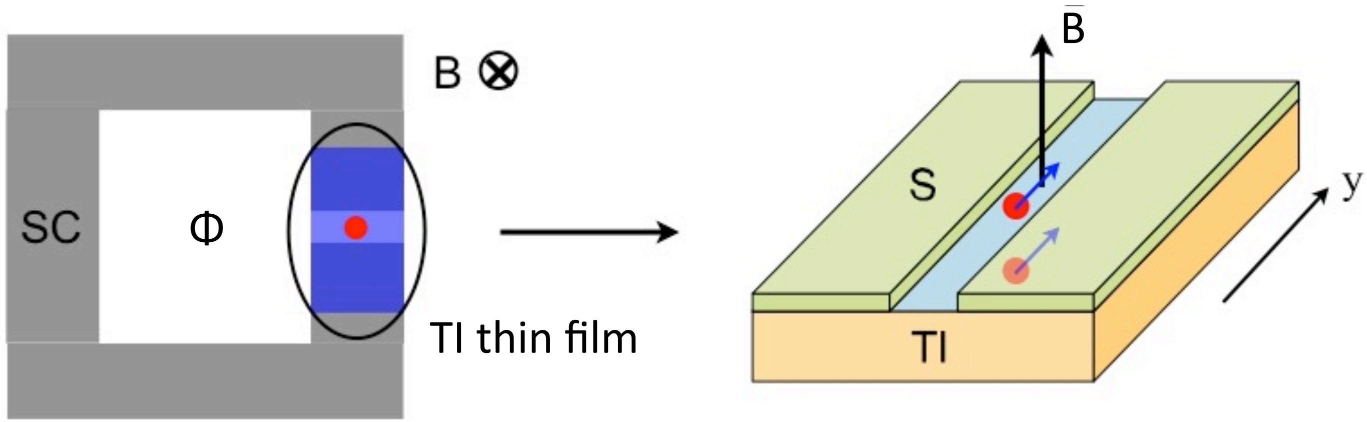


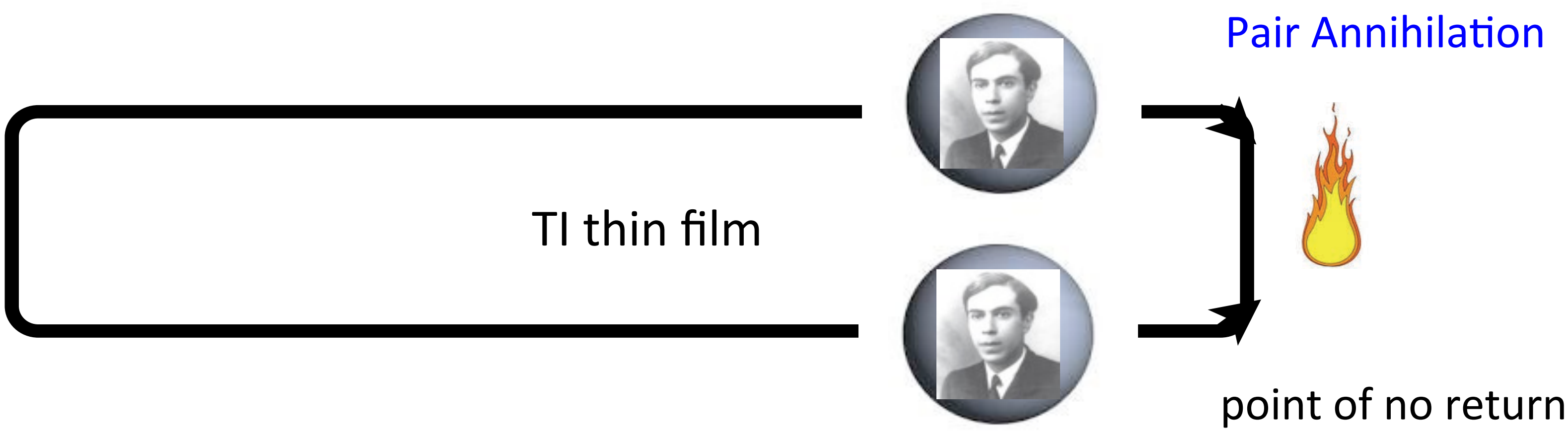




Majorana Pair Creation & Annihilation

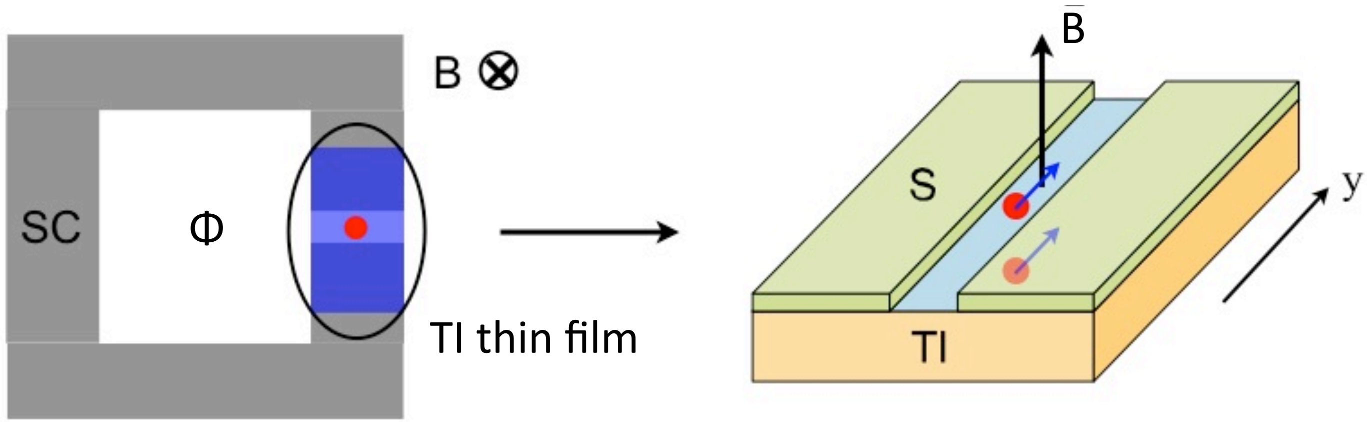
LF & Potter, PRB 13





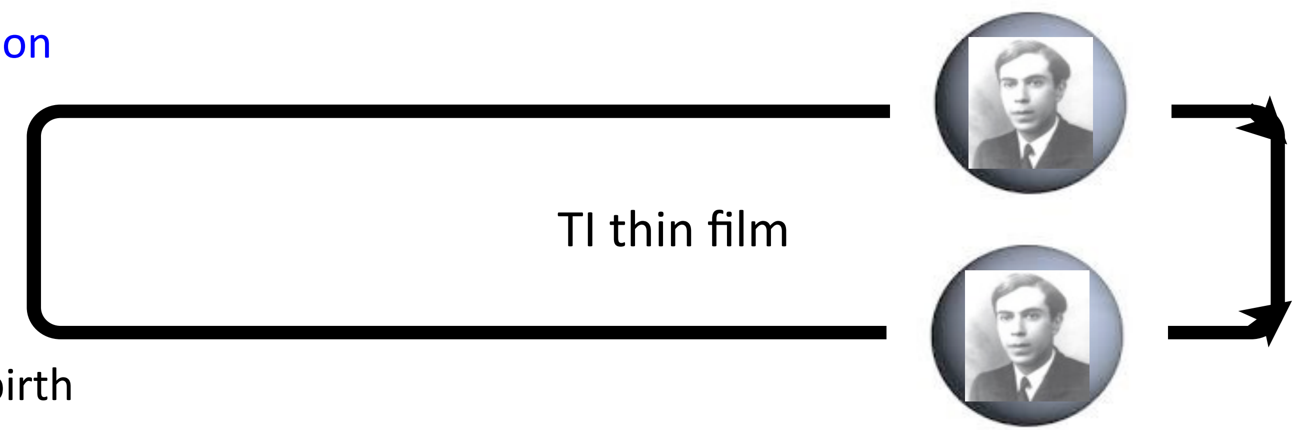
Majorana Pair Creation & Annihilation

LF & Potter, PRB 13



Pair Creation

place of rebirth

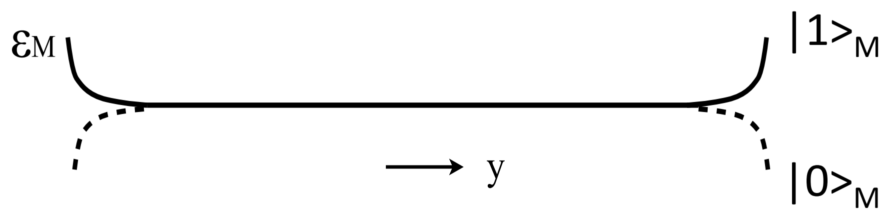


Signal of Pair Creation & Annihilation

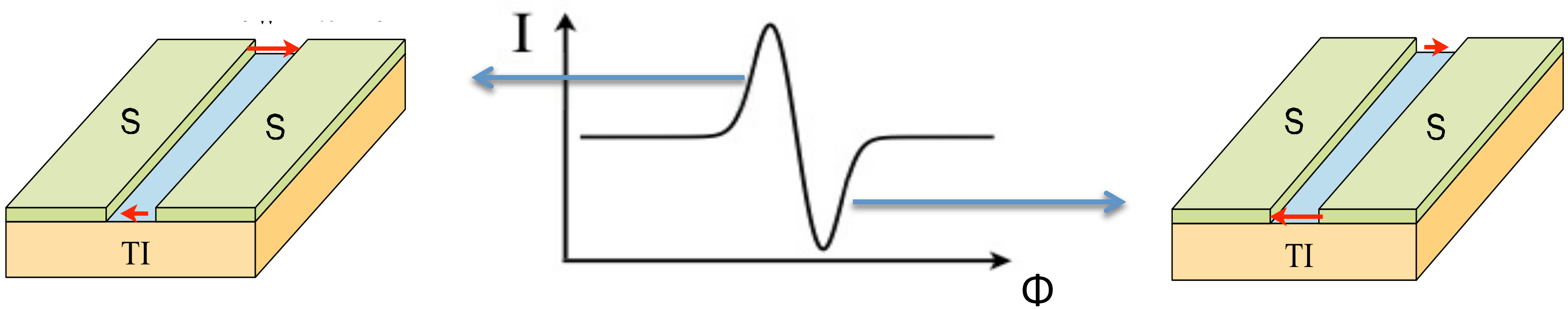
TI thin film:

Andrew Potter

Energy splitting of Majorana qubit depends on position y and thus flux Φ:



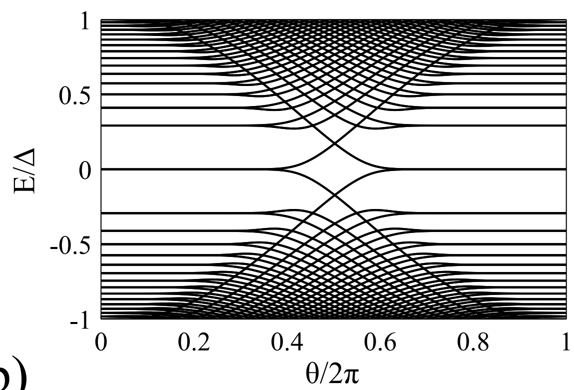
Supercurrent spikes with peak-dip structure: $I = \dfrac{\partial E}{\partial \Phi}$

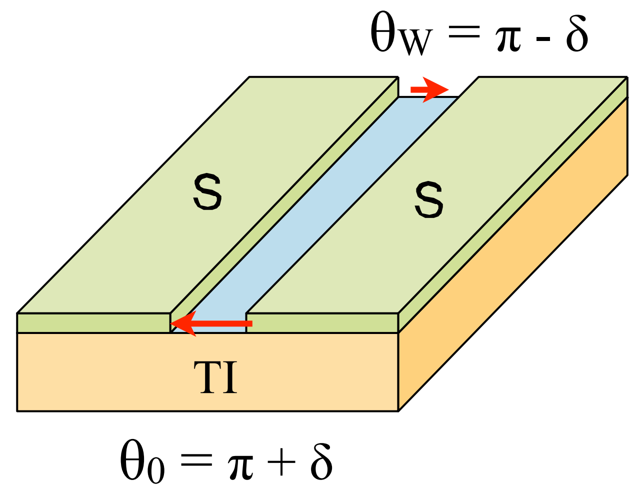


Magnitude of Majorana-derived Supercurrent

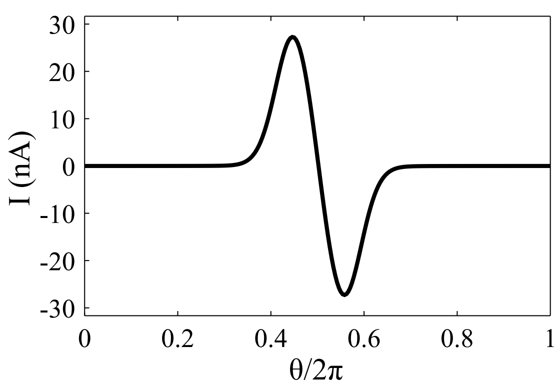
- short junction $L < \xi$: Josephson current is dominated by subgap Andreev states
- many Andreev states coexist with Majorana in a Josephson vortex

$\Phi = h/2e$





b)

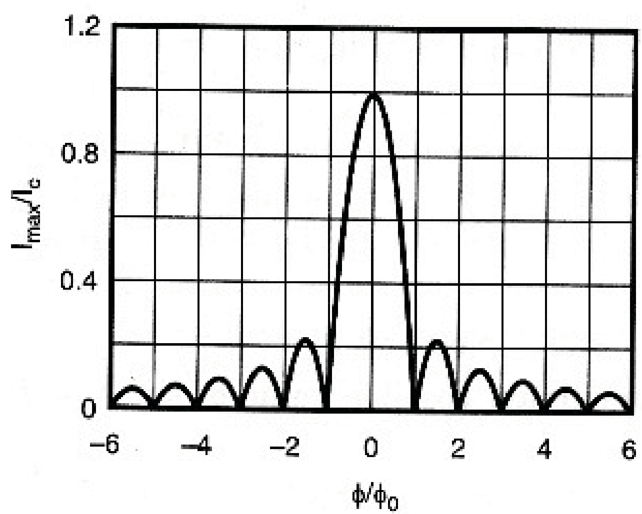


Maximum supercurrent is close to $e\Delta/h$.

- this result is largely independent of details
- the maximum amount of Josephson current carried by a single mode.

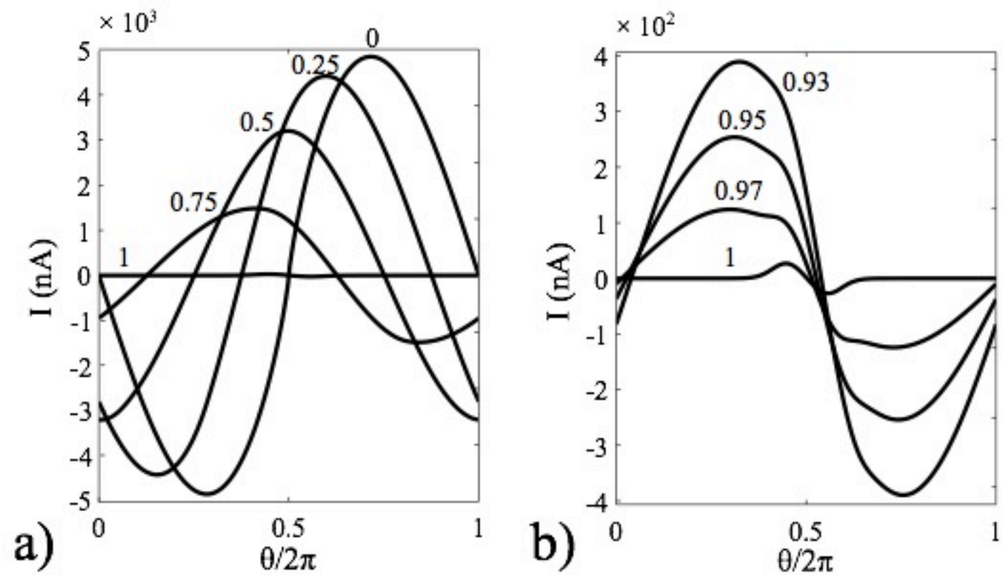
Sweet Spot: One Flux Quanta in Junction

- Supercurrent from metallic states in junction is greatly suppressed, and completely vanishes if junction is homogeneous (as seen from zero in Fraunhofer pattern)



- Sweet spot for isolating Majorana-derived supercurrent in TI JJ.

Current-Phase Relation



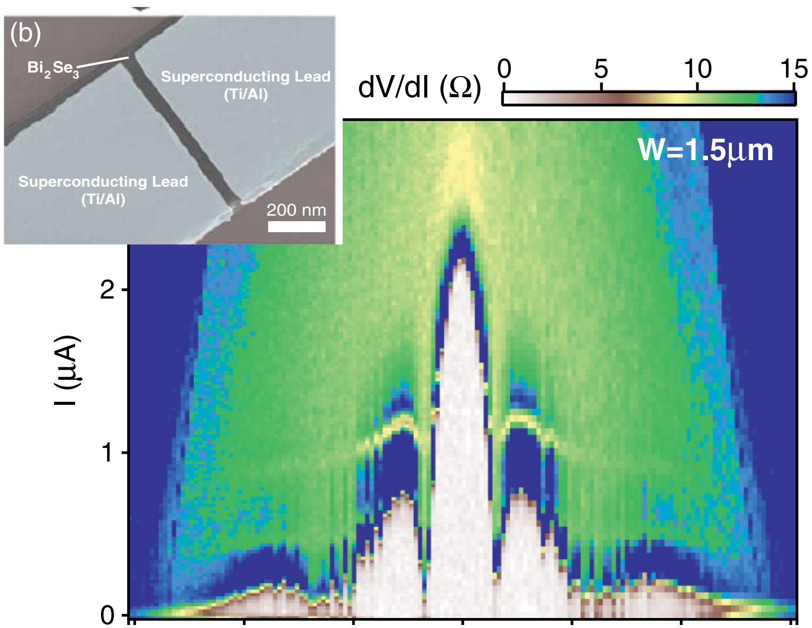
$$v_F = 4.2 \times 10^5 \mathrm{m/s}, \quad \mu = 10\mathrm{meV}$$

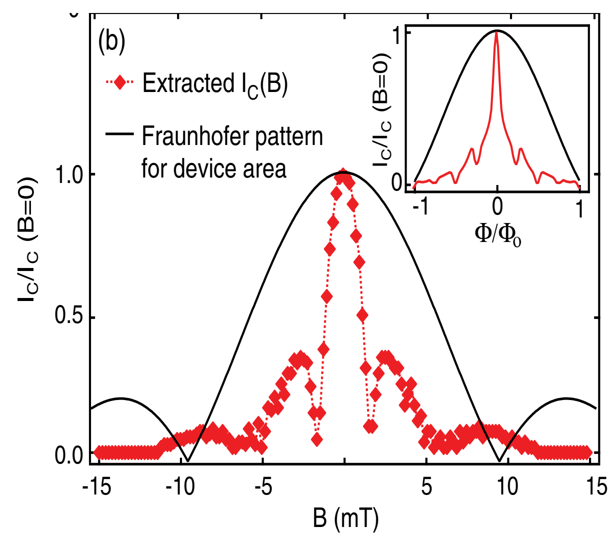
$$\Delta = 151\mu\mathrm{eV} \quad W = 2\mu\mathrm{m}$$

Majorana-derived supercurrent: 30nA

Experiment

Williams et al, PRL 12

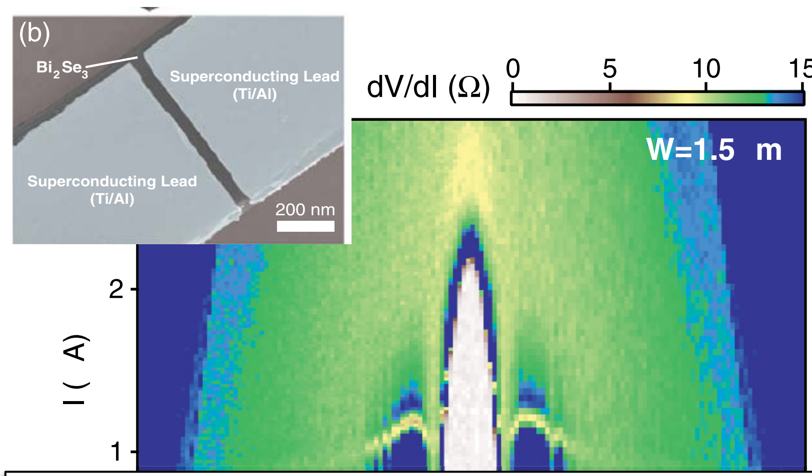


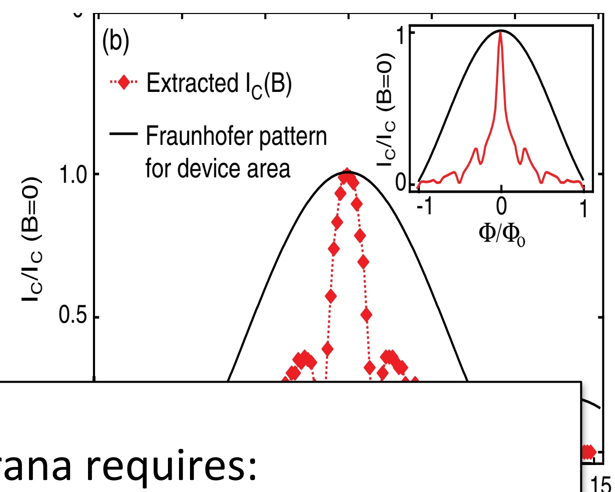


- lifted zeros in Fraunhofer pattern.
- residual supercurrent is too large to come from Majorana, consistent with current from side surface in thick TI film Moore, 12

Experiment

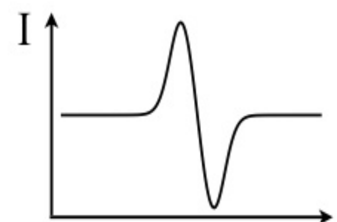
Williams et al, PRL 12





to detect pair creation/annihilation of Majorana requires:

1. low temperature, resolve small supercurrent
2. suppress metallic modes
3. thin TI film
4. current-phase meaurement



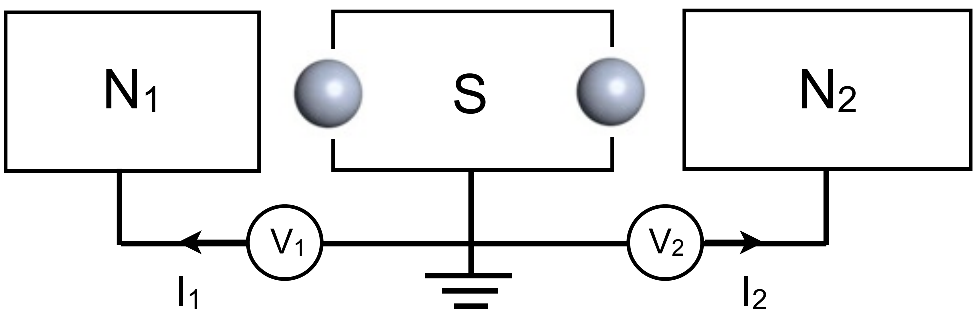
Exotic Properties of Majorana Modes

- Particle = Anti-particle: pair annihilation and production
- Non-locality: two spatially separated Majorana modes form one qubit
- Non-Abelian statistics: braiding changes Majorana qubit

How to Detect the Non-Local Majorana Qubit?

Perhaps this?

N-S-N junction



under conditions: 1. superconductor is grounded.

2. two Majorana modes have vanishing wavefunction overlap.

results from BTK theory:

- Andreev reflections at N_1-S and N_2-S are <u>independent</u>
- $I_1(V_1)$ and $I_2(V_2)$ are uncorrelated: e.g. $I_2=0$ if $V_2=0$ irrespective of V_1

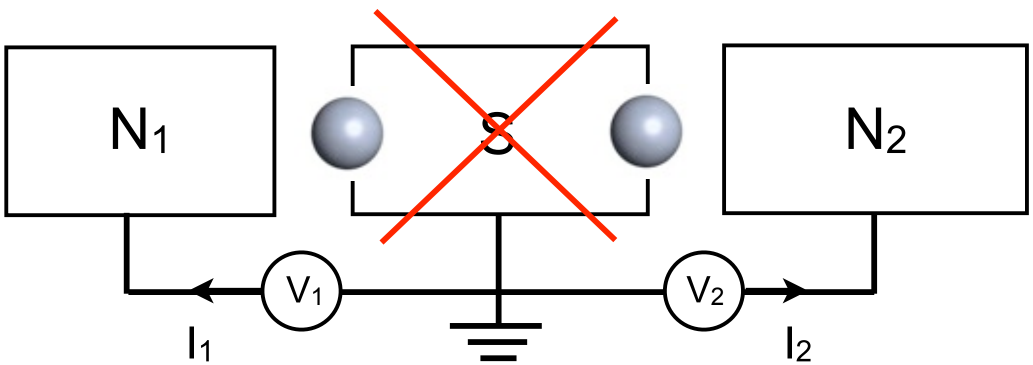
reason: N-S conductance is determined by **local** Andreev reflection at the interface.

Ahkmerov, Nilsson, Beenakker, 09; Bolech & Demler 07

How to Detect the Non-Local Majorana Qubit?

Perhaps this?

N-S-N junction



under conditions: 1. superconductor is grounded.

2. two Majorana modes have vanishing wavefunction overlap.

N - grounded S - N junction = two N-S junctions in parallel:
does not detect the Majorana qubit

Mission Impossible?

PRL 104, 056402 (2010) PHYSICAL REVIEW LETTERS week ending 5 FEBRUARY 2010

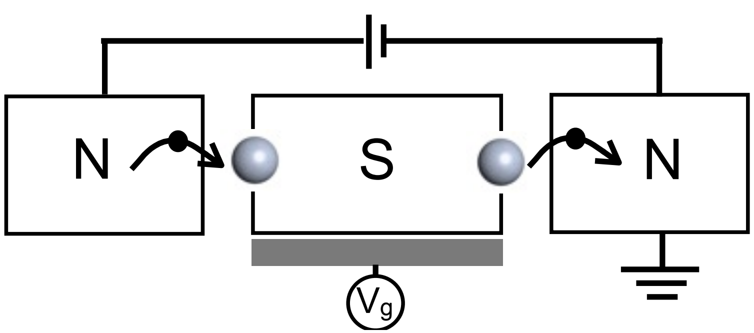
Electron Teleportation via Majorana Bound States in a Mesoscopic Superconductor

Liang Fu
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
(Received 23 October 2009; published 2 February 2010)

work, we predict a striking nonlocal phase-coherent electron transfer process by virtue of tunneling in and out of a pair of Majorana bound states. This teleportation phenomenon only exists in a mesoscopic superconductor because of an all-important but previously overlooked charging energy. We propose an experimental setup to detect this phenomenon in a superconductor–quantum-spin-Hall-insulator–-

new ingredient:

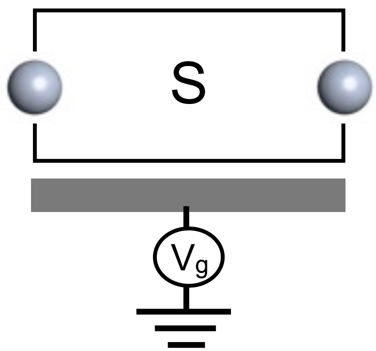
charging energy due to long-range Coulomb interaction

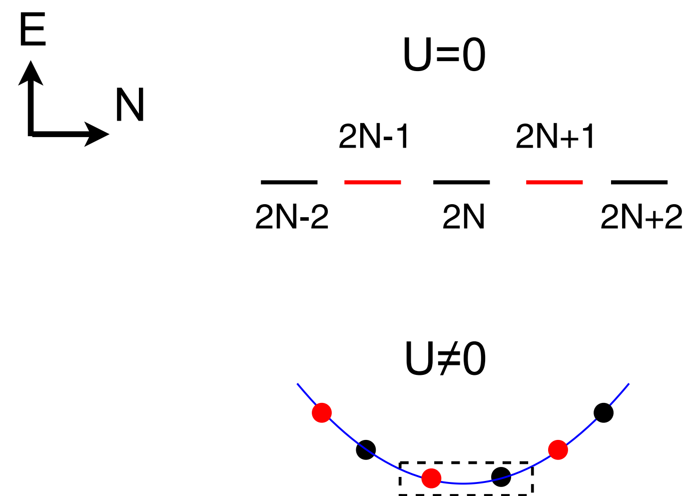


direct consequences of nonlocal nature of Majorana state

Charging Energy in Superconductor with Majoranas

Energy spectrum of topological superconductor with **two** Majorana modes present:





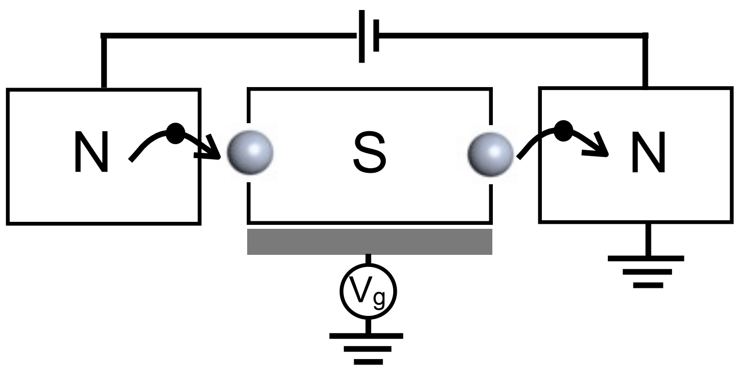
$$|G\rangle_{2N} = |0\rangle_1 \otimes |0\rangle_2 ... \otimes |0\rangle_M$$

$$|G'\rangle_{2N+1} = |0\rangle_1 \otimes |0\rangle_2 ... \otimes |1\rangle_M$$

- U=0: ground states with Majorana qubit $|0>_M$ and $|1>_M$ are degenerate and have different electron number parity: even-odd degeneracy

- U≠0: $E(N) = U(N-N_0)^2$ for both even and odd N

Nonlocal Transport via Majorana Qubit



Working conditions:
small bias below charging energy,
low temperature below tunneling strength

Weak tunneling and small bias limit (universal regime):

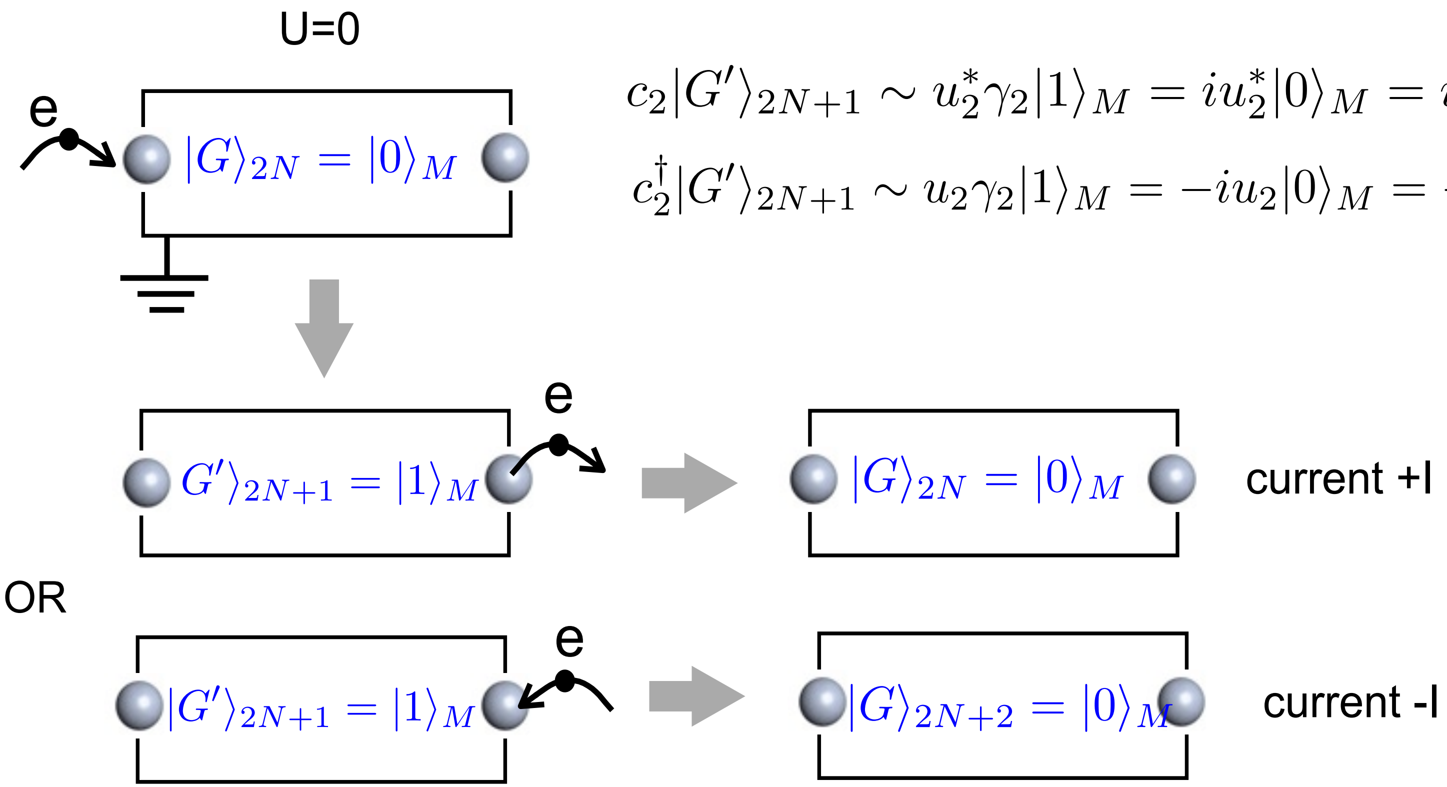
analyze the **slow** tunneling process in steps, work with **low-energy states** only, and calculate transmission amplitude of incident electron.

Tunneling Process

Weak tunneling and small bias limit (universal regime):

$$c_2|G'\rangle_{2N+1} \sim u_2^*\gamma_2|1\rangle_M = iu_2^*|0\rangle_M = iu_2^*|G\rangle_{2N}$$

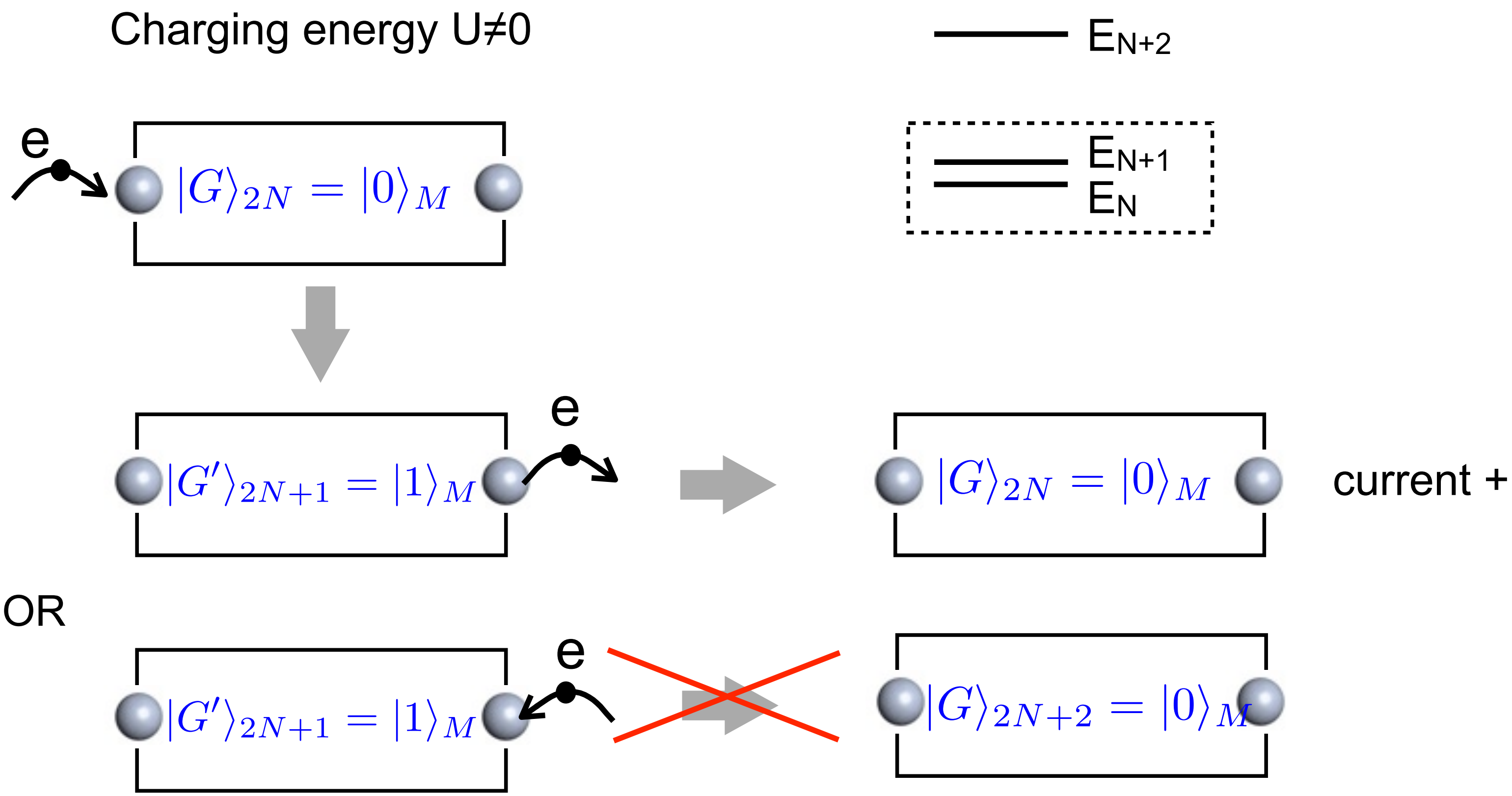
$$c_2^\dagger|G'\rangle_{2N+1} \sim u_2\gamma_2|1\rangle_M = -iu_2|0\rangle_M = -iu_2|G\rangle_{2N+2}$$



Total current is zero because (i) Majorana mode is equal superposition of electron and hole (ii) condensate reservoir absorbs Cooper pairs at zero energy cost.

Tunneling Process

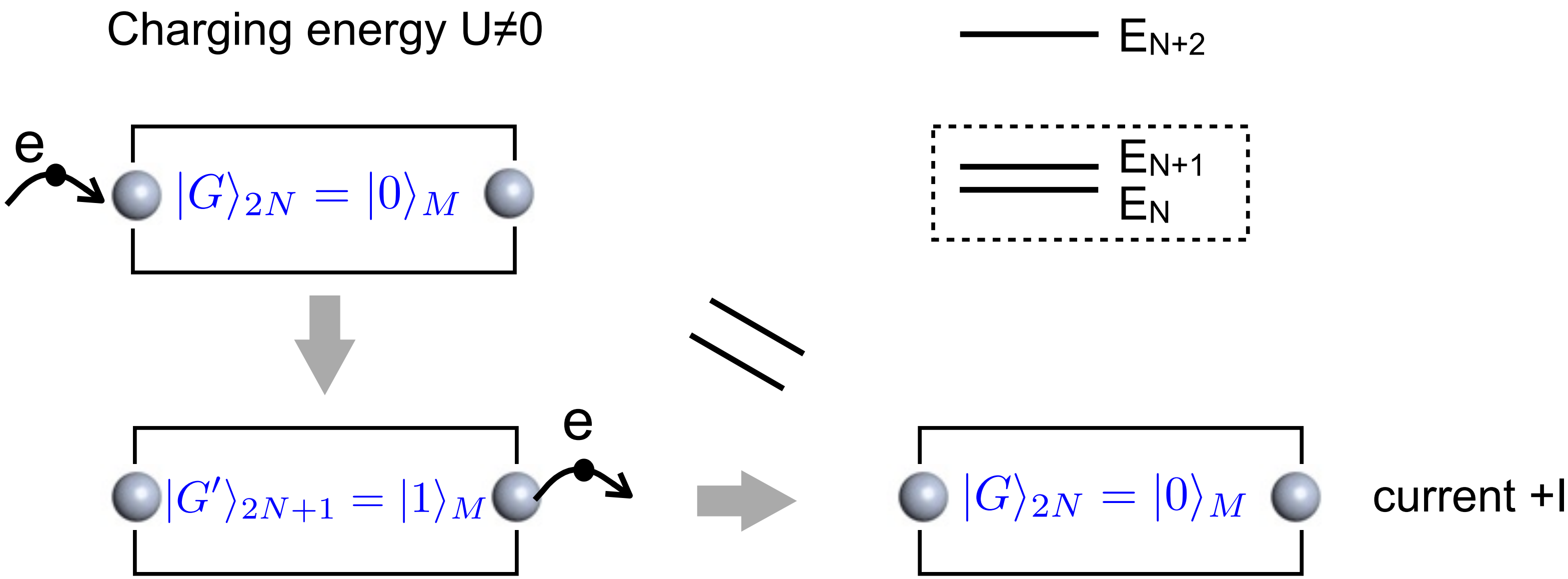
Weak tunneling and small bias limit (universal regime):



Charging energy removes degeneracy between different charge states in S, suppresses Andreev reflection and thus results in a nonzero conductance.

Tunneling Process

Weak tunneling and small bias limit (universal regime):



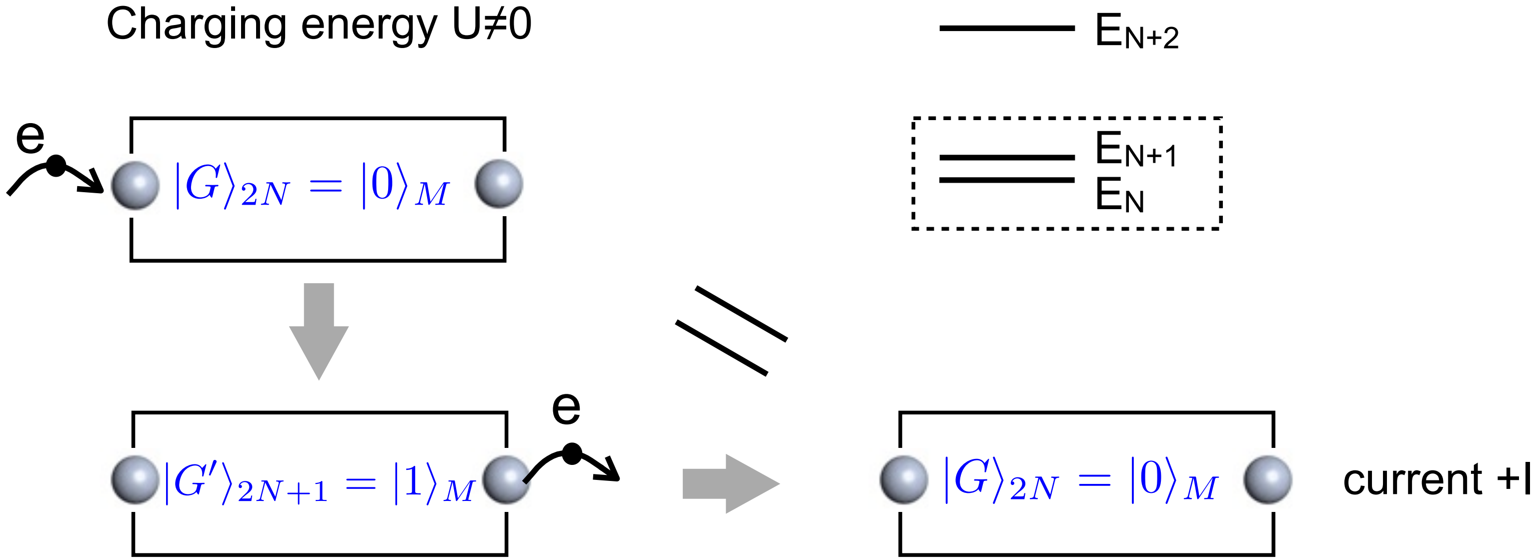
After completing a charge transfer via Majorana modes, the superconductor restores to the same ground state, because *two Majorana modes "share" one quantum state.*

This nonlocality enables electron to be added and subsequently removed from two ends of superconductor without leaving trace behind => elastic process

Tunneling at Off-Resonance

Weak tunneling and small bias limit (universal regime):

Charging energy U≠0



for bias smaller than detuning, $eV < E_{N+1} - E_N$

from second-order perturbation theory:

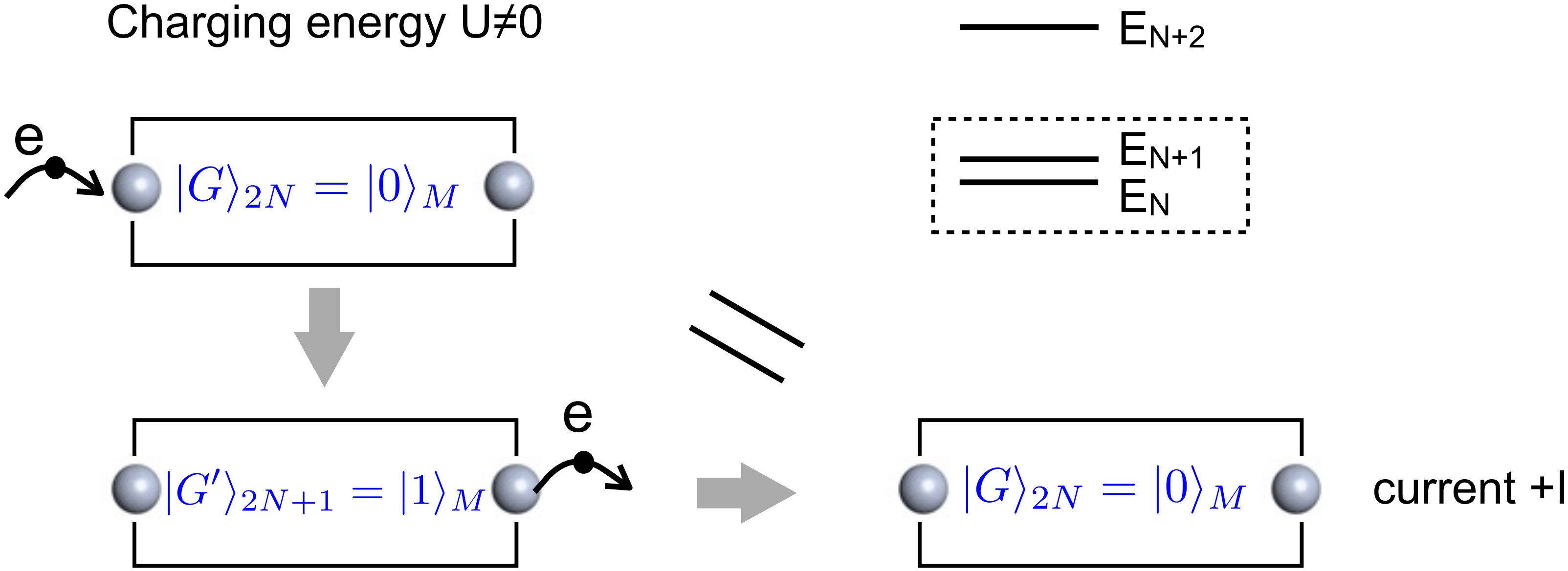
transmission amplitude = $\pm i\lambda_1\lambda_2(u_2^* u_1)$

Note the sign depends on Majorana qubit (here coincides with electron number parity)

Tunneling at Off-Resonance

Weak tunneling and small bias limit (universal regime):

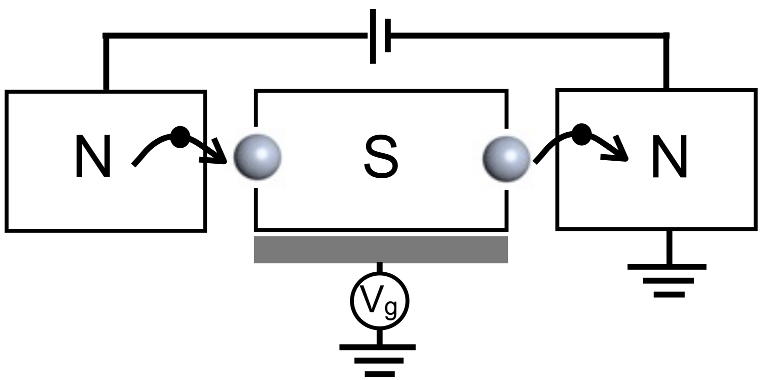
Charging energy U≠0

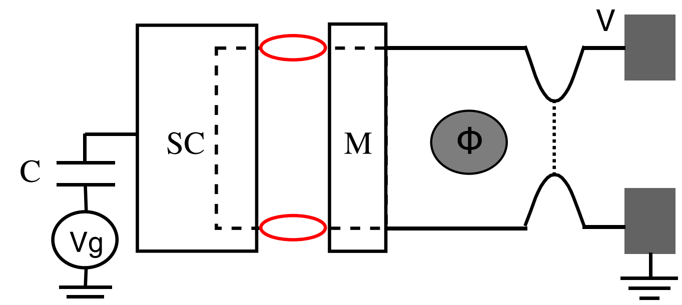


The problem of tunneling through two Majorana modes is mapped to tunneling through a single energy level.

because the two many-body ground states involved differ by charge one and opposite fermion parity

Quantum Teleportation via Majorana Modes





- two-terminal transport is phase-coherent

- conductance reaches e^2/h for symmetric resonant tunneling

- conductance and transmission phase shift is independent of what's inside S, such as distance between Majorana modes, fermion bath...

- phase shift (measured by interference) changes by π when Majorana qubit flips: measures fusion outcome of two Majorana modes *without moving them*.

direct consequences of nonlocal nature of Majorana state

feasible in quantum spin Hall state and nanowire.