Transport in non-Fermi liquids

Theory Winter School
National High Magnetic Field Laboratory, Tallahassee

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Talk online: sachdev.physics.harvard.edu

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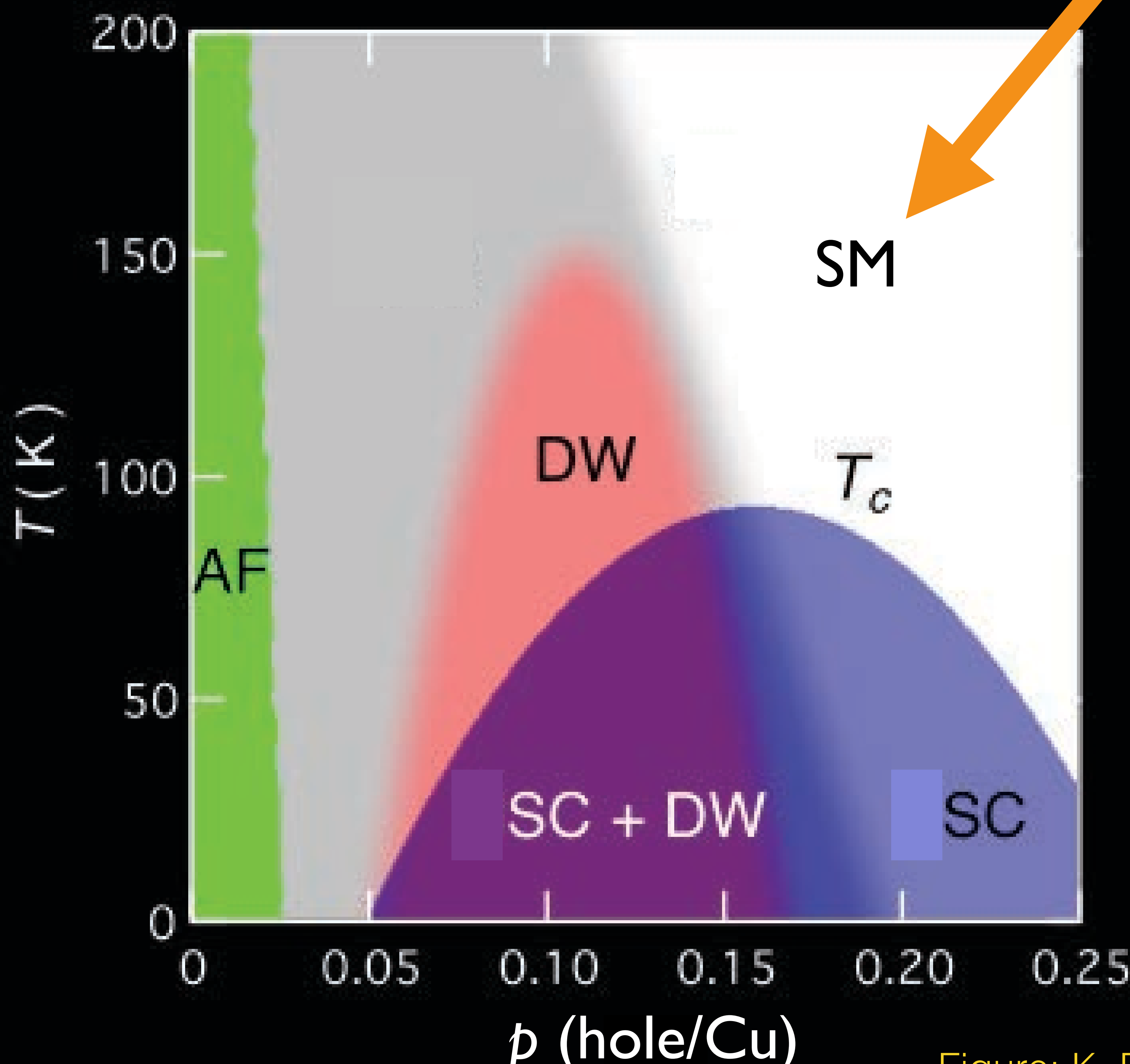
Quantum matter without quasiparticles

Strange metal



Entangled electrons lead to "strange" temperature dependence of resistivity and other properties

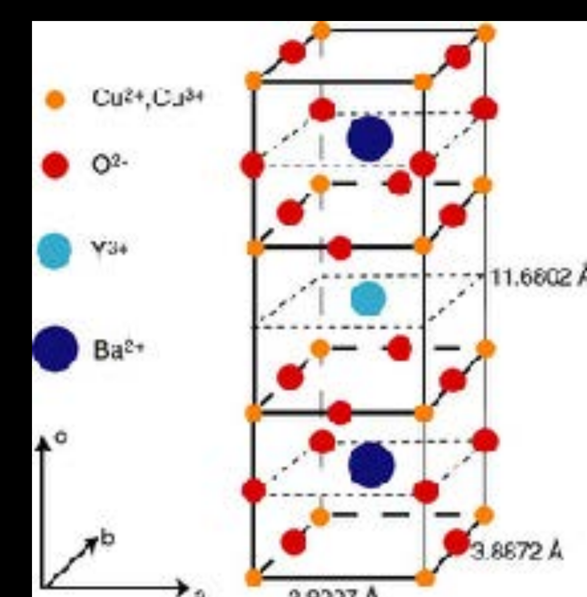
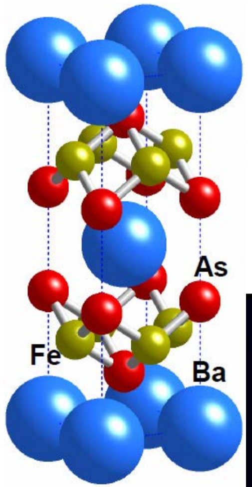
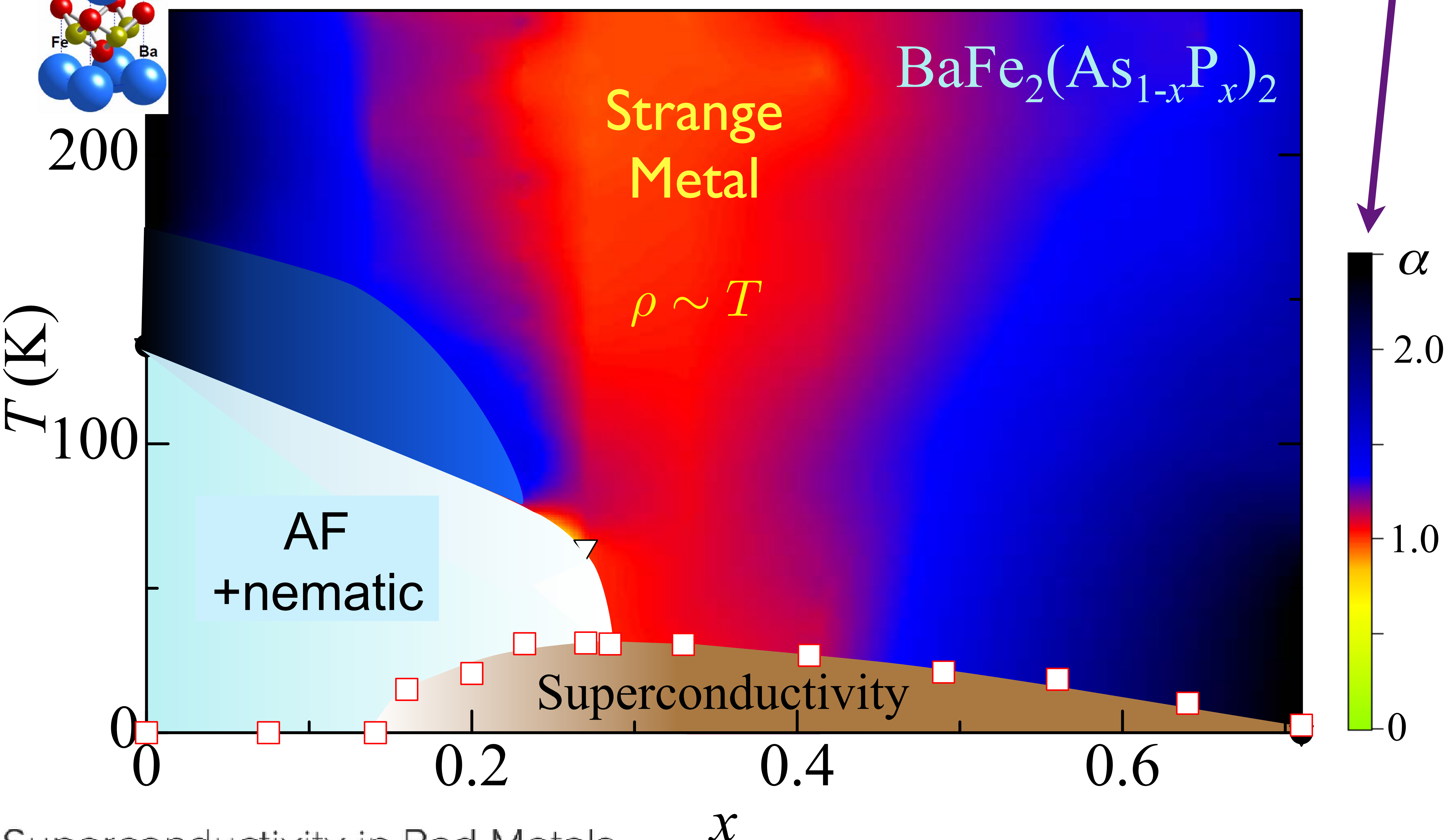


Figure: K. Fujita and J. C. Seamus Davis



Quantum matter without quasiparticles

Resistivity $\sim \rho_0 + AT^\alpha$



Superconductivity in Bad Metals

V. J. Emery and S. A. Kivelson

Phys. Rev. Lett. **74**, 3253 – Published 17 April 1995

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *PRB* **81**, 184519 (2010)

Ubiquitous

“Strange”,

“Bad”,

or “Incoherent”,

metal has a resistivity, ρ, which obeys

$$\rho \sim T,$$

and

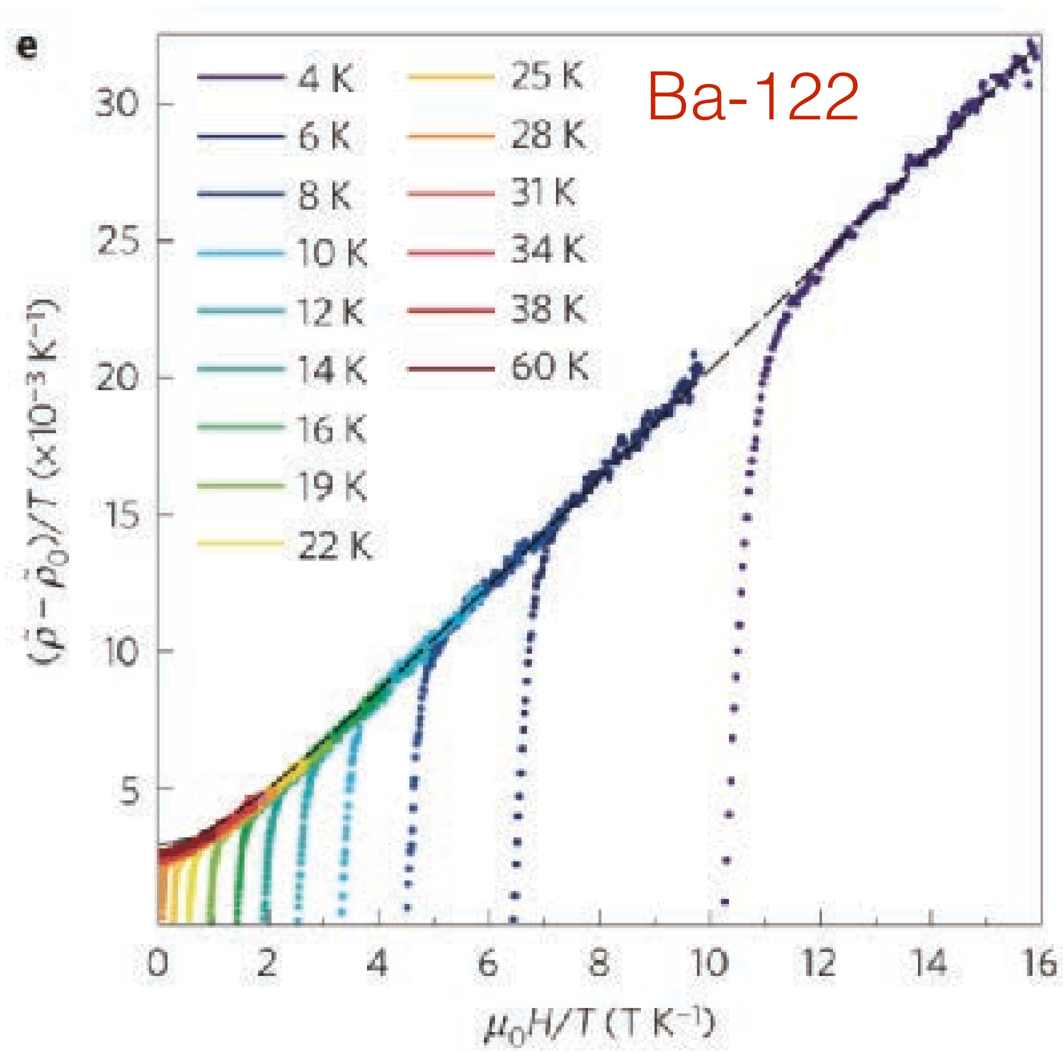
in some cases $\rho \gg h/e^2$

(in two dimensions),

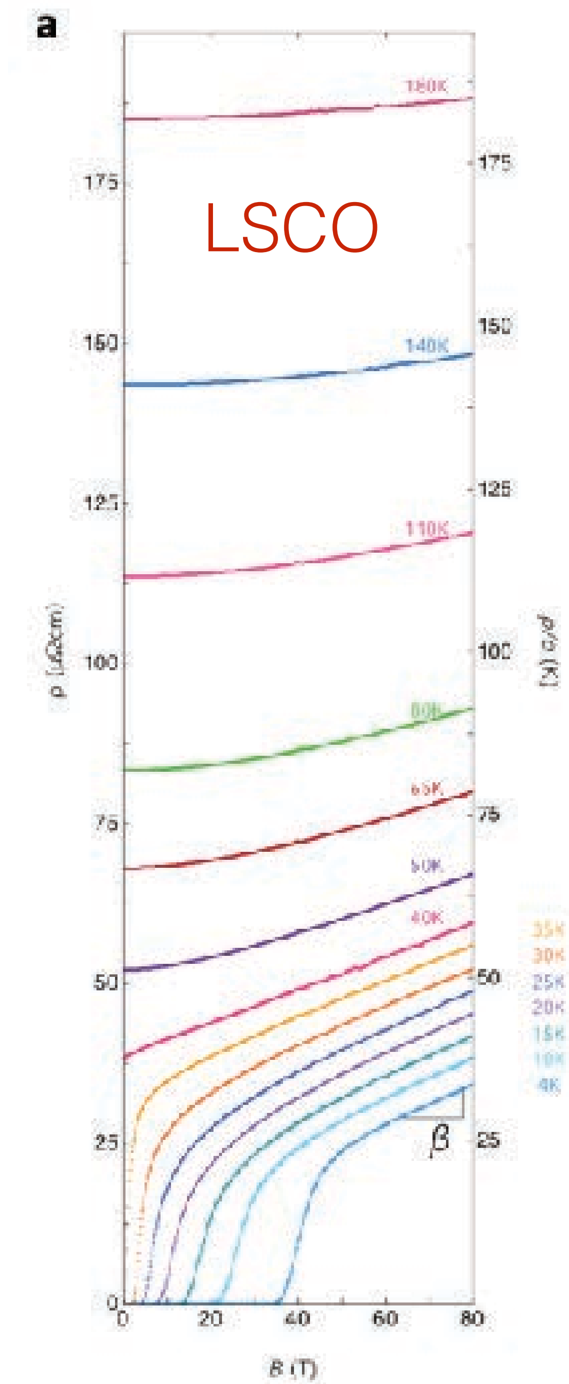
where h/e^2 is the quantum unit of resistance.

Strange metals just got stranger…

B-linear magnetoresistance!?



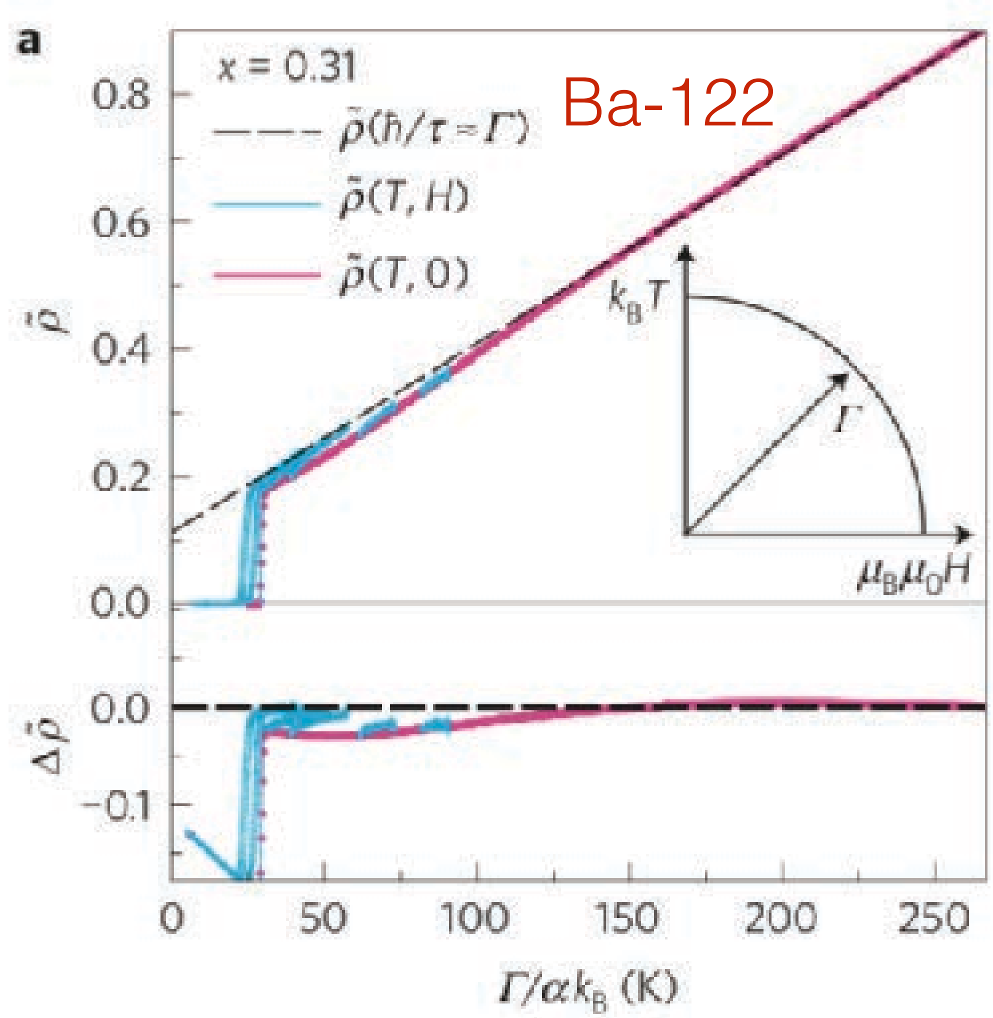
I. M. Hayes et. al., Nat. Phys. 2016



P. Giraldo-Gallo et. al., arXiv:1705.05806

Strange metals just got stranger…

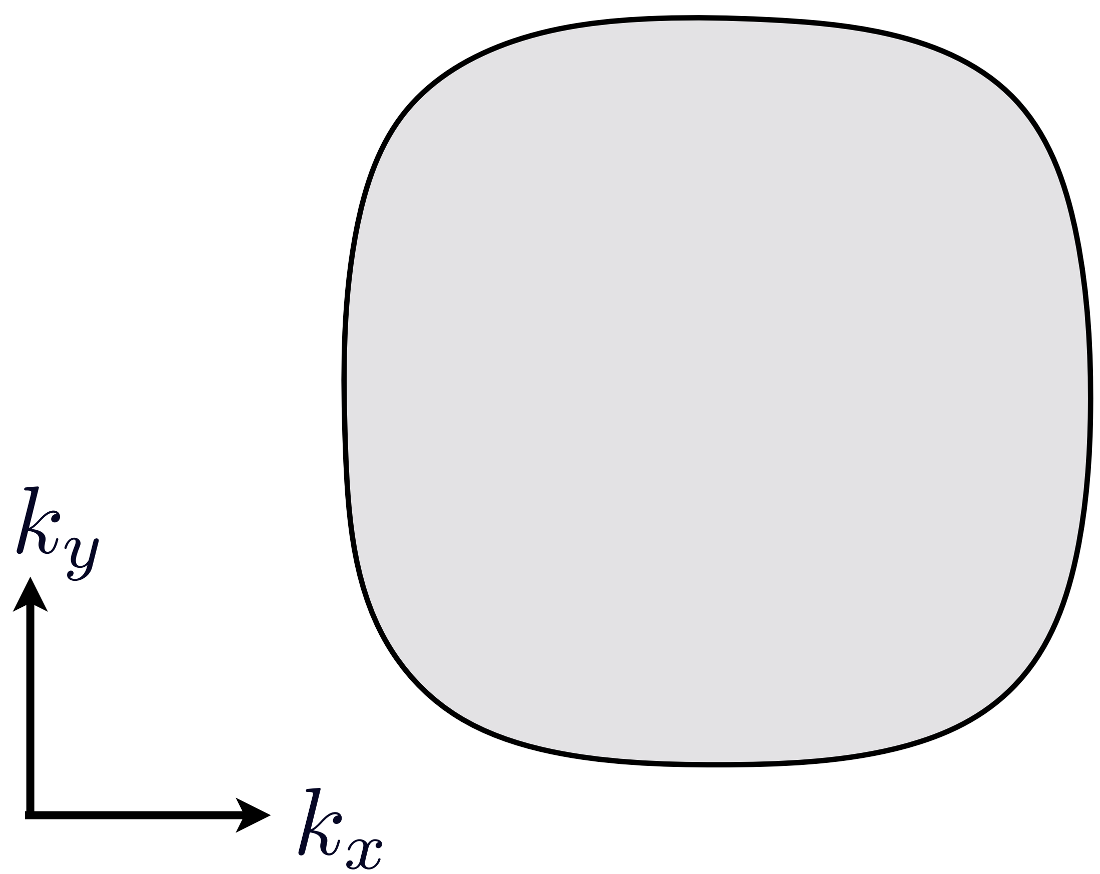
Scaling between B and T !?



$$\rho(H,T) - \rho(0,0) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2} \equiv \Gamma$$

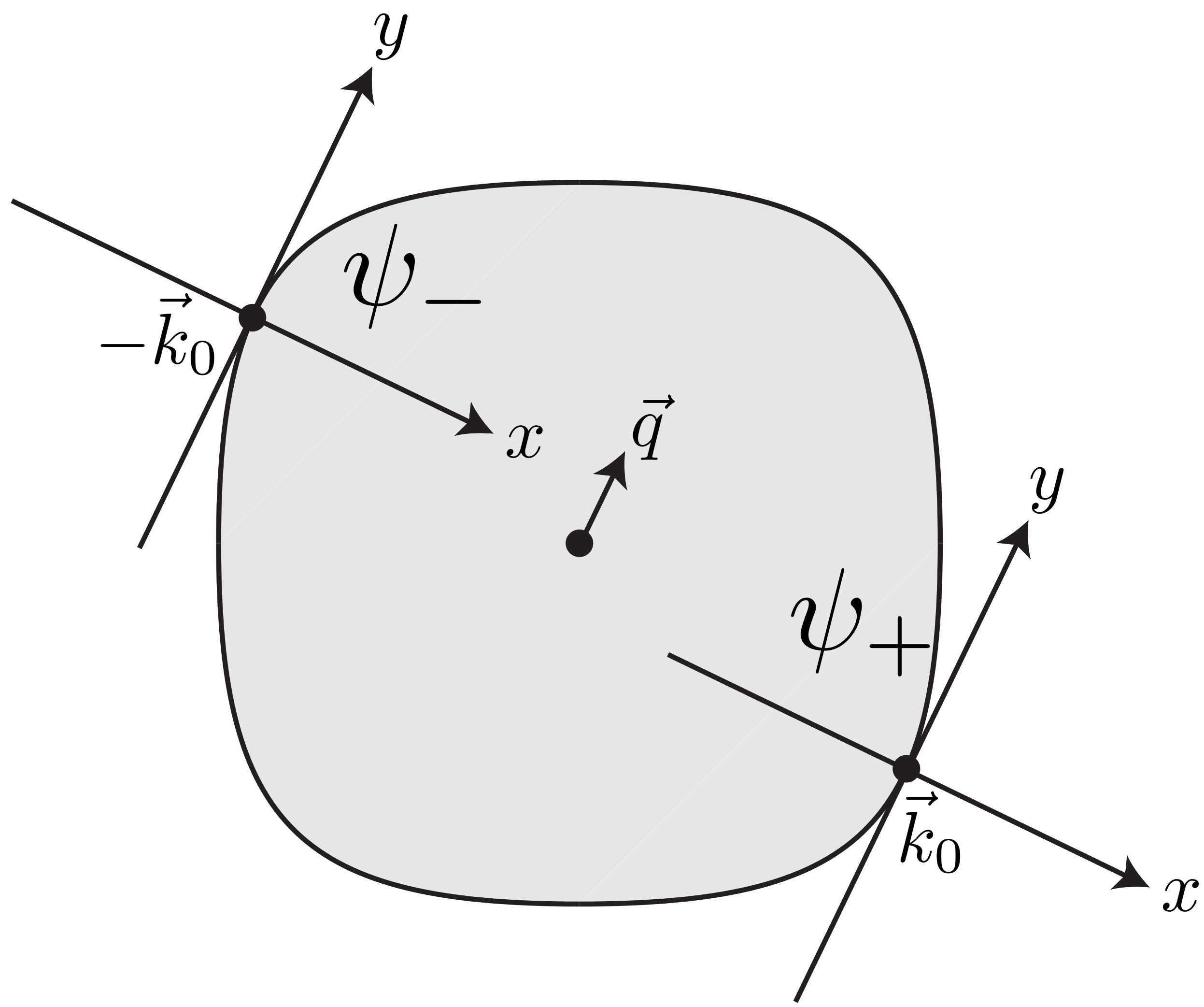
I. M. Hayes et. al., Nat. Phys. 2016

Fermi surface coupled to a gauge field



$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

Fermi surface coupled to a gauge field



$$\mathcal{L}[\psi_\pm, a] =$$

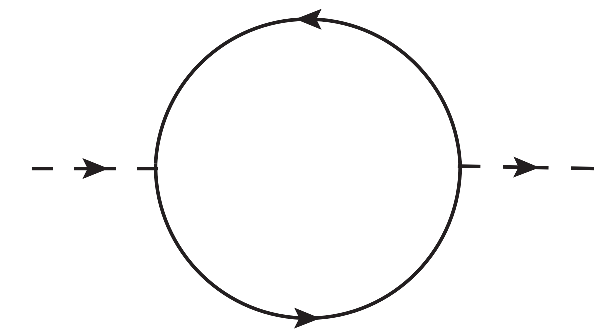
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_-$$

$$-a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Fermi surface coupled to a gauge field

One loop photon self-energy with N_f fermion flavors:

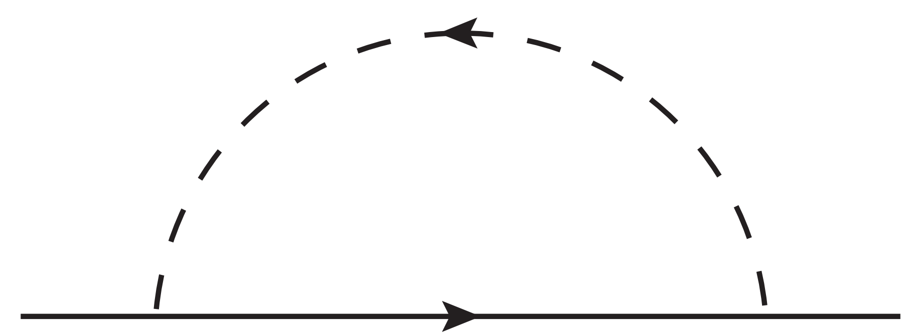


$$\Sigma_\phi(\vec{q},\omega) = N_f \int \frac{d^2k}{4\pi^2}\frac{d\Omega}{2\pi}\frac{1}{\left[-i(\Omega+\omega)+k_x+q_x+(k_y+q_y)^2\right]\left[-i\Omega-k_x+k_y^2\right]}$$

$$= \frac{N_f}{4\pi}\frac{|\omega|}{|q_y|}$$

Landau-damping

Electron self-energy at order $1/N_f$:



$$\Sigma(\vec{k},\Omega) = -\frac{1}{N_f}\int\frac{d^2q}{4\pi^2}\frac{d\omega}{2\pi}\frac{1}{\left[-i(\omega+\Omega)+k_x+q_x+(k_y+q_y)^2\right]\left[\frac{q_y^2}{g^2}+\frac{|\omega|}{|q_y|}\right]}$$

$$= -i\frac{2}{\sqrt{3}N_f}\left(\frac{g^2}{4\pi}\right)^{2/3}\mathrm{sgn}(\Omega)|\Omega|^{2/3}$$

No quasiparticles

- Breakdown of quasiparticles requires strong coupling to a low energy collective mode

- In all known cases, we can write down the singular processes in terms of a continuum field theory of the fermions near the Fermi surface coupled to the collective mode.

- In all known cases, the continuum critical theory has a conserved total (pseudo-) momentum, $\vec{P}$, which commutes with the Hamiltonian. This momentum may not be equal to the crystal momentum of the underlying lattice model.

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- As long as $\chi_{\vec{J},\vec{P}} \neq 0$ (where $\vec{J}$ is the electrical current) the d.c. resistivity of the critical theory is exactly zero. This is the case even though the electron self energy can be highly singular and there are no fermionic quasiparticles (many well-known papers on non-Fermi liquid transport ignore this point.)

- We need to include additional (dangerously) irrelevant umklapp corrections to obtain a non-zero resistivity. Because these additional corrections are irrelevant, it is difficult to see how they can induce a linear-in-T resistivity.

Theories of metallic states without quasiparticles in the presence of disorder

- Well-known perturbative theory of disordered metals has 2 classes of known fixed points, the insulator at strong disorder, and the metal at weak disorder. The latter state has long-lived, extended quasiparticle excitations (which are not plane waves).

- **Needed: a metallic fixed point at intermediate disorder and strong interactions without quasiparticle excitations.** Although disorder is present, it largely self-averages at long scales.

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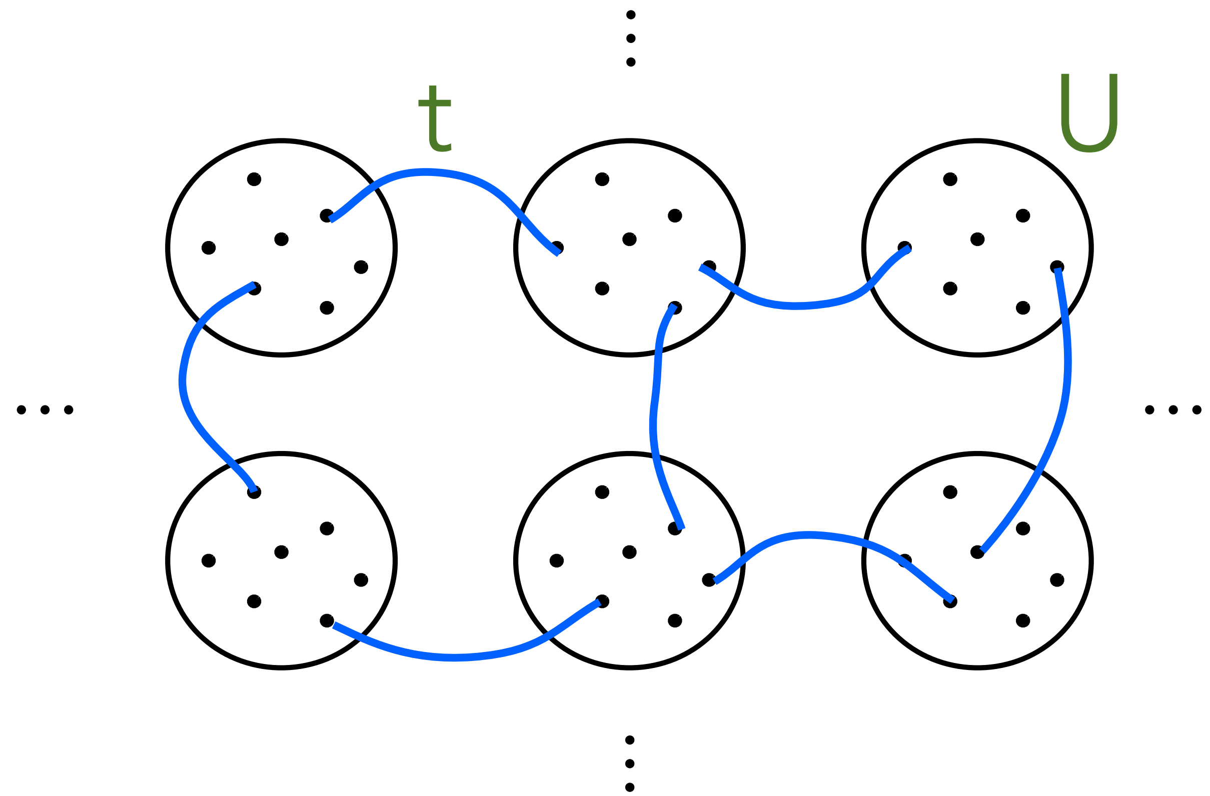
- **Needed: a metallic fixed point at intermediate disorder and strong interactions without quasiparticle excitations.** Although disorder is present, it largely self-averages at long scales.

- SYK models

arXiv:1705.00117 See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models

Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents



$$H = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c^{\dagger}_{ix} c^{\dagger}_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^{\dagger}_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

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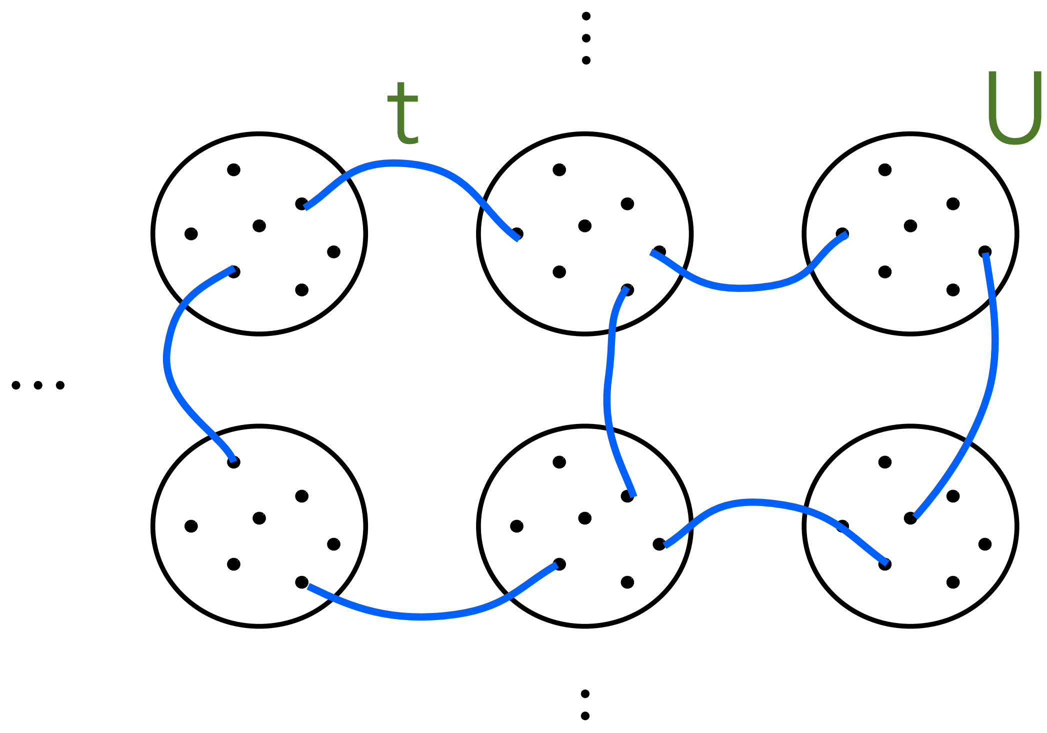
Self-consistent equations



$$G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - z t_0^2 G(i\omega_n),$$

$$\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau),$$

arXiv:1705.00117

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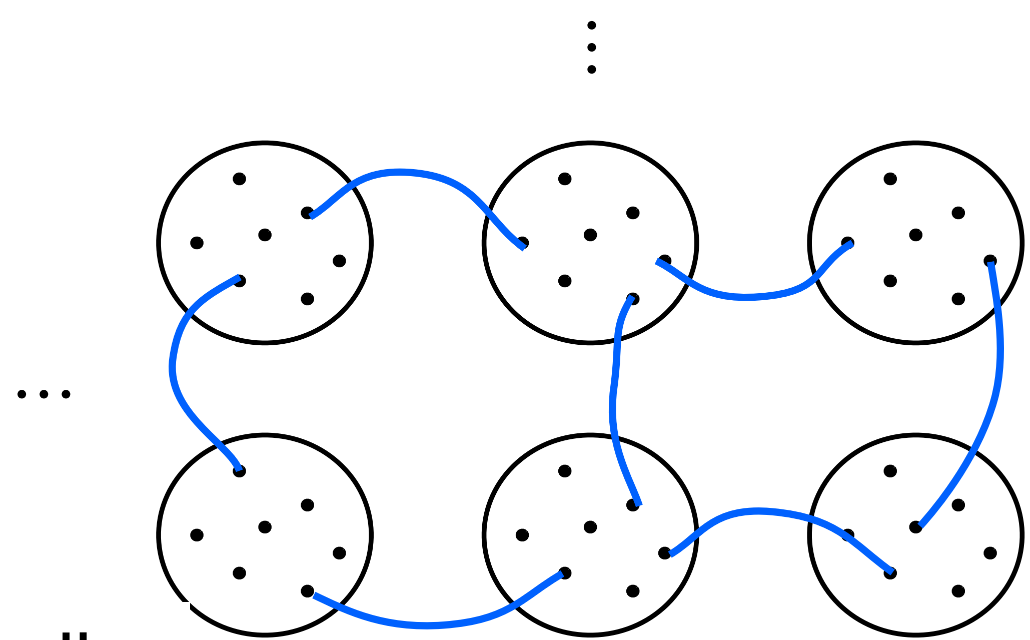
Coherence scale



$$G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - z t_0^2 G(i\omega_n),$$

$$\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau),$$

$$\bar{G}(i\bar{\omega}) = \tilde{t} G(i\omega)$$

Rescaling

$$\bar{\omega} = \frac{\omega}{\tilde{E}_c}, \quad \bar{\tau} = \tau \tilde{E}_c, \qquad \bar{G}(i\bar{\omega}) = \tilde{t} G(i\omega) \qquad \bar{\Sigma}(i\bar{\omega}) = \Sigma(i\omega)/\tilde{t} \qquad \tilde{t} = \left(\frac{z}{2}\right)^{\frac{1}{2}} t$$

$$\bar{G}(i\bar{\omega}) = \frac{\tilde{t}}{U} i\bar{\omega} - \bar{\Sigma}(i\bar{\omega}) \qquad \bar{\Sigma}(\bar{\tau}) = -\bar{G}(\bar{\tau})^2 \bar{G}(-\bar{\tau}) + 2\bar{G}(\bar{\tau}),$$

For t≪U, a single universal coherence scale appears $\tilde{E}_c = \frac{\tilde{t}^2}{U}$

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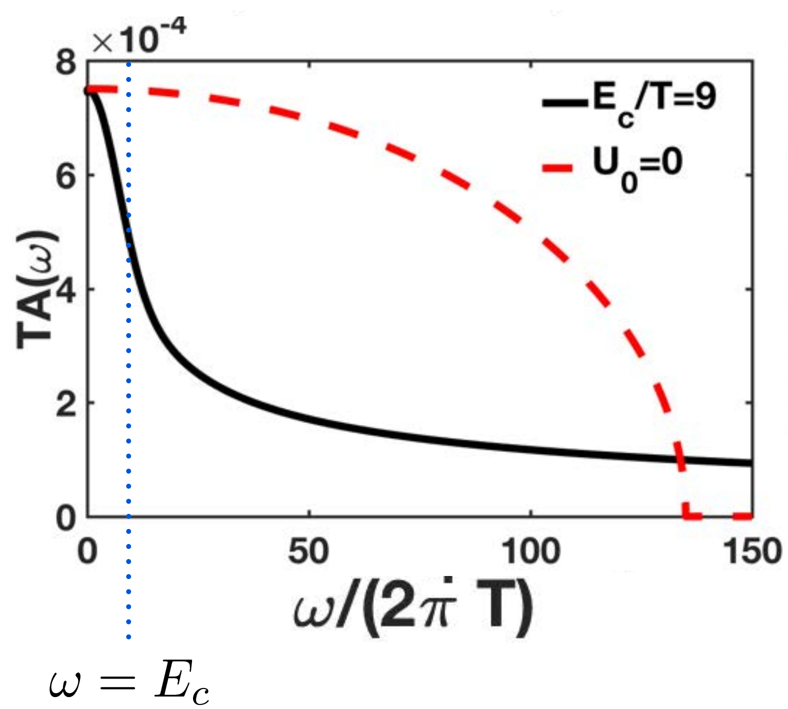
Coherence scale

We solve these equations in a real time Keldysh formulation numerically and determine asymptotics analytically.



Narrow "coherence peak" appears in spectral function: heavy quasiparticles form for $T \ll E_c$

Quasiparticle weight $Z \sim t/U$

arXiv:1705.00117

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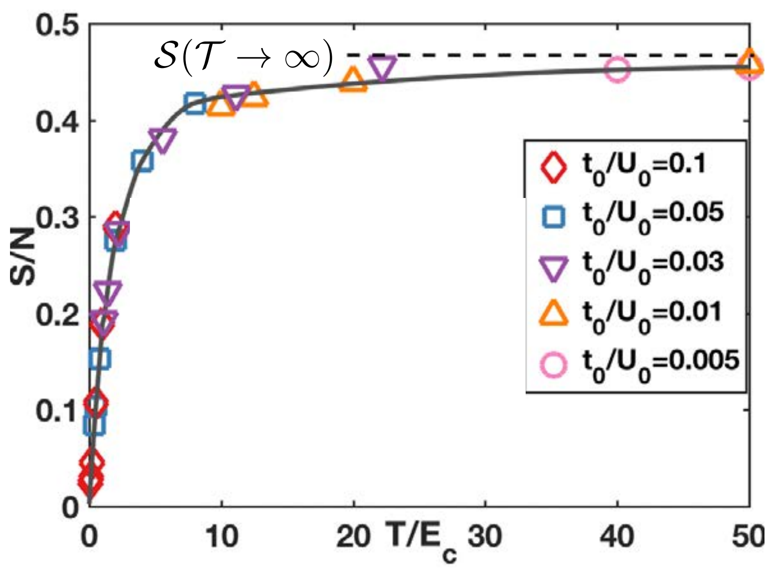
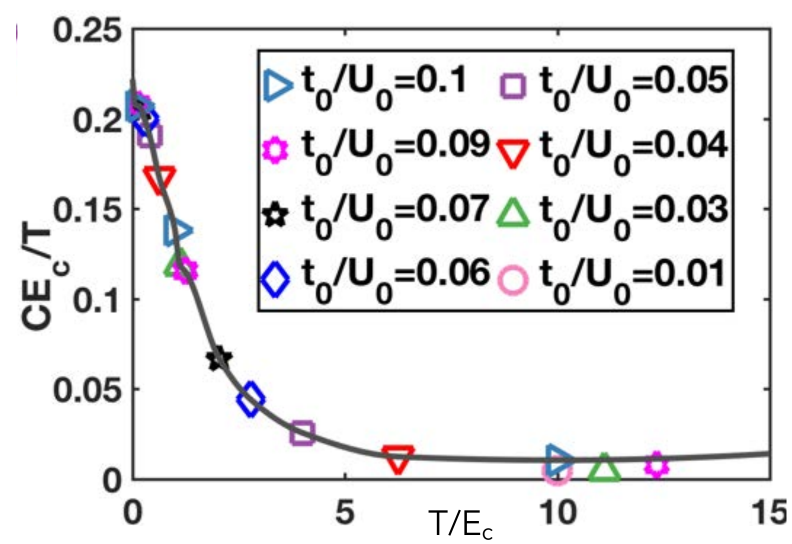
Entropy

Level repulsion: entropy is released for $T<E_c$!





Universal scaling forms

$$S/N = \mathcal{S}(T/E_c)$$

$$C/N = T/E_c \mathcal{S}'(T/E_c)$$

$$\gamma \equiv \lim_{T\to 0} \frac{C}{T} = \frac{\mathcal{S}'(0)}{E_c}$$

Sommerfeld enhancement

$$m^*/m \sim U/t$$

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Conductivity

From the Kubo formula, we have the conductivity

$$\mathrm{Re}[\sigma(\omega)] \propto t_0^2 \int d\Omega \frac{f(\omega+\Omega) - f(\Omega)}{\omega} A(\Omega) A(\omega+\Omega)$$

where $A(\omega) = \mathrm{Im}[G^R(\omega)]$.
At $T > E_c$ this yields

$$\sigma \sim \frac{e^2}{h} \frac{t_0^2}{U} \frac{1}{T}$$

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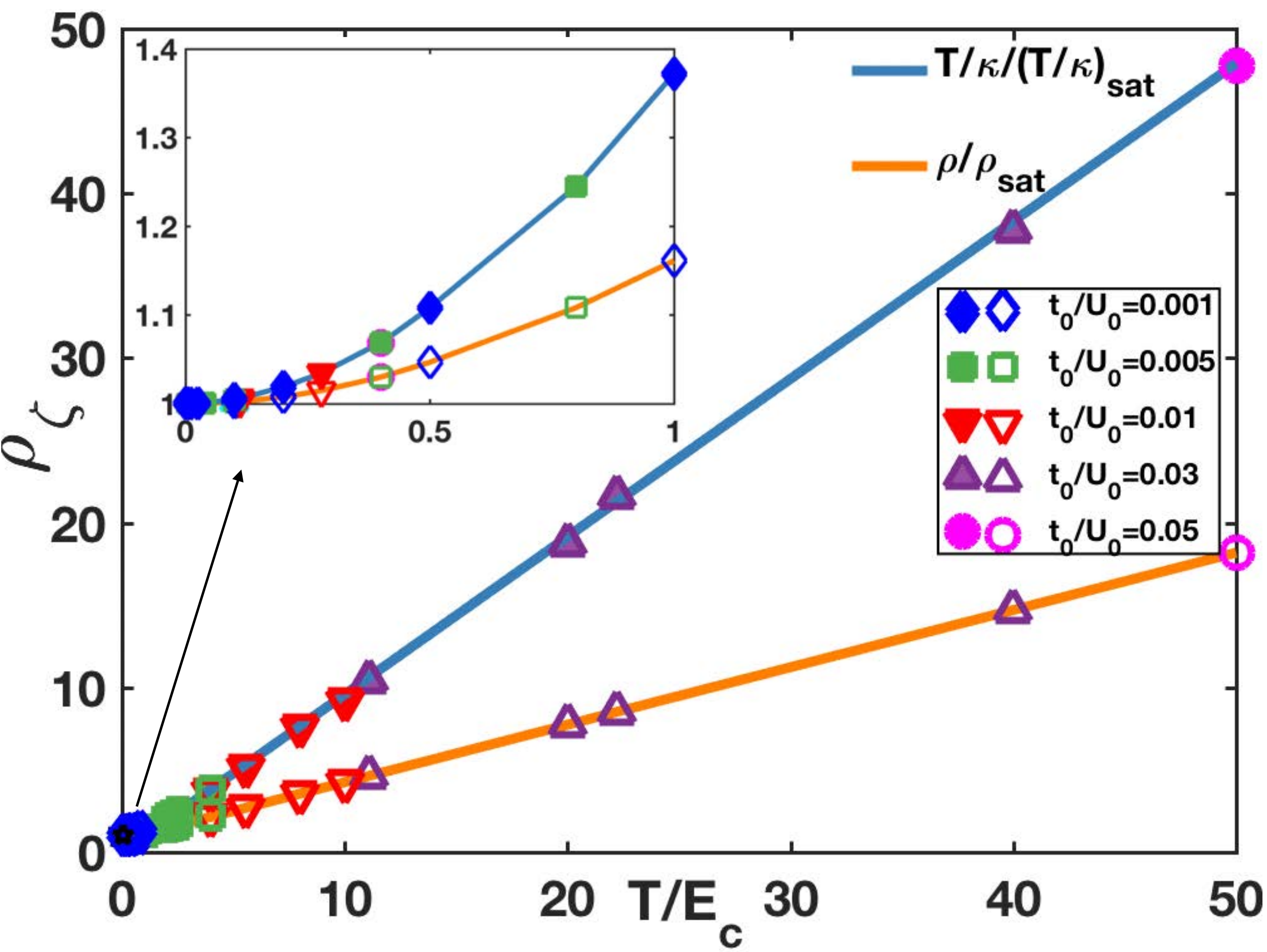
Low 'coherence' scale

$$E_c \sim \frac{t_0^2}{U}$$



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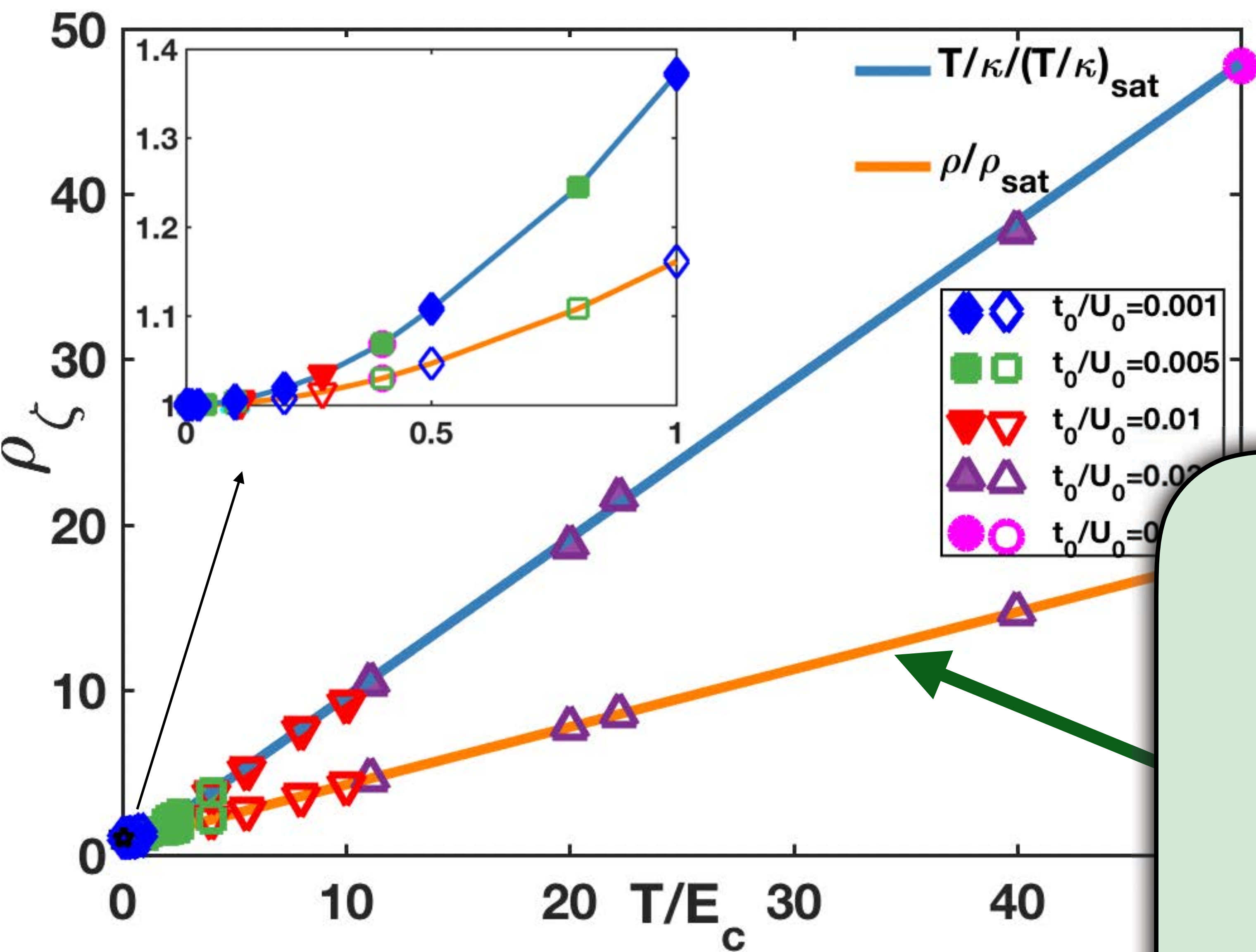

For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right) \,, \qquad s = s_0$$

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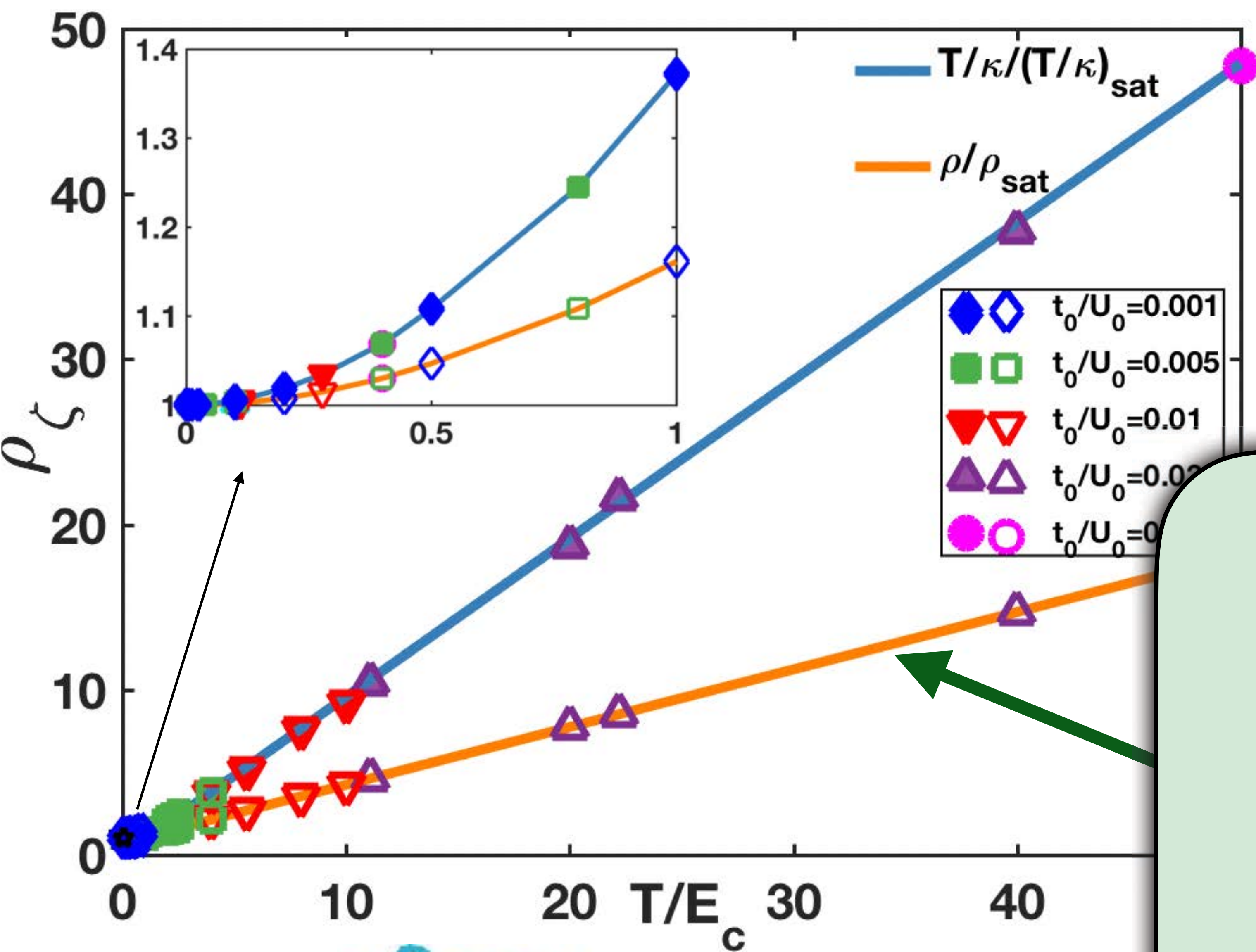

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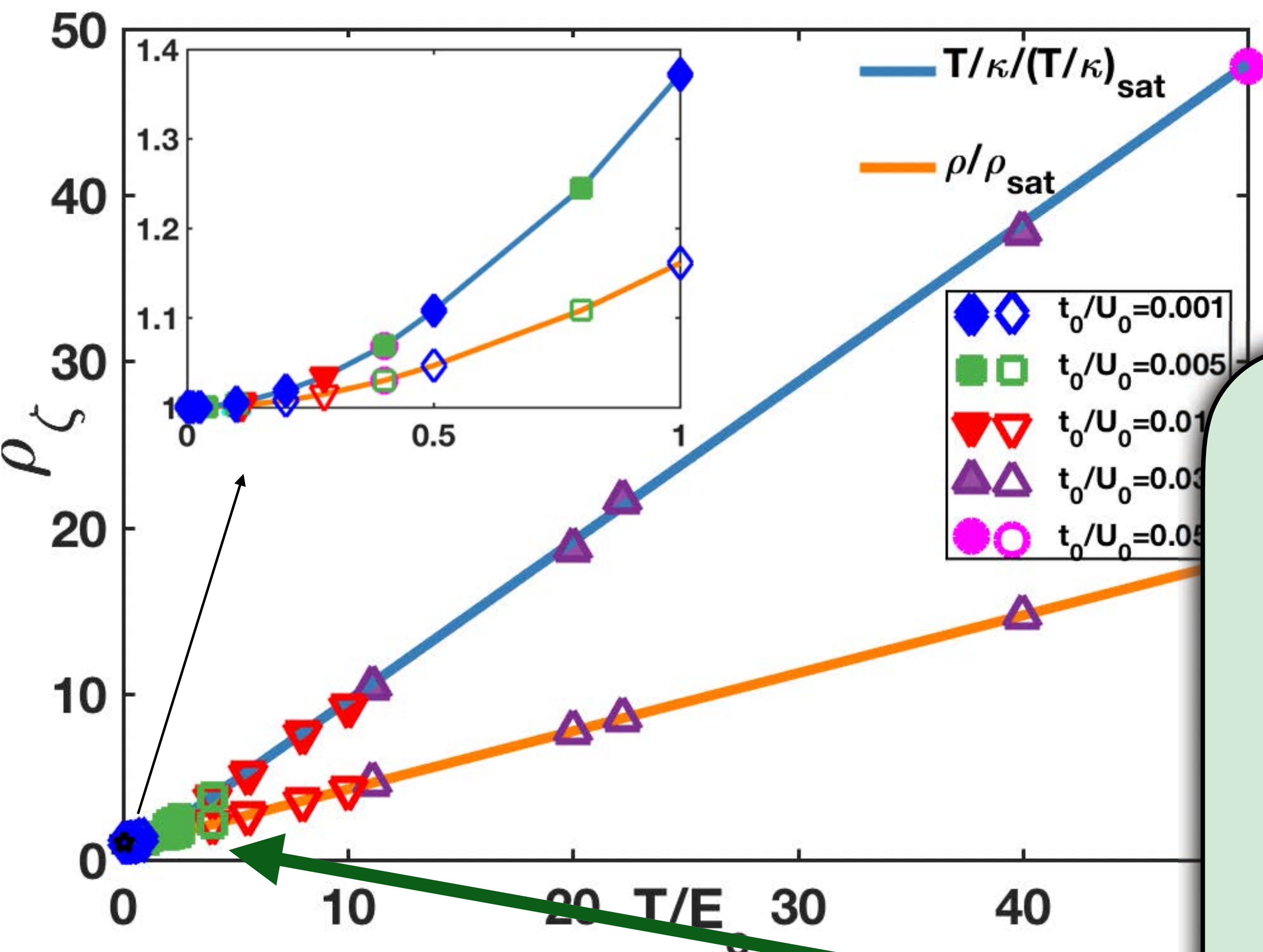


For $T < E_c$, the resistivity, ρ, and entropy density, s, are

$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

$$s \sim s_0\left(\frac{T}{E_c}\right)$$

arXiv:1712.05026

Title: Magnetotransport in a model of a disordered strange metal

Authors: Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev

Aavishkar Patel

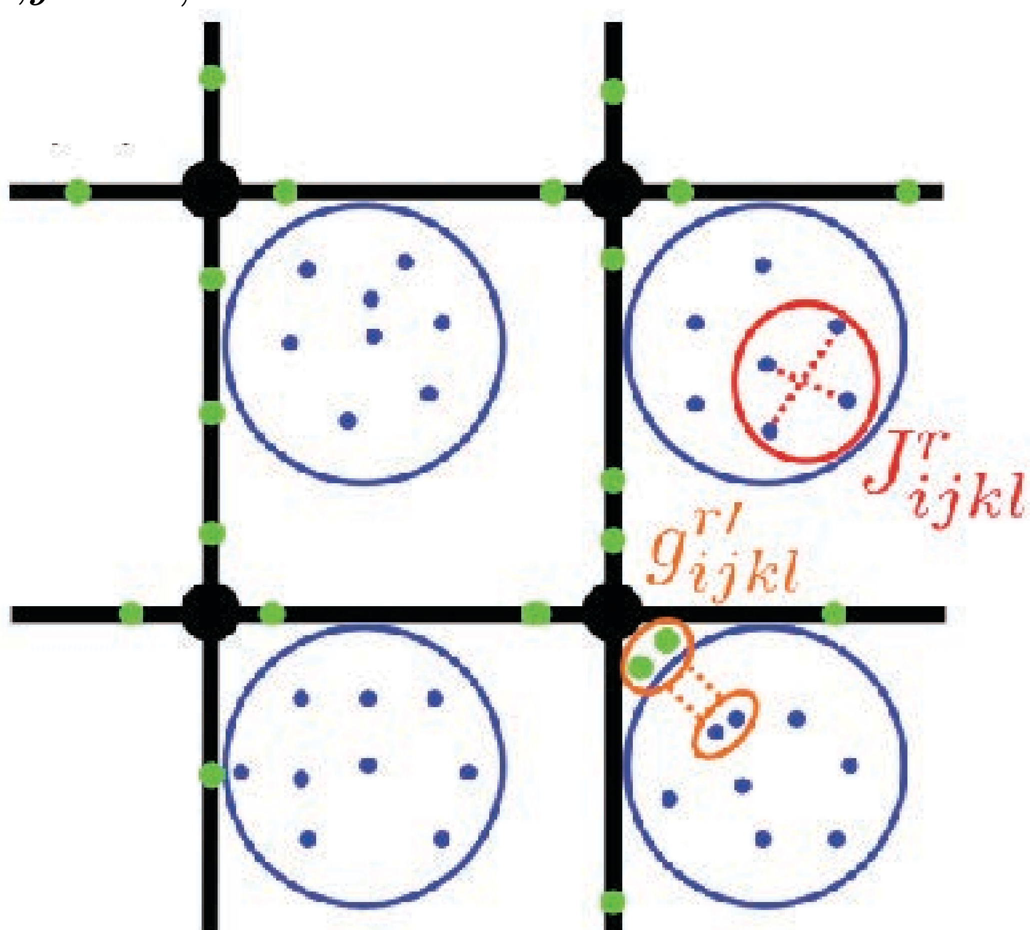
Infecting a Fermi liquid and making it SYK

Mobile electrons (c, green) interacting with SYK quantum dots (f, blue) with exchange interactions.

This yields the first model agreeing with magnetotransport in strange metals !

$$H = -t \sum_{\langle rr' \rangle;\ i=1}^{M} (c_{ri}^\dagger c_{r'i} + \text{h.c.}) - \mu_c \sum_{r;\ i=1}^{M} c_{ri}^\dagger c_{ri} - \mu \sum_{r;\ i=1}^{N} f_{ri}^\dagger f_{ri}$$

$$+ \frac{1}{NM^{1/2}} \sum_{r;\ i,j=1}^{N} \sum_{k,l=1}^{M} g_{ijkl}^{r} f_{ri}^\dagger f_{rj} c_{rk}^\dagger c_{rl} + \frac{1}{N^{3/2}} \sum_{r;\ i,j,k,l=1}^{N} J_{ijkl}^{r} f_{ri}^\dagger f_{rj}^\dagger f_{rk} f_{rl}.$$



A. A. Patel, J. McGreevy, D. P. Arovas and S. Sachdev, arXiv: 1712.05026

Infecting a Fermi liquid and making it SYK

$$\Sigma(\tau - \tau') = -J^2 G^2(\tau - \tau') G(\tau' - \tau) - \frac{M}{N} g^2 G(\tau - \tau') G^c(\tau - \tau') G^c(\tau' - \tau),$$

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)},$$

(*f* electrons)

$$\Sigma^c(\tau - \tau') = -g^2 G^c(\tau - \tau') G(\tau - \tau') G(\tau' - \tau),$$

$$G^c(i\omega_n) = \sum_k \frac{1}{i\omega_n - \epsilon_k + \mu_c - \Sigma^c(i\omega_n)}.$$

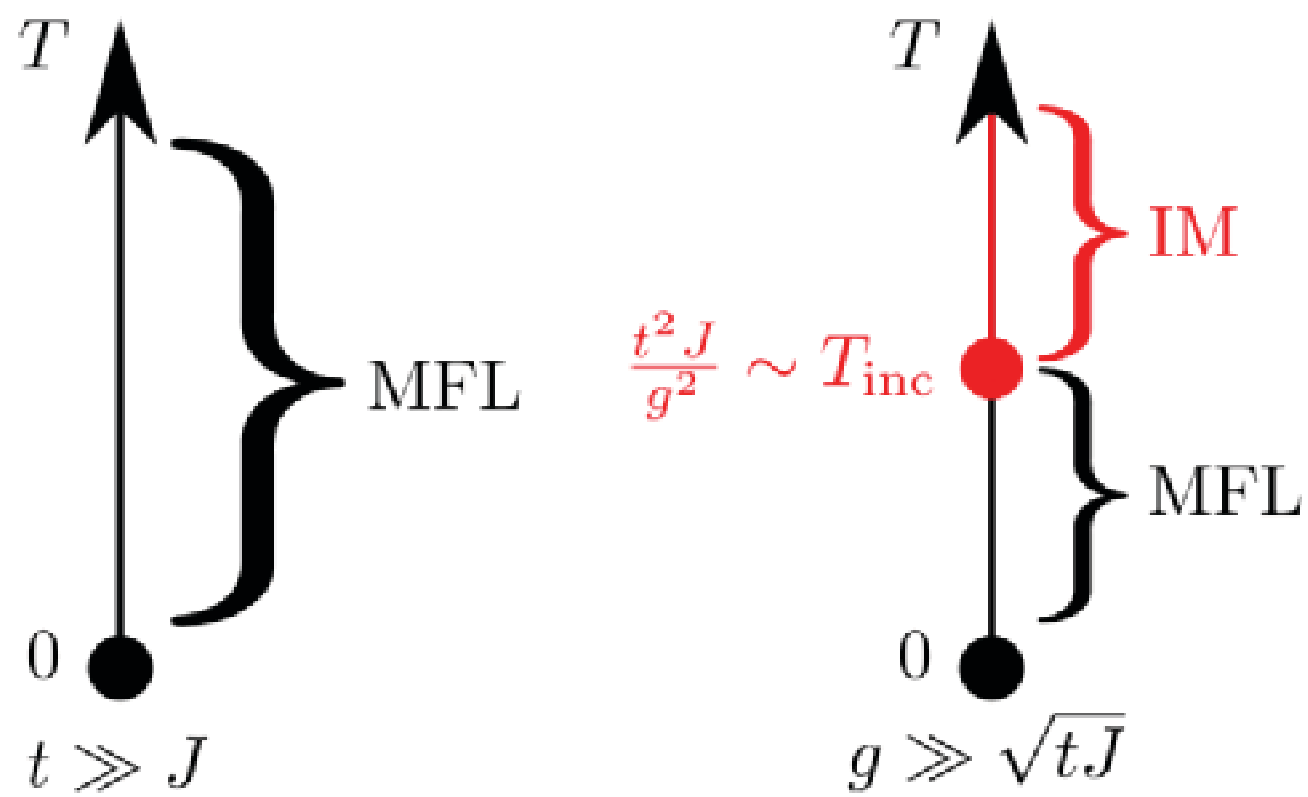
(*c* electrons)

Exactly solvable in the large *N*,*M* limits!

- Low-*T* phase: *c* electrons form a Marginal Fermi-liquid (MFL), *f* electrons are local SYK models

Infecting a Fermi liquid and making it SYK

- High-*T* phase: *c* electrons form an "incoherent metal" (IM), with local Green's function, and no notion of momentum; *f* electrons remain local SYK models

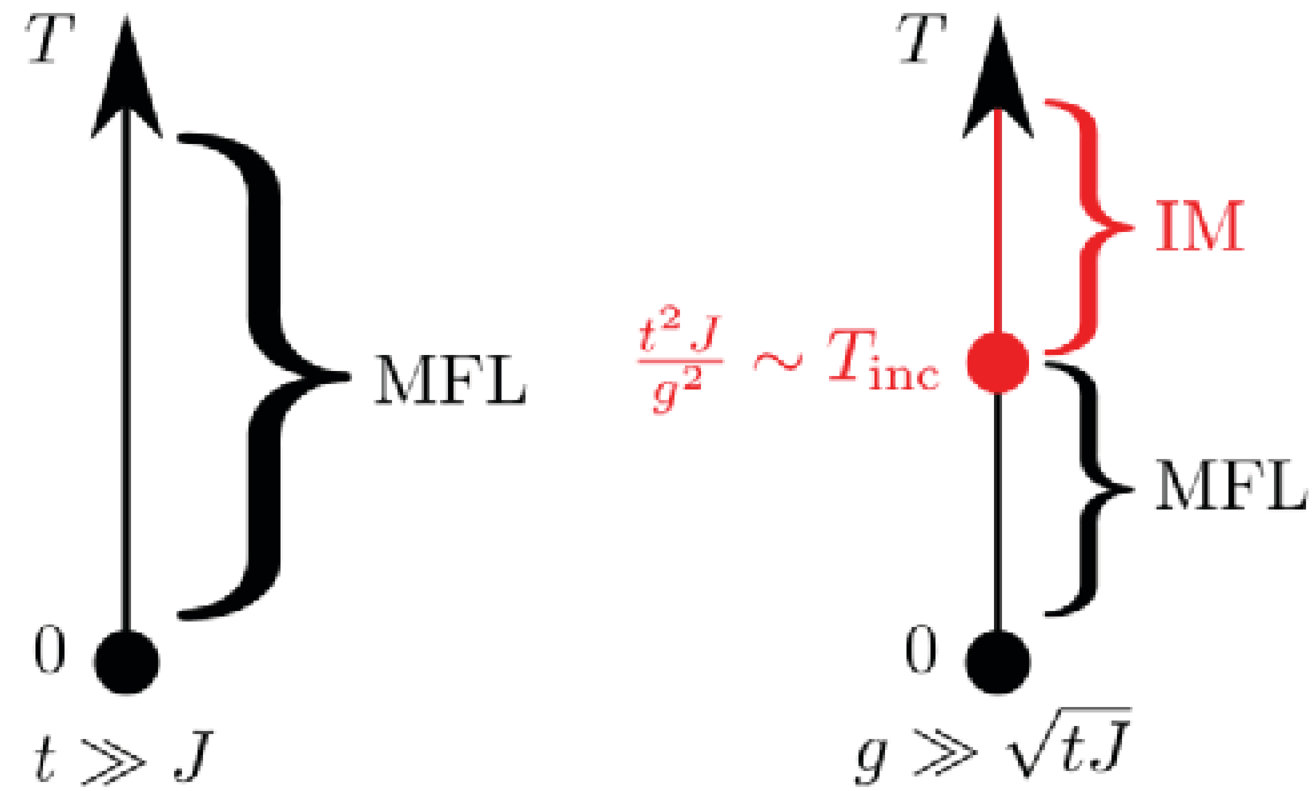


$$G^c(\tau) = -\frac{C_c}{\sqrt{1+e^{-4\pi\mathcal{E}_c}}}\left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2} e^{-2\pi\mathcal{E}_c T\tau},$$

$$G(\tau) = -\frac{C}{\sqrt{1+e^{-4\pi\mathcal{E}}}}\left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2} e^{-2\pi\mathcal{E}T\tau}, \quad 0 \le \tau < \beta$$

Infecting a Fermi liquid and making it SYK

- Low-T phase: c electrons form a Marginal Fermi-liquid (MFL), f electrons are local SYK models



$$\Sigma^c(i\omega_n) = \frac{ig^2\nu(0)T}{2J\cosh^{1/2}(2\pi\mathcal{E})\pi^{3/2}}\left(\frac{\omega_n}{T}\ln\left(\frac{2\pi T e^{\gamma_E-1}}{J}\right)+\frac{\omega_n}{T}\psi\left(\frac{\omega_n}{2\pi T}\right)+\pi\right),$$

$$\Sigma^c(i\omega_n) \to \frac{ig^2\nu(0)}{2J\cosh^{1/2}(2\pi\mathcal{E})\pi^{3/2}}\omega_n\ln\left(\frac{|\omega_n|e^{\gamma_E-1}}{J}\right), \quad |\omega_n| \gg T \quad (\nu(0)\sim 1/t)$$

Linear-in-*T* resistivity

Both the MFL and the IM are not translationally-invariant and have linear-in-*T* resistivities!

$$\sigma_0^{\mathrm{MFL}} = 0.120251 \times MT^{-1} J \times \left(\frac{v_F^2}{g^2}\right) \cosh^{1/2}(2\pi\mathcal{E}). \quad (v_F \sim t)$$

$$\sigma_0^{\mathrm{IM}} = (\pi^{1/2}/8) \times MT^{-1} J \times \left(\frac{\Lambda}{\nu(0) g^2}\right) \frac{\cosh^{1/2}(2\pi\mathcal{E})}{\cosh(2\pi\mathcal{E}_c)}.$$

[Can be obtained straightforwardly from Kubo formula in the large-*N,M* limits]

The IM is also a “Bad metal” with $\sigma_0^{\mathrm{IM}} \ll 1$

Magnetotransport: Marginal-Fermi liquid

- Thanks to large *N,M*, we can also exactly derive the linear-response Boltzmann equation for non-quantizing magnetic fields…

$$(1-\partial_\omega \mathrm{Re}[\Sigma^c_R(\omega)])\partial_t \delta n(t,k,\omega) + v_F \hat{k}\cdot \mathbf{E}(t)\; n_f'(\omega) + v_F(\hat{k}\times \mathcal{B}\hat{z})\cdot \nabla_k \delta n(t,k,\omega) = 2\delta n(t,k,\omega)\mathrm{Im}[\Sigma^c_R(\omega)],$$

$(\mathcal{B} = eBa^2/\hbar)$ (i.e. flux per unit cell)

$$\sigma_L^{\mathrm{MFL}} = M\frac{v_F^2\nu(0)}{16T}\int_{-\infty}^{\infty}\frac{dE_1}{2\pi}\mathrm{sech}^2\left(\frac{E_1}{2T}\right)\frac{-\mathrm{Im}[\Sigma^c_R(E_1)]}{\mathrm{Im}[\Sigma^c_R(E_1)]^2+(v_F/(2k_F))^2\mathcal{B}^2},$$

$$\sigma_H^{\mathrm{MFL}} = -M\frac{v_F^2\nu(0)}{16T}\int_{-\infty}^{\infty}\frac{dE_1}{2\pi}\mathrm{sech}^2\left(\frac{E_1}{2T}\right)\frac{(v_F/(2k_F))\mathcal{B}}{\mathrm{Im}[\Sigma^c_R(E_1)]^2+(v_F/(2k_F))^2\mathcal{B}^2}.$$

$$\sigma_L^{\mathrm{MFL}} \sim T^{-1}s_L((v_F/k_F)(\mathcal{B}/T)), \quad \sigma_H^{\mathrm{MFL}} \sim -\mathcal{B}T^{-2}s_H((v_F/k_F)(\mathcal{B}/T)).$$

$$s_{L,H}(x\to\infty)\propto 1/x^2, \quad s_{L,H}(x\to 0)\propto x^0.$$

Scaling between magnetic field and temperature in **orbital** magnetotransport!

Macroscopic magnetotransport in the MFL

- Let us consider the MFL with additional **macroscopic** disorder (charge puddles etc.)

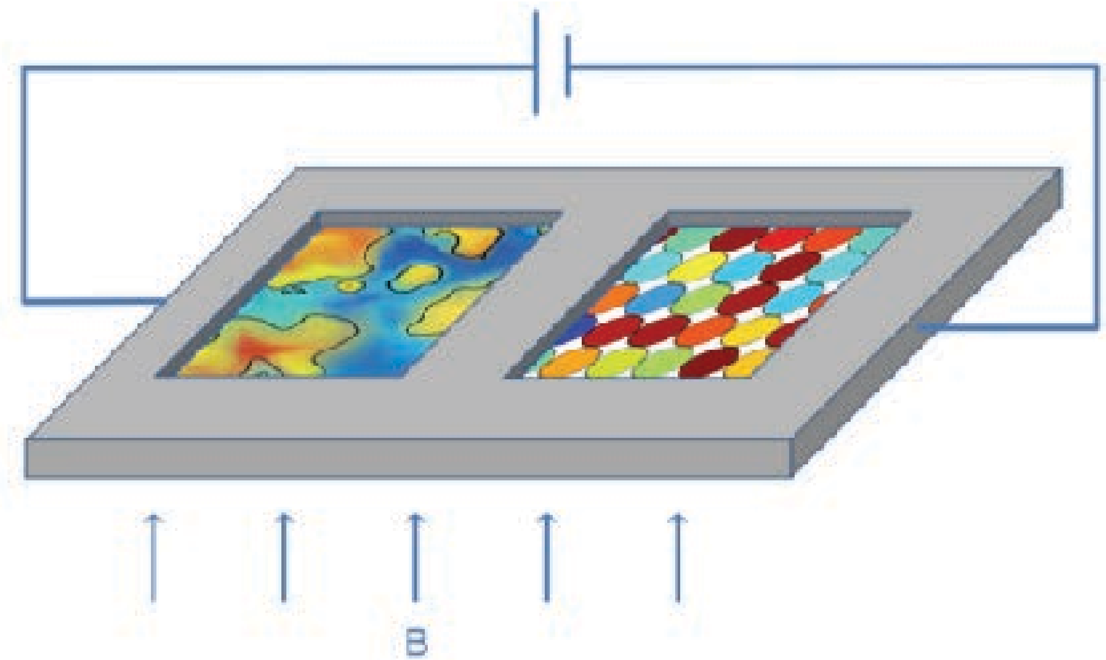


Figure: N. Ramakrishnan et. al., arXiv: 1703.05478

- No macroscopic momentum, so equations describing charge transport are just

$$\nabla \cdot \mathbf{I}(\mathbf{x}) = 0, \quad \mathbf{I}(\mathbf{x}) = \sigma(\mathbf{x}) \cdot \mathbf{E}(\mathbf{x}), \quad \mathbf{E}(\mathbf{x}) = -\nabla\Phi(\mathbf{x}).$$

- Very weak thermoelectricity for large FS, so charge effectively decoupled from heat transport.

Physical picture

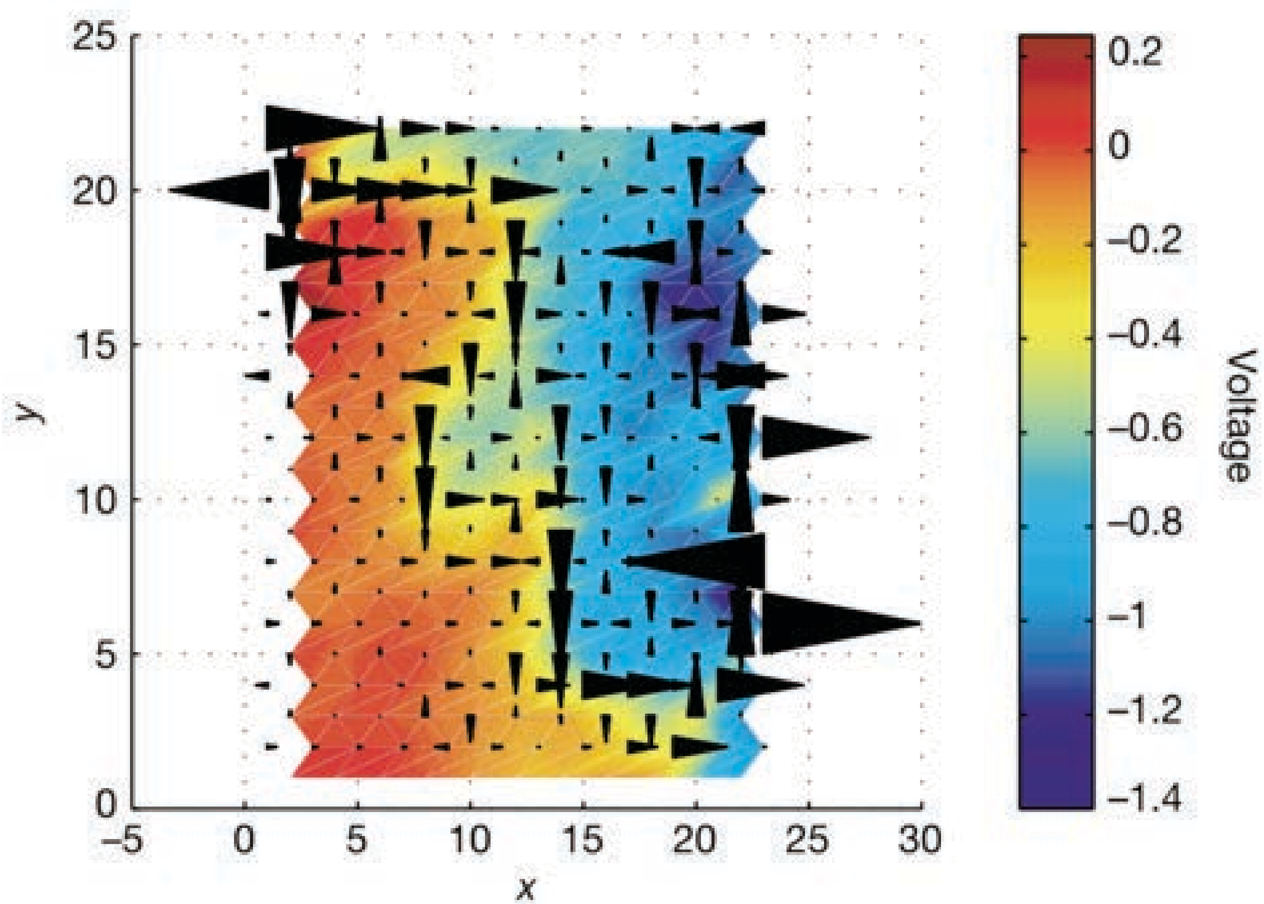
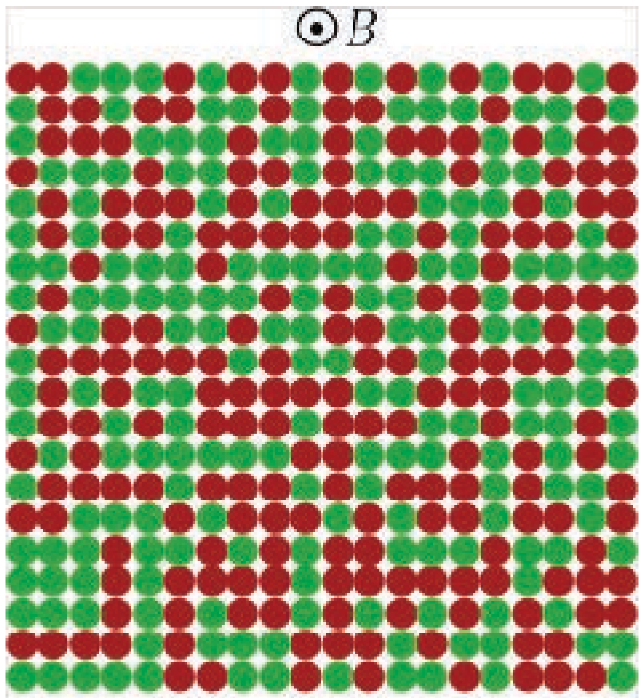


Figure 3 Visualization of currents and voltages at large magnetic field in a 10 × 10 random network of disks with radii 1 (arbitrary units), where the potential difference $U = -1$ V. The black arrows represent the currents, and arrow size depicts the magnitude of the current. The major current path is perpendicular to the applied voltage for a significant proportion of the time, which implies that the magnetoresistance is provided internally by the Hall effect, which is therefore linear in *H*.

- Current path length increases linearly with *B* at large *B* due to local Hall effect, which causes the global resistance to increase linearly with *B* at large *B*.

Exact numerical solution of charge-transport equations in a random-resistor network. (M. M. Parish and P. Littlewood, Nature 426, 162 (2003))

Solvable toy model: two-component disorder



- Two types of domains *a,b* with different carrier densities and lifetimes randomly distributed in approximately equal fractions over sample.
- Effective medium equations can be solved exactly

$$\left(\mathbb{I} + \frac{\sigma^a - \sigma^e}{2\sigma^e_L}\right)^{-1} \cdot (\sigma^a - \sigma^e) + \left(\mathbb{I} + \frac{\sigma^b - \sigma^e}{2\sigma^e_L}\right)^{-1} \cdot (\sigma^b - \sigma^e) = 0.$$

$$\rho^e_L \equiv \frac{\sigma^e_L}{\sigma^{e2}_L + \sigma^{e2}_H} = \frac{\sqrt{(\mathcal{B}/m)^2\left(\gamma_a\sigma^{\mathrm{MFL}}_{0a} - \gamma_b\sigma^{\mathrm{MFL}}_{0b}\right)^2 + \gamma_a^2\gamma_b^2\left(\sigma^{\mathrm{MFL}}_{0a} + \sigma^{\mathrm{MFL}}_{0b}\right)^2}}{\gamma_a\gamma_b(\sigma^{\mathrm{MFL}}_{0a}\sigma^{\mathrm{MFL}}_{0b})^{1/2}\left(\sigma^{\mathrm{MFL}}_{0a} + \sigma^{\mathrm{MFL}}_{0b}\right)},$$

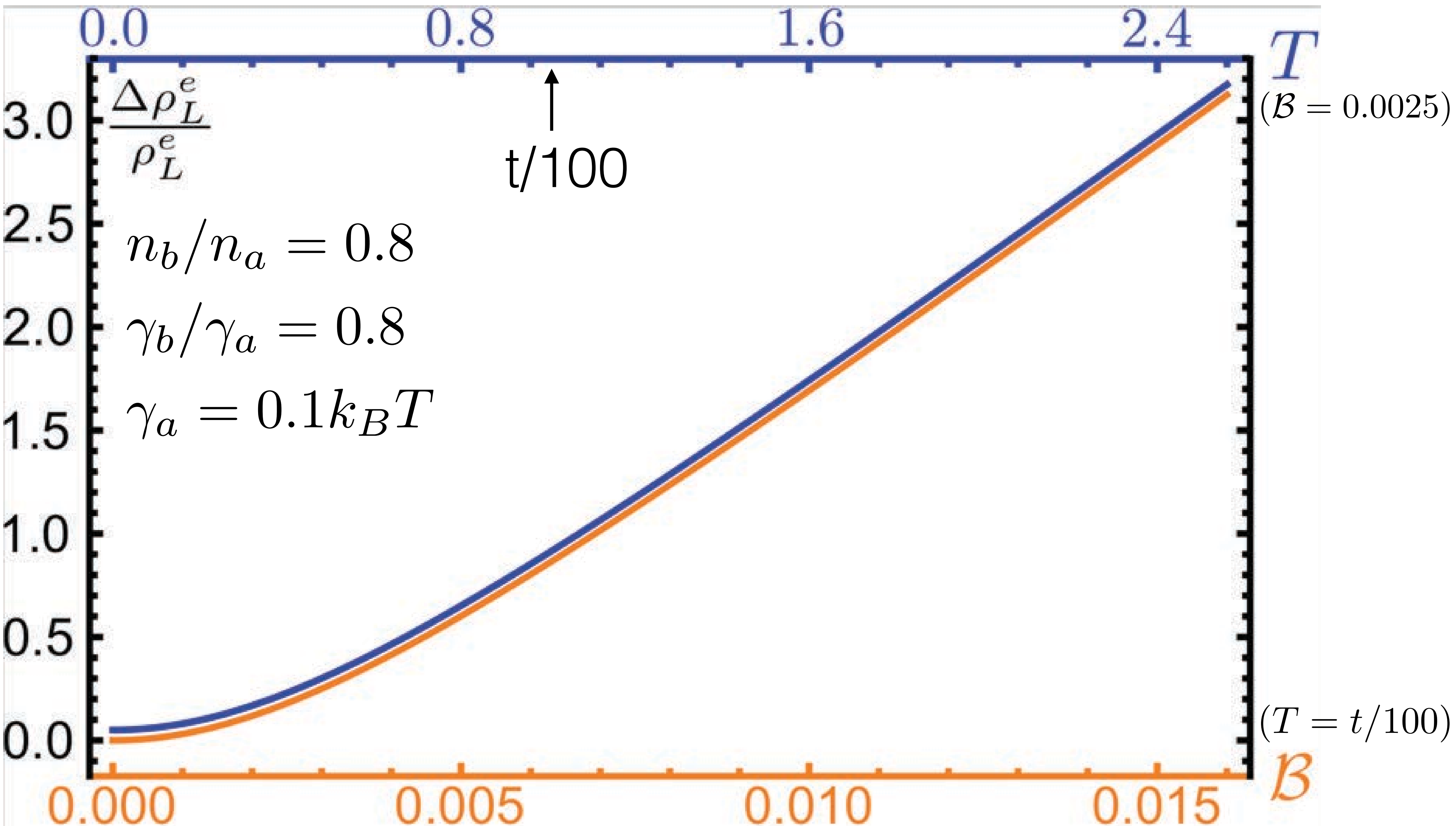
$$\rho^e_H \equiv -\frac{\sigma^e_H/\mathcal{B}}{\sigma^{e2}_L + \sigma^{e2}_H} = \frac{\gamma_a + \gamma_b}{m\gamma_a\gamma_b\left(\sigma^{\mathrm{MFL}}_{0a} + \sigma^{\mathrm{MFL}}_{0b}\right)}. \quad (m = k_F/v_F \sim 1/t)$$

$\gamma_{a,b} \sim T$ (i.e. effective transport scattering rates)

$$\rho^e_L \sim \sqrt{c_1 T^2 + c_2 B^2}$$

Scaling between *B* and *T* at microscopic orbital level has been transferred to global MR!

Scaling between B and T



~ 50 T (a = 3.82 A)

Magnetotransport in strange metals

- Engineered a model of a Fermi surface coupled to SYK quantum dots which leads to a marginal Fermi liquid with a linear-in-T resistance, with a magnetoresistance which scales as $B \sim T$.

- Macroscopic disorder then leads to linear-in-B magnetoresistance, and a combined dependence which scales as $\sim \sqrt{B^2 + T^2}$

- Higher temperatures lead to an incoherent metal with a local Green's function and a linear-in-T resistance, but negligible magnetoresistance.

- This simple two-component model describes a new state of matter which is realized by electrons in the presence of strong interactions and disorder.

- Can such a model be realized as a fixed-point of a generic theory of strongly-interacting electrons in the presence of disorder?

- Can we start from a single-band Hubbard model with disorder, and end up with such two-band fixed point, with emergent local conservation laws?

- Electrons in doped silicon appear to separate into two components: localized spin moments and itinerant electrons

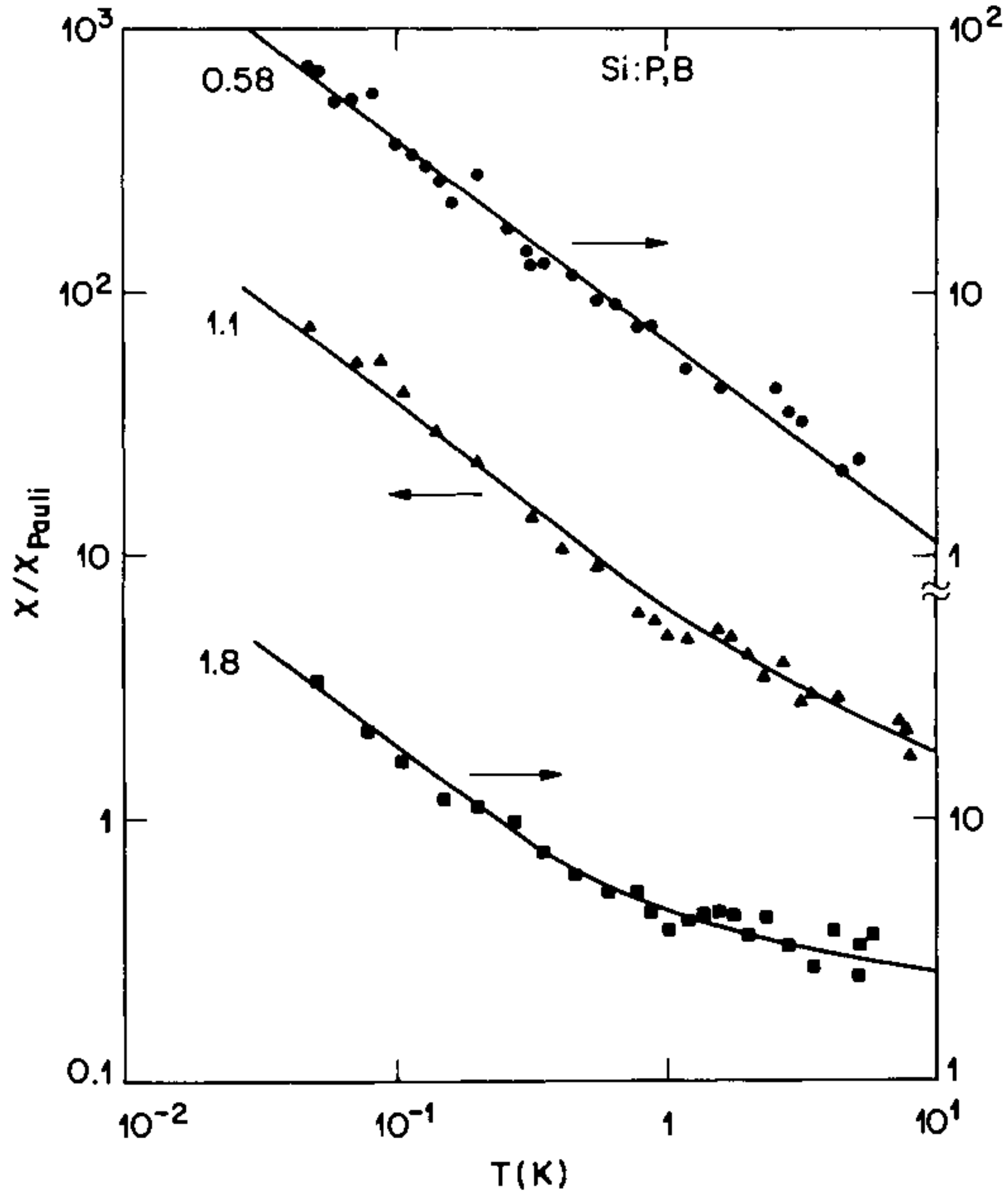
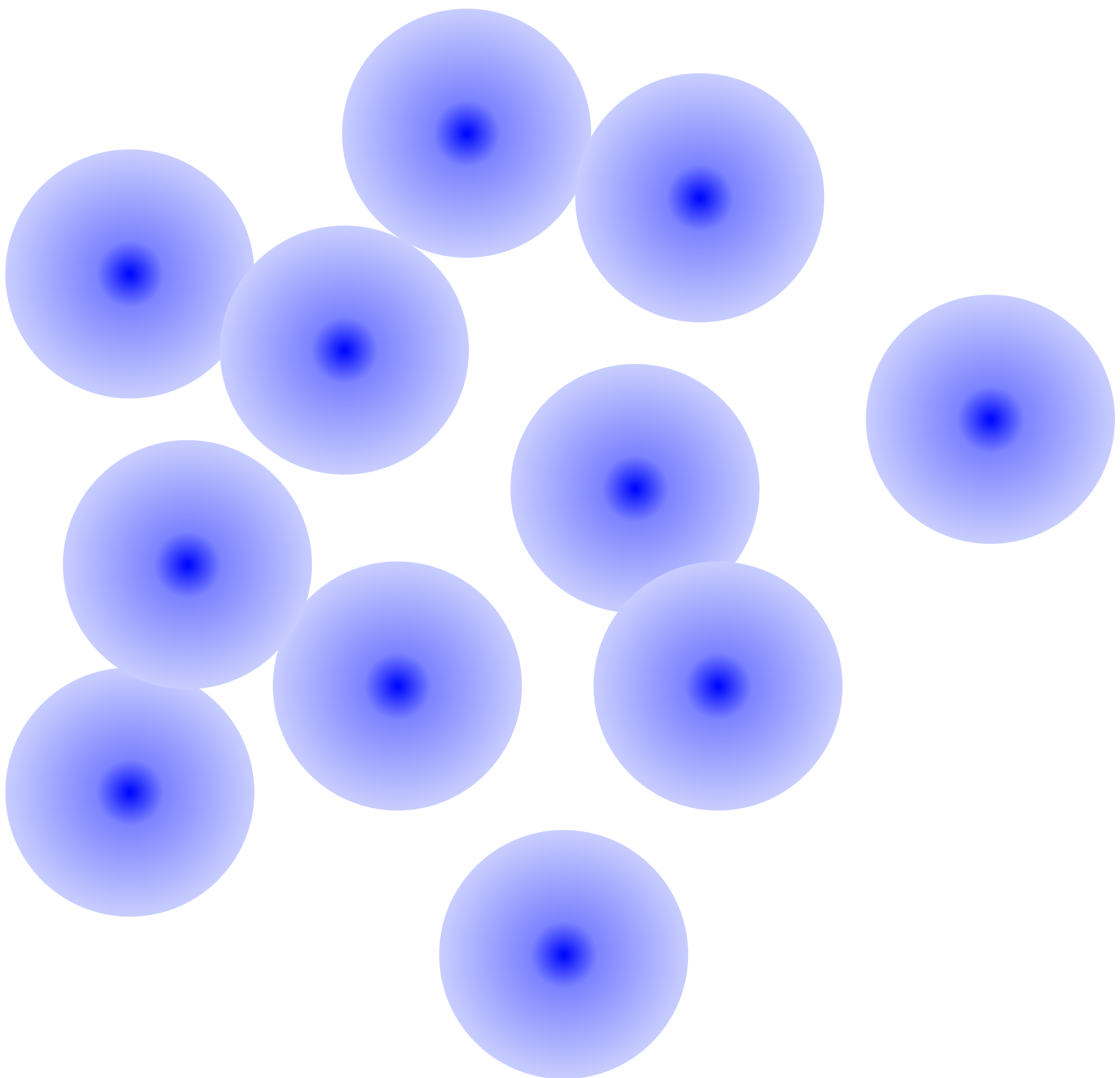


FIG. 1. Temperature dependence of normalized susceptibility χ/χ_{Pauli} of three Si:P,B samples with different normalized electron densities, n/n_c =0.58, 1.1, and 1.8. Solid lines through data are a guide to the eye.

M. J. Hirsch, D.F. Holcomb, R.N. Bhatt, and M.A. Paalanen, PRL **68**, 1418 (1992)



M. Milovanovic, S. Sachdev and R.N. Bhatt, PRL **63**, 82 (1989)
A.C. Potter, M. Barkeshli, J. McGreevy, T. Senthil, PRL **109**, 077205 (2012)